USPAS Accelerator Physics 2013
Duke University

Chapter 11: Space Charge Effects
Space Charge, Beam-Beam
(Electron Cloud in separate talk from Ecloud workshop)

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Debye Length

- Comes from thermodynamic description of large systems of moving charges
  \[ \lambda_D \equiv \left( \frac{\epsilon_0 k_B T}{Ne^2} \right)^{1/2} \]

- **Collisional regime**: dynamics dominated by binary collisions in close encounters (e.g. Touschek)
  - Single particle scattering and single particle effects
    \[ \sigma_{x,y,z} \ll \lambda_D \]

- **Self-field or space charge regime**: dynamics dominated by self-field over large distances compared to average particle separation
  - Collective effects, single-component cold plasmas
    \[ \sigma_{x,y,z} \gg \lambda_D \]
In space-charge dominated synchrotrons and low-energy beams (without bunch compression), we can approximate the beam as being cylindrical and effectively infinitely long.

\[ F(r) = q(E_r - \beta c B_\theta) \]

- Electric repulsion will be stronger than magnetic attraction
- At hyperrelativistic velocities these nearly balance (~no effect)
- Also seen by imagining electrostatic forces only in bunch frame
- Can compensate by shielding charges/currents (e.g. plasma)
Maxwell’s equations in cylindrical coordinates here are

\[ \nabla \cdot \vec{E} = \frac{1}{r} \frac{d}{dr} \left( r E_r \right) = \frac{q}{\epsilon_0} n(r) \quad \left( \nabla \times \vec{B} \right)_z = \frac{1}{r} \frac{d}{dr} (r B_\theta) = \frac{q}{c \epsilon_0} \beta n(r) \]

- We can integrate these directly

\[ \begin{align*}
E_r &= \frac{q}{\epsilon_0} \frac{1}{r} \int_0^r r n(r) \, dr \\
B_\theta &= \frac{\beta}{c} \frac{q}{\epsilon_0} \frac{1}{r} \int_0^r r n(r) \, dr = \frac{\beta}{c} E_r
\end{align*} \]

- These are simple if the beam has constant density
- Also simple if we treat these beams as having a radial Gaussian distribution

\[ n(r) = A \exp \left( \frac{-r^2}{2\sigma^2} \right) \quad A = \frac{N}{2\pi l \sigma^2} \]

\[ \text{Bunch length } l \quad \text{Bunch population } N \]

Relativistic \( \beta \)
Space Charge Force

- Calculating the force from these fields

\[ F(r) = q(E_r - \beta c B_\theta) \]

gives

\[ F(r) = \frac{N q^2}{2\pi \epsilon_0 l} \left( 1 - \frac{\beta^2}{r} \right) \left[ 1 - \exp \left( -\frac{r^2}{2\sigma^2} \right) \right] \]

- This is a defocusing force that is equal in both directions -- how does it affect our focusing and tune?

\[ \frac{d^2 y}{ds^2} = \frac{1}{\beta^2 c^2} \frac{d^2 y}{dt^2} = \frac{1}{\beta^2 c^2 m\gamma} \frac{F_y}{\gamma} \quad \frac{d^2 y}{d\theta^2} = \frac{R^2 F(y)}{\beta^2 \gamma m c^2} \]

\[ R : \text{average radius}, \ s = R\theta \]

\[ \frac{d^2 y}{d\theta^2} + Q^2 V y = \frac{R^2}{\beta^2 \gamma m c^2} F(y) \]

\[ r_0 \equiv q^2/(4\pi \epsilon_0 m c^2) \]
Equation of Motion

- We get something rather unpleasant for the expansion

\[
\frac{d^2 y}{d\theta^2} + Q_V^2 y = \frac{2 N r_0 R^2}{l \beta^2 \gamma^3} \left[ 1 - \exp \left( -\frac{y^2}{2 \sigma_V^2} \right) \right]
\]

- But we can expand the unpleasantness and ignore the higher order terms for now (which contribute nonlinearities)

\[
\left[ 1 - \exp \left( -\frac{y^2}{2 \sigma_V^2} \right) \right] \approx \frac{y}{2 \sigma_V^2} - \sum_{n=2}^{\infty} \frac{(-1)^n y^{2n-1}}{2^n n! \sigma_V^{2n}}
\]

Maclaurin series
Taylor series about 0

\[
\frac{d^2 y}{d\theta^2} + \left( Q_V^2 - \frac{N r_0 R^2}{l \sigma_V^2 \beta^2 \gamma^3} \right) y = 0
\]
That Messy Term

\[
\left[ 1 - \exp \left( -\frac{y^2}{2\sigma_V^2} \right) \right] \frac{y}{\sigma_V}
\]

Applicable for space charge and beam-beam

Linear for core of beam

VERY nonlinear for tails of beam
Linear Space Charge Tune Shift: Calculated

\[
\frac{d^2 y}{d\theta^2} + (Q_V + \delta Q_{sc})^2 y \approx 0
\]

\[
\delta Q_{sc} = -\frac{N r_0 R^2}{2lQ_V \sigma_V^2 \beta^2 \gamma^3} = -\frac{N r_0}{4\pi B_f \epsilon^*_{V,\text{rms}} \beta \gamma^2}
\]

- To get the last we’ve used a few other conversions

\[
\beta_V \approx \frac{R}{Q_V}
\]

\[
\pi \epsilon^*_{V,\text{rms}} = \pi \frac{\sigma_V^2}{\beta_V \beta \gamma}
\]

Bunching factor: 

\[
B_f \equiv \frac{l}{2\pi R} = \frac{I_{\text{ave}}}{I_{\text{peak}}}
\]

- \(\delta Q_{sc}\) is called the (vertical) Laslett tune shift
  - For hadron beams, this is a big effect at low energy, high N
  - Dominates high intensity boosters (FNAL, BNL, CERN)
  - Electrons escape most effects except for very low \(\gamma\)
Flat/Round Space Charge Tune Shift

- This tune shift is symmetric between H,V for round beams
  - e.g. most hadron beams
- For elliptical or ribbon beams one can show that the proper calculation gives

\[
\delta Q_{sc, V} = -\frac{\beta V N r_0}{2\pi B_f \sigma_V (\sigma_H + \sigma_V) \beta^2 \gamma^3}
\]

- The horizontal is just given by reversing H and V
- This is true for most synchrotron electron beams

- This tune shift is different for different parts of the beam
  - Commonly called an incoherent tune shift
  - Compare to coherent tune shift given by, e.g., quadrupoles
Head and Tail Space Charge

- Differential defocusing in head and tail of beam from space charge is a source of some (reversible) emittance growth.

\[ \varepsilon_{rms} = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}^2} = \sqrt{\langle x'^2 \rangle \langle x''^2 \rangle - \langle xx' \rangle^2} \approx \sin(\sqrt{2ks}z) \]

Ferrario, CAS Greece 2011
Messy Nonlinearities

\[
\left[ 1 - \exp \left( -\frac{y^2}{2\sigma_y^2} \right) \right] = \frac{y}{2\sigma_y^2} - \sum_{n=2}^{\infty} \frac{(-1)^n}{2^n n!} \frac{y^{2n-1}}{\sigma_y^{2n}}
\]

- Note that space charge also drives many nonlinearities
  - First order is quadrupole (tune shift) so higher orders are octupole (nonlinear tune shift), dodecapole, …
  - Beam size is natural scaling parameter
- How high can we run $\delta Q_{sc}$?
  - It spreads beam across resonances, so not more than 1!
  - Large amplitude particles shift away from resonance though
  - Incoherent tune spread across emittance of beam
  - Some facilities (FNAL Booster, AGS) run up to 0.7!
- Note that space charge is distributed through the accelerator
  - This makes it quite computationally expensive to model
Space Charge Compensation

- How to compensate for space charge?
- Create neutral plasma by co-propagating opposite charge beam
  - Studied at e.g. CERN PS Booster (Aiba), FNAL Booster
  - Matching beams compensate tune shift and resonances
  - Ion recombination not expected to be substantial
  - Electron columns with strong solenoidal fields (Shiltsev, FNAL, ‘07)
- Inject small mix of electronegative gases
  - Very low energy high flux ion beam (Dudnikov, FNAL/BINP)
- Low energy electrons: Use focusing solenoidal field in gun
  - Developed by Carlsten (LANL, ’94) for 1.3 GHz photoinjector
  - Now routinely incorporated into high-brightness photoinjector design
  - Also incorporate superconducting RF half-cell: faster acceleration
  - Ion back-bombardment problem on RF photocathodes (Pozdeyev)
11.4: Longitudinal Space Charge Defocusing

- Longitudinal space charge is most easily calculated in the center of mass frame of the beam
  - Then boost fields to lab frame with Lorentz transformation
  - Voltage for uniform beams, in the beam center of mass frame
    \[ V_{cm} = \frac{qg}{4\pi\epsilon_0} \frac{N}{\gamma l_{beam}} \quad g \equiv \left( 1 + 2 \ln \frac{b}{a} \right) \]
  - For a more realistic longitudinal distribution (parabolic)
    \[ V_{cm} = \frac{qg}{4\pi\epsilon_0} \frac{3N}{2\gamma l_{beam}} \left[ 1 - \left( \frac{2z_{cm}}{\gamma l_{beam}} \right)^2 \right] \]
  - Boosting back and finding the longitudinal force
    \[ F_{||} = \frac{3}{\pi} \frac{g}{\epsilon_0 l_{beam}^2} \frac{Nq^2}{\gamma^2} z \]
- Modifies synchrotron frequency, important near transition
- Creates bunch lengthening in low energy electron beams
In this Letter we study the evolution of an initial periodic modulation in the temporal profile of a relativistic electron beam under the effect of longitudinal space-charge forces. Linear theory predicts a periodic exchange of the modulation between the density and the energy profiles at the beam plasma frequency. For large enough initial modulations, wave breaking occurs after $1/2$ period of plasma oscillation leading to the formation of short current spikes. We confirm this effect by direct measurements on a ps-modulated electron beam from an rf photoinjector. These results are useful for the generation of intense electron pulse trains for advanced accelerator applications.
11.3: Beam-Beam Force

- The beam-beam force is very similar to space charge
  - Except now use properties of colliding beam
  - Oppositely charged beams focus, same sign defocus
  - Velocities are opposite so sign of force from B term reverses

\[ F(r) = q(E_r + \beta c B \theta) \]

\[ F(r) = \frac{Nq^2}{2\pi\varepsilon_0 l} \frac{1 + \beta^2}{r} \left[ 1 - \exp\left( -\frac{r^2}{2\sigma^2} \right) \right] \]

- We also only integrate the beam-beam kick over the bunch length as a short kick, since the interaction is short
  - We can then use the same short kick approximations as before for multipoles
Beam-Beam Force (cont)

\[ F(r) = \frac{Nq^2}{2\pi\varepsilon_0 l} \frac{1 + \beta^2}{r} \left[ 1 - \exp \left( -\frac{r^2}{2\sigma^2} \right) \right] \]

- Assume particles are ultrarelativistic (\(\beta \sim 1\)) and take the leading term to find linear focusing and thus a tune shift

\[ F(y) \approx \frac{Nq^2}{2\pi\varepsilon_0 l\sigma^2_V} y \]

- The counterrotating bunch acts like a short lens of length \(l/2\)

\[ \delta p_y = F(y)\delta t = F(y)(l/2\beta c) = \frac{Nq^2}{4\pi\varepsilon_0 \beta c\sigma^2_V} y \left( \frac{p}{\beta \gamma mc} \right) = \frac{Nr_0 p}{\gamma \sigma^2_V} y \]

\[ \Delta y' = \frac{\delta p_y}{p} = \frac{Nr_0}{\gamma \sigma^2_V} y \]

Thin quadrupole!
Beam-Beam Force (cont)

\[ \Delta y' = \frac{\delta p_y}{p} = \frac{N r_0}{\gamma \sigma_V^2} y \]

- We can use the thin quadrupole formula we derived on Friday to derive the corresponding linear tune shift
  - Reverse sign of tune change for oppositely charged beams!

\[ \delta Q = -\frac{\beta_y \Delta k}{4\pi} \]

\[ \Delta Q_{bb} = -\frac{N_{IP} N r_0 \beta^*_V}{4\pi \gamma \sigma_V^2} = -\frac{N_{IP} N r_0}{4\pi \gamma \epsilon_{V,rms}} \]

- \( N_{IP} \) is the number of interaction points
- Tune shift independent of beta function at collision point
- \( N \) and emittance are properties of opposing beam
- For elliptical beams we can similarly generalize to

\[ \Delta Q_{V, bb, elliptical} = -\frac{N_{IP} N r_0 \beta^*_V}{2\pi \gamma \sigma_V (\sigma_H + \sigma_V)} \]
Beam-Beam Simulation

- Most beam-beam simulations are done using a **weak-strong** model
  - Assume the beam creating the kick is static or quasistatic
  - Beam-beam kick is then just a thin (very) nonlinear lens
  - Scales as particle partitioning of weak beam
  - Quick: you did this in the Java lab!

- But we can have two-beam collective effects
  - The two beams are two coupled oscillators
  - Both coherent and incoherent effects (strong nonlinearities)
  - This requires a **strong-strong** model where both distributions must evolve together consistently over time
  - Can be very computationally expensive
    - Similar to large detailed space charge calculations
Beam-Beam Coherent Effects

- Colliding beams are not always aligned on center
  - Offsets of beam centers can easily produce coherent coupled oscillations between the beams
  - Can give rise to $\pi$ and $\sigma$ oscillator modes

W. Fischer et al, “Observation of Coherent Beam-Beam Modes in RHIC”, 2002
Long Range Beam-Beam

- Recall that our force is very nonlinear at large amplitudes
  - There are good reasons to separate colliding beams to moderate or large amplitudes
  - e.g. short-spaced bunch trains in long interaction regions with crossing angle (LHC with minimum bunch spacing)

- This introduces beam-beam crossings at angles and nonlinearities from parasitic beam-beam kicks
- Drives designs to large crossing angles for large separation
- Luminosity optimization drives need for crab cavities
Beam Beam vs Space Charge

- Beam-beam and space charge tune shifts look similar!
  - Their nonlinear terms also look similar
    \[
    \delta Q_{sc} = -\frac{N r_0 R^2}{2 l Q_V \sigma_V^2 \beta^2 \gamma^3} = -\frac{N r_0}{4\pi B_f \epsilon^*_V,\text{rms} \beta \gamma^2}
    \]
    \[
    \Delta Q_{V,\text{bb}} = -\frac{N_{IP} N r_0 \beta_V^*}{4\pi \gamma \sigma_V^2} = \frac{N_{IP} N r_0}{4\pi \epsilon^*_V,\text{rms} \beta \gamma^2}
    \]
  - Naively we would expect their behavior to be similar too
    - Space charge is distributed through the accelerator
      - Nearly phase-averages out many nonlinear behaviors
    - But beam-beam is localized in phase
      - Emphasizes nonlinear behaviors, little phase averaging
    - Beam-beam tune shift limits are more like $5-7 \times 10^{-3}/\text{IR}$
      - e.g. RHIC polarized proton collisions are beam-beam limited
Beam-Beam Compensation

- Similar to space charge, it’s possible to compensate beam-beam behaviors with similar nonlinear magnetic fields
  - Local compensation preferred to compensate nonlinearities near source
  - Electron lens (Tevatron, Shiltsev et al)
  - Wires (RHIC, Fischer et al)

- Hollow electron beam collimation
  - Not exactly beam-beam but doesn’t fit many other places
  - Requires creation of hollow electron beam at source/gun
  - Used to limit halo of high energy hadron beams
  - Surround core of beam with circular beam of electrons
  - Creates nonlinear forces that eliminate halo, do not disturb core
    - See, e.g., http://arxiv.org/abs/1202.1512 (Stancari et al)