# More Lattice Optics and Insertions

T. Satogata: June 2011 USPAS Accelerator Physics

## 1 Doglegs

### 1.1 Simple dogleg

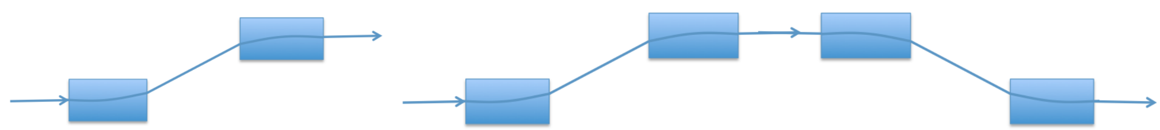

Figure 1: Layouts of dogleg (left) and chicane (right) magnet arrangements, using dipoles to displace the beam in one dimension or around obstacles. Chicane-like motion can be created with four dipole correctors, and is called a four-bump.

A **dogleg** is a pair of equal and opposite strength dipole bends separated by a drift. This is used to "move the beam to the side" without changing the design trajectory direction. Incoming and outgoing design trajectories are parallel since the bends are equal and opposite strength. A dogleg is pictured in the left of Fig. 1.

The 6x6 linear dipole transport matrix of bend angle $\theta$ and bend radius $\rho$ (with length $L_{\rm dipole} = \rho\theta$) is given by Conte/MacKay Eqn. (3.102):

$$\mathbf{M}_{\rm dipole}(\rho,\theta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & -\rho(1-\cos\theta) & 0 & 0 & 1 & -\rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.1}$$

One comment is worth inserting here about coordinate systems. Note that if $\rho, \theta > 0$, positive displacement points **out** from the circle of the dipole curvature. This will become important when we want to understand the sign of the dispersion in this dogleg. It also can become very confusing in systems of multiple synchrotrons that rotate in different directions such as AGS and the RHIC Blue ring, or both rings of a collider. If we want "up" to be $+x$ in Fig. 1, the first dipole bends with angle $-\theta$, and the second dipole is a dipole that bends with angle $+\theta$!

Inserting a drift of length $L$ between two dipoles of equal and opposite bend angle and equal bend radius produces a dogleg. Be careful about the order of matrix multiplication or

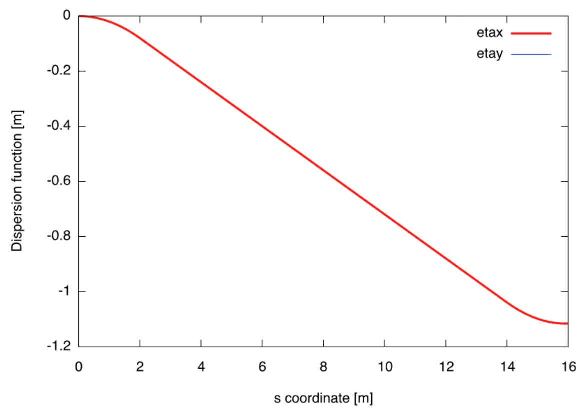


Figure 2: Dispersion generated by a dogleg bend with $L = 14$ m and dipole kicks of $\theta = 0.08$ rad. $L\theta = 1.12$ m, and agrees quite well with the pictured horizontal dispersion. Note that dispersion is only introduced in the bending plane; dipoles are sources (and sinks) of dispersion.

you will get the sign of the dispersion wrong!

$$\begin{aligned}\mathbf{M}_{\text{dogleg}} &= M_{\text{dipole}}(\rho,\theta)M_{\text{drift}}M_{\text{dipole}}(\rho,-\theta) \qquad (1.2)\\ &= \begin{pmatrix} 1+\frac{L}{\rho}\cos\theta\sin\theta & L\cos^2\theta & 0 & 0 & 0 & -L\cos\theta\sin\theta \\ -\frac{L}{\rho^2}\sin^2\theta & 1-\frac{L}{\rho}\cos\theta\sin\theta & 0 & 0 & 0 & \frac{L}{\rho}\sin^2\theta \\ 0 & 0 & 1 & L+2\rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{L}{\rho}\sin^2\theta & -L\sin\theta\cos\theta & 0 & 0 & 1 & L(1+\sin^2\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\end{aligned}$$

This matrix makes sense: it becomes a drift of length $L$ as $\theta \to 0$, the total vertical drift length is $L + 2L_{\text{dipole}}$, and there is a bit of paraxial focusing that one expects from dipoles. For weak dipoles and to first order in $\theta$,

$$\mathbf{M}_{\text{weak dogleg}} = \begin{pmatrix} 1+\frac{L\theta}{\rho} & L & 0 & 0 & 0 & -L\theta \\ 0 & 1-\frac{L\theta}{\rho} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L+2\rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -L\theta & 0 & 0 & 1 & L \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (1.3)$$

If the optics entering the chicane are dispersion-free ($\eta = \eta' = 0$), then the dispersion at the end of the dogleg is $\eta = -L\theta$, with $\eta' = 0$. There is about as much dispersion added by a dogleg as transverse horizontal displacement. Note the sign: the dispersion added by the dogleg is negative!

This makes sense because the transverse kick from the (thin, small angle) dipoles is

$$\begin{aligned}\Delta x'(\delta = 0) &= \frac{BL}{(B\rho)} = \frac{q}{p}BL \qquad (1.4)\\ \Delta x'(\delta) &= \frac{q}{p(1+\delta)}BL \approx \frac{q}{p}BL(1-\delta) = (1-\delta)\Delta x'(\delta = 0) \qquad (1.5)\end{aligned}$$

A small momentum offset of $+\delta$ reduces the dipole kick by a factor of delta, and this is magnified to a transverse offset from design at the end of the dogleg by $-\delta L\theta$. Fig. 2 shows the dispersion generated by a horizontal dogleg, showing that the above dispersion calculation works.

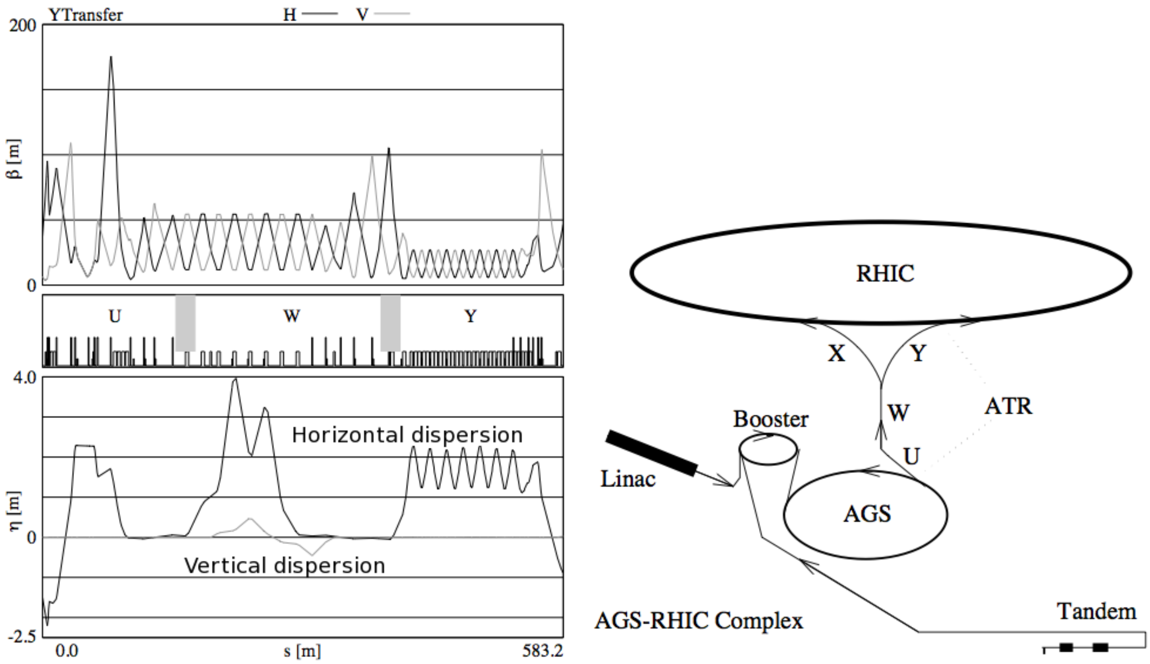


Figure 3: Design optics for the AGS to RHIC Transfer Line (ATR) at BNL [1]. Optics functions are plotted on the left, and the facility layout is on the right.

### 1.2 Zero-dispersion dogleg

There are times when we want a dogleg but don't want any residual dispersion in the bend plane. For example, we may want to dogleg vertically in a transfer line transport between two accelerators (e.g. the AGS to RHIC transfer line, Fig. 3) or into multiple recirculation arcs (e.g. CEBAF). We don't want any residual vertical dispersion because the only way we can correct it is with additional vertical dipoles. So how do we dogleg with two dipoles but no vertical dispersion?

The key is to make a dispersion "two-bump", akin to a steering two-bump. We want the dispersion contributions from the two dipoles to cancel, which means we will need to introduce quadrupoles. $\pi$ insertions, discussed earlier, are particularly useful, where the net transport is simply $M_{\pi\,\mathrm{ins}} = -I$, as they tend to cancel the effects of elements placed on either side of them.

We can then grind out the transport matrices to find

$$\mathbf{M}_{\pi\,\mathrm{dogleg}} = M_{\mathrm{dipole}}(\rho,\theta)(-I)M_{\mathrm{dipole}}(\rho,-\theta) \tag{1.6}$$

$$= \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -2\rho\theta & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2\rho\theta \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{1.7}$$

Zero input dispersion $(\eta, eta')$ gives zero output dispersion. In fact, many other effects of the dipoles cancel out, like their drift lengths in the $M_{12}$ term, and you basically are left with the transport of the $\pi$ insertion.

In general, any transport that has net phase advance of $\pi$ (or a multiple) will be achromatic — that is, zero dispersion at the entrance will map to zero dispersion at the exit. This fact is quite commonly used when designing bending arcs that match into dispersion-free straight sections.

Also note the $M_{56}$ term in this dogleg. $M_{56}$ is important in machines like CEBAF or recirculating linac FELs where the coupling between momentum spread and bunch length is extremely important; often one works to tune the accelerator to a condition where $M_{56} = 0$. (For example, the CEBAF operating envelope for $M_{56}$ from each recirculation arc is about 0.5 m.

### 1.3 CEBAF splitters/recombiners

As mentioned before, CEBAF is a recirculating linac with separated return arcs that transport beam at various energies via vertical separation. Matching into each of these arcs requires vertical separation, so vertical doglegs are appropriate. As seen in [2] (linked on the class website) and as we discussed earlier, though, the $\pi$ insertion creates a problem: large beta functions in the horizontal (non-bending) plane. The traditional dogleg is seen in Fig 5a of the above reference. This ended up creating high sensitivies to magnet errors, large required magnet strengths, and unacceptably large beam sizes in the transport of the original CEBAF design.

To lower this beta function, the final CEBAF design instead uses a "staircase" pair of vertical doglegs, with $\pi/2$ phase advance in each dogleg. This results in less space for the horizontal beta function to grow, and maintains the beta functions in both planes within acceptable ranges. The staircase recombiner optics is shown in Fig 5b of the CEBAF paper. Note that you can see in both cases that the vertical dispersion has the shape of a single period of a sine wave. This is a hallmark of a "two-bump" system, whether in orbit or dispersion control.

## 2 Chicane

A **chicane** is an assembly of two doglegs in opposite directions; chicanes usually return the design trajectory to the same original path. They are sometimes also called bypasses, since they can be used to steer the design trajectory around obstacles such as large magnets. They can also be used for bunch decompression since higher momentum particles are laterally diverted less by the chicane, and thus have a shorter path length than lower momentum particles through $M_{56}$.

## 3 Low-Beta Insertion

Recall that the luminosity in a collider is inversely proportional to the beam sizes:

$$L = \frac{f_{\text{coll}} N_1 N_2}{4\pi\sigma_x\sigma_y} \tag{3.1}$$

where the colliding beam sizes are assumed to be equal with RMS width $(\sigma_x, \sigma_y)$ and $N_{1,2}$ are the number of particles per colliding bunch. To increase the luminosity, we want to reduce the beam sizes. However, simple quads focus the beam in one direction only at the expense of defocusing the beam in the other. This is fine (even preferable) in electron-positron ring colliders, where the beam naturally has "ribbon" shape ($\sigma_y << \sigma_x$) and we can focus horizontally while defocusing vertically to avoid very strong vertical beam-beam forces. In this case, one can use pairs of quad doublets, similar to the $\pi/2$ insertion, as shown in the text and in Fig. 4.

As shown in the text, we can find a solution to these optics that produces specific optics at the interaction point in the center of the insertion. Often the optics are designed such that $\beta^\star$ (the beta function at the interaction point) is minimized, and then $\alpha^\star = 0$.

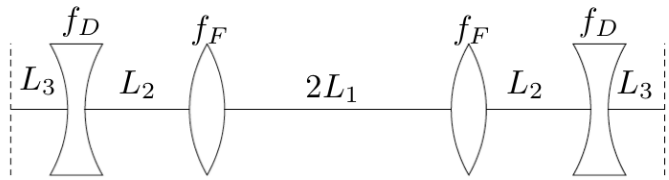


Figure 4: Low beta insertion for a single plane using quadrupole doublets.

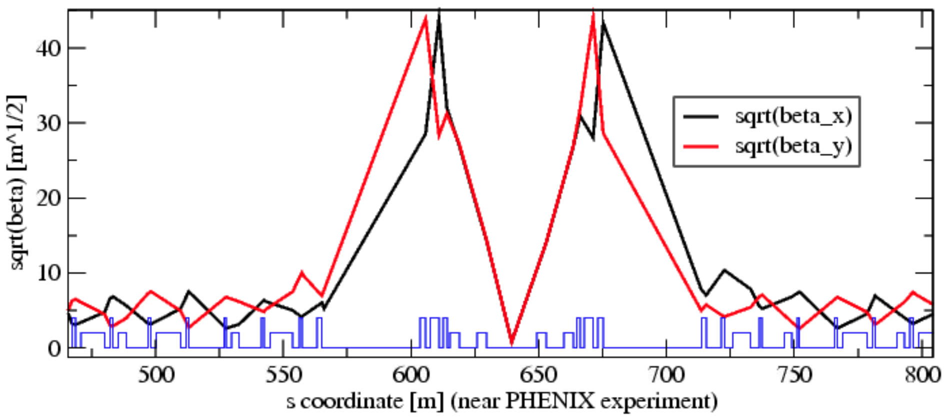


Figure 5: The low beta insertion area for recent RHIC collision optics. $\sqrt{\beta}$ is commonly shown in many optics programs because it is proportional to the expected beam size.

Waldo showed some optics for the RHIC lattice earlier where there are three quadrupoles in a "triplet" formation to either side of the collision point. Three quadrupoles are enough to provide matching optics that constrain $\alpha_x^\star = \alpha_y^\star = 0$ and produce $\beta_x^\star = \beta_y^\star$. Recent low-beta collision optics in RHIC are shown in Fig. 5.

## 4 Mobius Insertion

The Möbius insertion was design by Dick Talman at Cornell to provide a by-definition fully coupled equal-emittance optics for electron-positron collisions at CESR [3]. This design included an insertion that symmetrically exchanged horizontal and vertical motion every turn, producing an accelerator with only one tune degree of freedom instead of two and round beams rather than flat beams typically found in electron and positron machines.

The trick is to match the insertion to points in the lattice where $\alpha_x = \alpha_y$ and $\beta_x = \beta_y$ at each end of the insertion, with betatron phase advances that differ by $\pi$ in both planes. A matrix of this form would then be

$$M_{\mathbf{erect}} = \begin{pmatrix} T & 0 \\ 0 & -T \end{pmatrix} \tag{4.1}$$

where the $T$ are block $2 \times 2$ single-plane (uncoupled) matrices. A rotation of all elements in the insertion by 45 degrees around the longitudinal axis then produces an insertion that still matches, but with the transport

$$M_{\mathbf{mobius}} = \begin{pmatrix} 0 & T \\ T & 0 \end{pmatrix} \tag{4.2}$$

which fully exchanges the horizontal and vertical motion in a symmetric way through the insertion.

The "tunes" for this accelerator $Q_{1,2}$ are related to the unrotated lattice tunes $Q_{x,y}$ by

$$Q_{1,2} = \frac{Q_x + Q_y}{2} \mp \frac{1}{4} \tag{4.3}$$

A Möbius insertion was built and commissioned for CESR, but strong betatron and synchrobetatron resonances have limited its usefulness.

## 5 Bates Bend

A "Bates Bend" is a 4-dipole chicane split in two by a 180-degree dipole, as shown in Fig. 6.

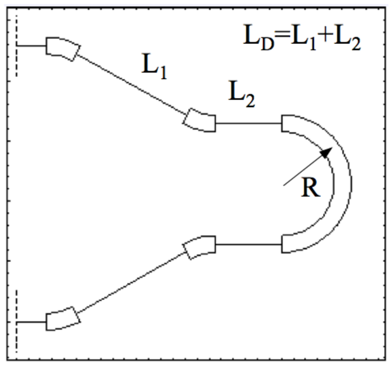


Figure 6: A Bates Bend.

Bates bends include sextupoles and are used for nonlinear bunch compression, such as at Bates lab (where they originated) and in the recirculation ends of the Jefferson Lab recirculating linac FEL. They are particularly useful because they accomplish complicated gymnastics in longitudinal phase space without requiring complicated multi-harmonic RF systems.

I'll show slides from Pavel Evteshenko from the DIPAC 2011 conference at DESY. I'll see if I can write up a summary of Bates Bends for the class website.

## References

[1] W.W. MacKay et al., "AGS to RHIC Transfer Line, Design and Commissioning", Proc. of the 1996 European Particle Accelerator Conference, Sitges, Spain.

[2] D. Douglas, R.C. York, and J. Kewisch, "Optical Design of the CEBAF Beam Transport System", Proc. of the 1989 Particle Accelerator Conference, Chicago, IL.

[3] S. Henderson, R. Talman, et al., "Investigation of the Möbius Accelerator at CESR", Proc. of the 1999 Particle Accelerator Conference, New York, NY; R. Talman, "A Proposed Möbius Accelerator", Phys. Rev. Lett **74**, 1590-3 (1995).