Introduction To Accelerator Physics Homework 1

Due date: Tuesday September 6, 2011

(Need help? Email satogata at jlab.org)

1 Basic Relativity

(10 points) Show that the incremental increase dE in total particle energy E is related to the incremental increase dp in particle momentum p through

$$\frac{dE}{E} = \beta^2 \frac{dp}{p}$$

where the total energy $E = \gamma mc^2$ and particle momentum is $p = \beta\gamma mc$.

2 RHIC energy and current

Gold ions $^{197}Au^{+77}$ (A=197) are injected into the Brookhaven Alternating Gradient Synchrotron (AGS) with a kinetic energy of 1.03 GeV/nucleon. We are using a magnet cycle the same magnet cycle that would extract protons at a kinetic energy of 22.9 GeV for use in the Relativistic Heavy Ion Collider (RHIC). The circumference of the AGS is 807.1 m, and the rest mass of a gold ($^{197}Au^{+77}$) ion is 183.434 GeV/c^2.

(a) (4 points) What is the velocity of the injected gold ions?

(b) (4 points) What is the corresponding kinetic energy for $^{197}Au^{+77}$ ions extracted from the AGS for RHIC using this magnet cycle? Assume the AGS B field at extraction is the same for protons or gold. (Patently not true in the real AGS.)

(c) (2 points) Why does the beam current increase even though the circulating charge stays constant during acceleration?

3 Collider vs Fixed Target

(10 points) Show that the total energy for a head-on collision of two identical particles, each with a center of mass energy of $\gamma_{cm}mc^2$, is equivalent to a fixed target collision where one particle is at rest and the other is moving such that its relativistic γ is

$$\gamma = 2\gamma_{cm}^2 - 1$$

(Flip the page...)

4 Lithium lens (Conte-Mackay 4-1; yes, you can do it)

(10 points) A Lithium lens of length l and radius a has a current I flowing through its end caps with uniform current density as pictured in Fig. 1. Consider a beam of antiprotons with momentum p. What is the focal length of this lens for the antiprotons? Does the Lithium lens current need to flow from left to right or right to left for the lens to focus antiprotons? Recall that the focal length for a focusing lens is defined as the distance at which incoming paraxial rays converge.

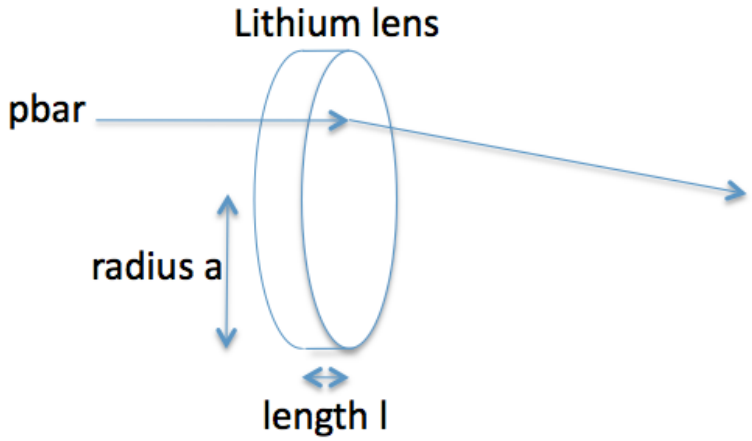

Figure 1: Lithium lens diagram. A uniform current I is applied through the end caps of the Lithium cylinder to create a focusing lens.