USPAS Graduate Accelerator Physics Final Exam

Due date: Friday June 24, 2011

1 Twiss Parameters from a One-Turn Matrix

Given the following 1-turn matrix that transforms motion from $s = 0$ to $s = C$ around the circumference of an accelerator,

$$M = \begin{pmatrix} -1.05746 & -3.59421 & 0.00000 & 0.00000 & 0.00000 & 35.44680 \\ 0.00189 & -0.93923 & 0.00000 & 0.00000 & 0.00000 & -0.82369 \\ 0.00000 & 0.00000 & 1.72622 & -72.45113 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.05149 & -1.58161 & 0.00000 & 0.00000 \\ -0.80399 & -36.25338 & 0.00000 & 0.00000 & 1.00000 & -50.03916 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix}$$

(a) Find the fractional part of the horizontal betatron tune, Q_x.

(b) Find the Twiss functions (β, α, γ) for the horizontal plane at this point, $s = 0$.

(c) Find the horizontal dispersion functions η and η'_x at this point, $s = 0$.

2 Maximum Phase Advance

Calculate the maximum possible phase advance for a drift.

3 Design of a FODO from Existing Quads and Beam

You inherit a set of identical quadrupoles from another accelerator lab that are each 0.5 m long. They have a 7 cm bore radius, and a maximum pole tip field of 1.1 T.

(a) What is the minimum quadrupole focal length for one of these magnets for a $p = 15$ GeV proton beam?

(b) You need to build a FODO transport line with these magnets to transport this beam with a phase advance per cell of 60 degrees in each plane. How far apart do you need to place consecutive focusing/defocusing quadrupoles? (Assume you power the quadrupoles to maximum field.)

(c) What are the maximum and minimum beta functions of this FODO lattice?

(d) You want to transport $\pm 4\sigma$ of the beam through this line without scraping. What is the largest RMS emittance of the beam that you can transport through this FODO lattice in this way?

(e) Bonus: What current do each of these quadrupoles need at maximum field if each pole is wrapped by 40 turns of conductor?

4 Low-Beta Triplet

(a) A matrix M transports beam from one location to another, where M is written in a very general form:

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{4.1}$$

Demonstrate that the focal length of this transport is a/c. (Hint: you do not need to know anything about parameterization of M into beta functions to derive this result.)

(b) Consider the quadrupole triplet pictured in Fig. 1, where the quads can be treated as thin, the focusing quadrupoles have equal focal length $f = 10$ m, and the lengths between the quadrupoles are equal with $L = 20$ m. Find the defocusing quadrupole focal length d such that this triplet has equal focal length in both horizontal and vertical planes.

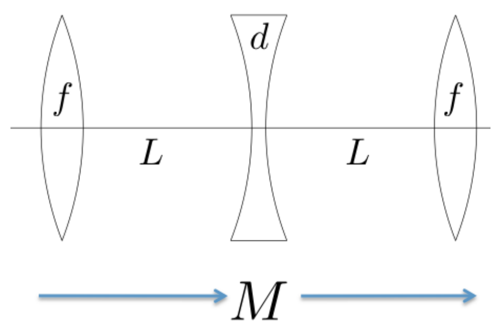

Figure 1: A triplet magnet lattice, with equal thin focusing quadrupole focal lengths f, thin defocusing quadrupole strength d, and equal drift lengths L.

5 Synchrotron Motion Spectrum

The following longitudinal Schottky spectrum was acquired for a RHIC beam of $^{197}\text{Au}^{79+}$ ions (i.e. fully stripped ions), when only a single storage cavity was powered. Calculate the RF voltage in the RF cavity. Assume the following RHIC parameters: γ_{tr}=22.8, circumference 3833.845 m, RF cavity harmonic number $h = 7 \times 360 = 2520$, and $U_s = 100$ GeV/nucleon at constant energy.

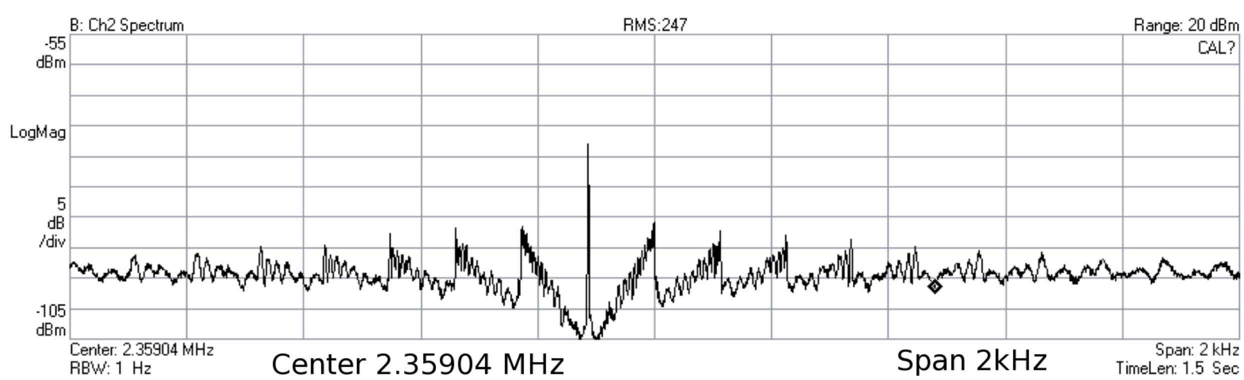

Figure 2: Longitudinal Schottky spectrum for Problem 5. The spectrum shown here extends over a range of 2 kHz, and shows a central revolution harmonic peak surrounded by several synchrotron frequency sidebands.