

Your name: _____

Please show your work, write neatly, write units, and box your answers.

1. A charge $q_1=1.0$ C is separated from another charge $q_2=-1.0$ C by a distance $d=30$ cm. The force between them is attractive and has magnitude $F = kq_1q_2/d^2$ where $k = 9.0 \times 10^9$ N m²/C².

- (a) (2 points) If we move apart the charges so the force between them is $1/9^{\text{th}}$ of the original force, what is the new distance between them?

Solution: Call the new distance between the charges d_2 . The new force between the charges at distance d_2 is $1/9$ of the original force F , so we have

$$\frac{F}{9} = \frac{kq_1q_2}{d_2^2} \Rightarrow F = 9\frac{kq_1q_2}{d_2^2}$$

The original force $F = kq_1q_2/d^2$, so

$$F = \frac{kq_1q_2}{d^2} = 9\frac{kq_1q_2}{d_2^2} \Rightarrow d_2^2 = 9d^2 \Rightarrow d_2 = 3d = \boxed{90 \text{ cm} = d_2}$$

- (b) (3 points) What is the *magnitude* of the electric field E at a point halfway between the two charges? Remember to write down the proper units.

Solution: For electric field, $E = kq/r^2$. Here $r = d/2 = 15$ cm=0.15 m. The electric field from the positive charge is therefore

$$E(q_1) = \frac{kq_1}{r^2} = \frac{(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.0 \text{ C})}{(0.15 \text{ m})^2} = 4.0 \times 10^{11} \text{ N/C}$$

Recall that electric field points away from positive charges and towards negative charges, so the electric fields from each of these charges both point towards the negative charge and therefore add up. They are also the same magnitude, so the total electric field is just twice the above: $E(\text{tot})=2E(q_1)=8.0 \times 10^{11} \text{ N/C}$.

Some students might have interpreted this question to use the distance from part (a) in this calculation. That distance is three times the original distance, so the electric field is reduced by a factor of 9 from the previous answer; that is, the field scales the same way as the force. Using $r = 45$ cm = 0.45 m and following the above calculation, one gets $E(\text{tot})=8.9 \times 10^{10} \text{ N/C}$.

2. An proton of charge $q_p = 1.6 \times 10^{-19}$ C is located at the origin. Another proton is located at the point $x = -4$ nm, $y = -3$ nm (1 nm is 10^{-9} m).

- (a) (2 points) What is the magnitude of the electric force on the proton at the origin?

Solution: If we call the distance between the protons r , then

$$r^2 = (3 \text{ nm})^2 + (4 \text{ nm})^2 \Rightarrow r = \sqrt{(3 \text{ nm})^2 + (4 \text{ nm})^2} = \sqrt{25 \text{ nm}^2} = 5 \text{ nm} = 5 \times 10^{-9} \text{ m}$$

We can then calculate the magnitude of the Coulomb force:

$$F = \frac{kq_p^2}{r^2} = \frac{(9.0 \times 10^9 \text{ Nm}^2/\text{C}^2)(1.6 \times 10^{-19} \text{ C})^2}{(5 \times 10^{-9} \text{ m})^2} = \boxed{9 \times 10^{-12} \text{ N} = F}$$

- (b) (3 points) What are the F_x and F_y components (in the \hat{i} and \hat{j} directions respectively) of the electric force on the proton at the origin?

Solution: The Coulomb force here is repulsive since both charges are the same sign, so the force will point up and to the right; it will have positive x and y components. Using similar triangles or observing that the triangle here is 3:4:5, we can find

$$F_x = (4/5)F = \boxed{7.4 \times 10^{-12} \text{ N} = F_x}$$
$$F_y = (3/5)F = \boxed{5.5 \times 10^{-12} \text{ N} = F_y}$$