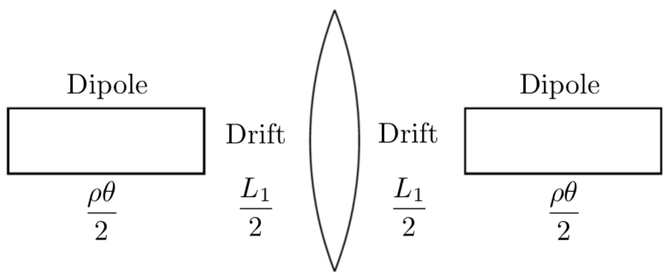
Introduction To Accelerator Physics Homework 4

Due date: Thursday November 3, 2011

(Need help? Email `satogata at jlab.org`)

1 Double Bend Achromat (30 points)



Consider the arrangement of magnets shown in Fig. 1, where two sector bend dipole magnets are equally spaced around a single focusing quadrupole of focal length f. This system is known as a **double bend achromat** (DBA), and this problem goes through the steps to calculate the quadrupole focusing strength f required for this system to be an achromat. Each dipole bends by angle $\theta/2$ and has bending radius ρ, and so has length $L = \rho\theta/2$. Each drift has drift length L_1. This system is very similar to the xkcd cartoon shown on page four of the slides, although here both dipoles bend in the same direction.

(a) (5 points) Since we are calculating aspects of a system with dipoles and dispersion, we will need to use 3×3 matrices as in the class notes, where the third coordinate is fractional momentum deviation $\delta \equiv (p - p_0)/p_0$. For a sector dipole of bend angle $\theta/2$ and bend radius ρ, the 3×3 transport matrix is

$$M_{\text{dipole}} = \begin{pmatrix} \cos(\theta/2) & \rho\sin(\theta/2) & \rho[1-\cos(\theta/2)] \\ -\frac{1}{\rho}\sin(\theta/2) & \cos(\theta/2) & \sin(\theta/2) \\ 0 & 0 & 1 \end{pmatrix} \tag{1.1}$$

Show that for weak dipoles ($\theta \to 0$ with $\rho\theta$ constant), this matrix can be written to first order in θ as

$$M_{\text{dipole}} = \begin{pmatrix} 1 & L & L\theta/4 \\ 0 & 1 & \theta/2 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.2}$$

(b) (8 points) Write out the five 3×3 matrices for this system and calculate their product. This matrix will have the form

$$M_{\text{DBA}} = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix} \tag{1.3}$$

Hints: When the system is left-right symmetric like this, you should end up with $C = S'$. D and D' should be proportional to θ since they should go to zero if the dipoles become drifts ($\theta \to 0$).

(c) (4 points) Using the equations on p. 6 of the slides (or 5.87–8 of the text), show that the periodic solution for η' is $\eta' = 0$.

(d) (5 points) Show that the periodic solution for η is 0 if the quadrupole focal length is

$$f = \frac{L + L_1}{4} \tag{1.4}$$

Under these conditions, the dispersion η and its derivative η' will be zero on both sides of this group of magnets. This is the property for an optical system (or accelerator magnet system) to be **achromatic**.

(e) (8 points) For the case when this system is achromatic, calculate the maximum dispersion $\hat{\eta}$, which will be located at the center of the quadrupole.

2 RHIC RF Calculations (C&M 7–3) (20 points)

(a) (4 points) Calculate the synchrotron tune Q_s for RHIC for fully ionized $^{197}\mathrm{Au}^{79+}$ (gold) ions where

$$\begin{aligned}
\gamma_{\text{r,injection}} &= 10.4 \\
\gamma_{\text{tr}} &= 22.8 \\
C \text{ (circumference)} &= 3833.845 \text{ m} \\
h \text{ (harmonic number)} &= 360 \\
\phi_s \text{ (synchronous phase)} &= 0^\circ \\
mc^2 &= 197 \times 0.93113 \text{ GeV} \\
Z \text{ (atomic number)} &= 79 \\
A \text{ (atomic weight)} &= 197 \\
V_{\text{rf}} &= 300 \text{ kV}
\end{aligned}$$

(b) (2 points) What is the synchrotron frequency?

(c) (4 points) For a synchronous phase of $\phi_s = 5.5^\circ$, how much energy does the synchronous particle gain per turn?

(d) (6 points) How long would it take to accelerate a particle from the injection $\gamma_{\text{r,injection}}$ to $\gamma_r = 107.4$ (or 100 GeV/nucleon)? Ignore the jump in phase that is necessary at transition energy, $\gamma_r = \gamma_{\text{tr}}$.

(e) (4 points) Plot the synchrotron frequency as a function of energy or γ_r.