Introduction to Accelerator Physics Homework 5

Due date: Tuesday December 6, 2011

(Need help? Email satogata at jlab.org)

This homework is oriented towards nonlinear dynamics examples, and is therefore as much a homework as a lab. Open up your favorite (Java-enabled) web browser and go to the URL http://www.toddsatogata.net/2011-USPAS/Java/. Within this directory are some simple Java programs and tutorials that are meant to demonstrate aspects of accelerator nonlinear dynamics. Please record a picture or two from each question and turn them in with your answers to the homework.

1 The Henon Map (10 points, 2 points each)

http://www.toddsatogata.net/2011-USPAS/Java/henon.html

Open the Henon map Java example in the URL above and play with it a bit. Clicking in the black area at the center of the screen will "launch" particles to be tracked through a simple motion that is similar to a linear ring with a single sextupole. The normalized coordinates (x_N, x'_N) are displayed once per iteration of the map in a Poincaré section. This is among the simplest of nonlinear maps (a rotation with a quadratic nonlinear kick), so it has been extensively studied by dynamicists. Set the number of iterations to 1000 and hit return; this will track 1000 "turns" for each particle that you launch into this accelerator.

(a) Set $b_2 = 0$ and track a particles for several different tune values Q. Why does the motion always appear pretty much the same? What happens when you set the tune to a low-order rational number, like 0.25?

(b) Keep $Q = 0.25$ and start raising b_2. Around what values of b_2 do you start seeing something new happening for large-amplitude particles? Are the tunes for these particles increasing or decreasing from 0.25?

(c) Set the tune near $Q = 1/3$ and raise the sextupole strength to about $b_2 = 0.2$. Compare the motion to motino that we discussed in class. What happens when you move the tune to the other side of $Q = 1/3$, and why?

(d) Set the tune to $Q = 0.252$ and gradually raise b_2 from near zero by increments of 0.2 from 0–2. How does the phase space change?

(e) Set the sextupole strength to $b_2 = 0.5$ and find the separatrix. Change the number of iterations to 100 and track near and away from the separatrix. From this can you infer the period of the particle motion around the separatrix?

2 The Henon Map with Damping (10 points)

`http://www.toddsatogata.net/2011-USPAS/Java/henon2.html`
In electron rings, there is synchrotron radiation damping, so particles tend to damp to the closed orbit. This can be included in the nonlinear dynamics simulation, along with sources of noise, to show how particles can damp into resonance islands instead of onto the closed orbit. Set the number of

(a) (4 points) Set the tune Q to 0.254 and D to 0.001. Launch a variety of particles to observe that you have damping turned on. Here the damping is rather strong. About how many turns does it take for the particle amplitude (radius) to decrease by a factor of 2?

(b) (6 points) Gradually raise b_2 from 0 to 1. How does the phase space change now? Note that the resonance islands can become attractors even in the presence of relatively strong damping, so they are really stable fixed points. At what value of b_2 do you start to observe resonance islands on the edges of the screen?

3 Octupoles and Decapoles (20 points)

`http://www.toddsatogata.net/2011-USPAS/Java/odo.html`
We have looked at sextupoles, so what about octupoles and decapoles? This exercise will also introduce Poincaré plots in action-angle coordinates instead of the normalized (x, x') coordinates we have been using. There is also *tune modulation* in this simulation that we should turn off; set the number of iterations to about 200, q to 0, Tm to 1, b1b to 0, and b1q to 0.

(a) (4 points) Set the tune $Q = 0.198$, and b_3 and b_4 (octupole and decapole strengths) to zero. Tracking particles now gives horizontal lines, where the horizontal axis is phase (going from 0 to 2π), and the vertical axis is action J. These horizontal lines are really just circles that have been "flattened out" in a plot of cylindrical coordinates. If you change the number of iterations to 20 (thus plotting only 20 turns for each launched particle), why are there five discrete groupings of dots plotted?

(b) (8 points) Set $b_3 = 0.002$ (the octupole strength) and $b_4 = 0.001$ (the decapole strength), and the number of iterations to 400. How has the phase space changed? Locate the fifth-order resonance islands and estimate the amplitude of the fixed points if the vertical scale ranges from 0–1. Vary b_3 from 0.001 to 0.004, keeping b_4 constant, and plot the fixed point amplitudes vs b_3.

(c) (8 points) Set b_3 back to 0.002 and now vary b_4 from 0.0005 to 0.003 while keeping b_3 constant. Plot the width of the fifth order resonance island widths (separatrix widths) vs b_4.