USPAS Graduate Accelerator Physics Homework 5

Due date: Tuesday January 22, 2013

1 Dispersion Function with RF

(10 points) Consider a ring with a thin rf cavity whose linear transfer matrix just after the cavity is given by

$$\mathbf{M} = \begin{pmatrix} C & S & 0 & D \\ C' & S' & 0 & D' \\ E & F & 1 & G \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & Q & 1 \end{pmatrix}$$

Show that the dispersion functions are still given by

$$\eta = \frac{(1-S')D + SD'}{2(1-\cos\mu)}, \quad \text{and} \quad \eta' = \frac{(1-C)D' + C'D}{2(1-\cos\mu)}.$$

Hint: The eigenvector equation of Eq. 5.86 must be modified to allow for momentum compaction:

$$\mathbf{M} \begin{pmatrix} \eta \\ \eta' \\ 0 \\ 1 \end{pmatrix} \delta = \begin{pmatrix} \eta \\ \eta' \\ 0 \\ 1 \end{pmatrix} \delta + \begin{pmatrix} 0 \\ 0 \\ \Delta L \\ 0 \end{pmatrix}.$$

2 RHIC Longitudinal Parameters with Au

(a) (3 points) Calculate the synchrotron tune for RHIC for fully stripped $^{197}\mathrm{Au}^{79+}$ (gold ions)

$$\begin{aligned} \gamma_{\mathrm{inj}} &= 10.4 \\ \gamma_{\mathrm{tr}} &= 22.8 \\ L &= 3834 \text{ m} \\ h &= 360 \\ \phi_s &= 0^\circ \\ mc^2 &= 197 \times 0.93113 \text{ GeV} \\ Z &= 79 \quad \text{(protons)} \\ A &= 197 \quad \text{(neutrons + protons)} \\ V_{\mathrm{rf}} &= 300 \text{ kV}. \end{aligned}$$

(b) (3 points) What is the synchrotron frequency?

(c) (3 points) For a synchronous phase of $\phi_s = 5.5^\circ$, how much energy does the synchronous particle gain per turn?
(Flip the page...)

(d) (3 points) How long would it take to accelerate to $\gamma = 107.4$ (100 GeV/nucleon)? Assume that the phase jump at transition has been performed correctly (i.e., ignore it).

(e) (3 points) Plot the synchrotron frequency as a function of energy.

3 Power Loss in Cavity Walls

(10 points) Show Equation (9.16) in the text, that the RF power loss in the conducting walls of an RF cavity is

$$\langle P_{\text{loss}} \rangle = \frac{R_s}{2} \int_{\text{S}} |H_{\parallel}|^2 \, dS$$

by averaging the power loss inside the wall over one cycle and using Ampere's Law to relate field components of the electric and magnetic fields in the surface of the conductor.