USPAS Graduate Accelerator Physics Homework 4

Due date: Friday January 18, 2013

1. Natural Chromaticity of FODO lattice.
Consider a ring made of N identical FODO cells with equally spaced quadrupoles. Assume that the two quadrupoles are both of length l_q, but their strengths may differ. Calculate the natural chromaticities for this machine, and show that for short quadrupoles,

$$\xi_N \simeq -\frac{2\tan\frac{\mu}{2}}{\mu},$$

where μ is the betatron phase advance per cell.

2. Coupled matrix with equal tunes.
Show that the coupled matrix

$$\mathbf{M} = \begin{pmatrix} \cos\mu & \sin\mu + \frac{a^2}{\sin\mu} & a & 0 \\ -\sin\mu & \cos\mu & 0 & -a \\ a & 0 & \cos\mu & \sin\mu + \frac{a^2}{\sin\mu} \\ 0 & -a & -\sin\mu & \cos\mu \end{pmatrix}$$

is symplectic and has equal tunes for the two oscillation modes.

3. Double bend achromat.
An achromatic bend (the double bend achromat) may be made from two dipoles with a horizontally focusing quadrupole between them. The transfer matrix through the achromat is of the form:

$$\mathbf{M} = \mathbf{B}(\theta)\mathbf{L}\left(\tfrac{1}{2}\mathbf{Q}\right)\left(\tfrac{1}{2}\mathbf{Q}\right)\mathbf{LB}(\theta).$$

a) Use thin the lens approximation for quads and small angle approximation for bends to find the dispersion in the middle of the quad. Write the focal length in terms of the drift length and bend parameters.

$$\text{Hint:} \quad \begin{pmatrix} \eta_c \\ 0 \\ 1 \end{pmatrix} = \left(\tfrac{1}{2}\mathbf{Q}\right)\mathbf{LB}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

b) Show that the dispersion is again zero ($\eta = \eta' = 0$) after the bend.