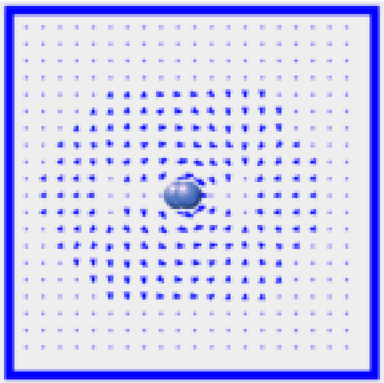
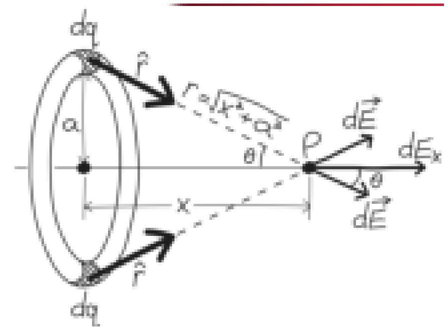


University Physics 227N/232N
Old Dominion University



More Electrostatics
20:3-4 Dipoles, Charge Distributions, …

Dr. Todd Satogata (ODU/Jefferson Lab)

satogata@jlab.org

http://www.toddsatogata.net/2014-ODU

Friday, January 24 2014

Happy Birthday to Michio Kaku, Kristen Schaal, John Belushi,
Neil Diamond, and Dan Shechtman (Nobel: quasicrystals)!

Announcements

- Evening Problem Solving Sessions
  - An opportunity to develop your problem-solving skills!
  - Run by Eric Stacy, estac003@odu.edu, Physics Learning Center, Mondays and Tuesdays 7-9 PM.
  - Similar sessions held 12:30-1:30 on Tuesdays in the SCALE-UP classroom (by your valiant TA, Fred Miller).

- Public Talk: The Physics of Football
  - North Cafeteria, Webb Center, Tuesday Jan 28 7 PM
  - Tim Gay, University of Nebraska, Lincoln
    - Has consulted with NFL Films, ESPN, New York Times, and others
  - RSVP to 757-683-3116 or http://www.odu.edu/univevents (TGL14) if you plan to attend.

Happy Birthday to Michio Kaku, Kristen Schaal, John Belushi, Neil Diamond, and Dan Schechtman!

Summarizing Last Week: 20.1-4

(so long ago!)

- Electromagnetic forces dominate your experience
- Protons/electrons are equally and oppositely charged
  - Rule: Same sign charges attract; like sign charges repel

- Charge units: $\text{elementary charge } e = 1.6 \times 10^{-19}\ \text{C}$
- Coulomb's Force Law:

$$\vec{F}_{12} = \frac{k q_1 q_2}{r^2}\hat{r} \qquad k = 9.0 \times 10^9\ \text{N m}^2/\text{C}^2$$

- Electric Field:

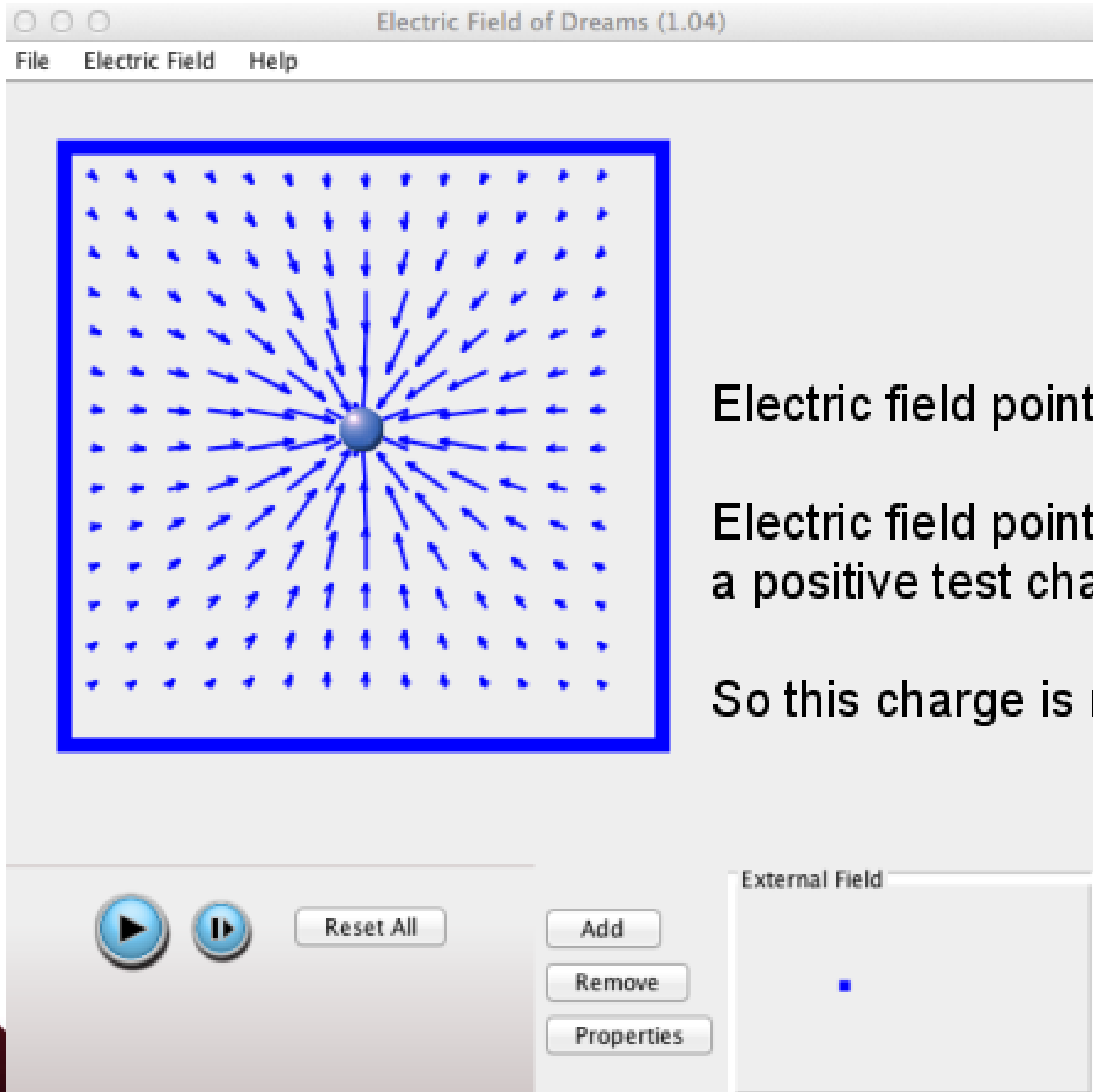
$$\vec{E} = \frac{kq}{r^2}\hat{r} \qquad k = 9.0 \times 10^9\ \text{N m}^2/\text{C}^2$$

- Superposition: Effects of individual charges add together

$$\vec{E}_{\text{total}} = \sum_k \vec{E}_n = \sum_n \frac{k q_n}{r_n^2}\hat{r}_n$$

Review: Electric Field (of Dreams): One Charge

http://phet.colorado.edu/en/simulation/efield



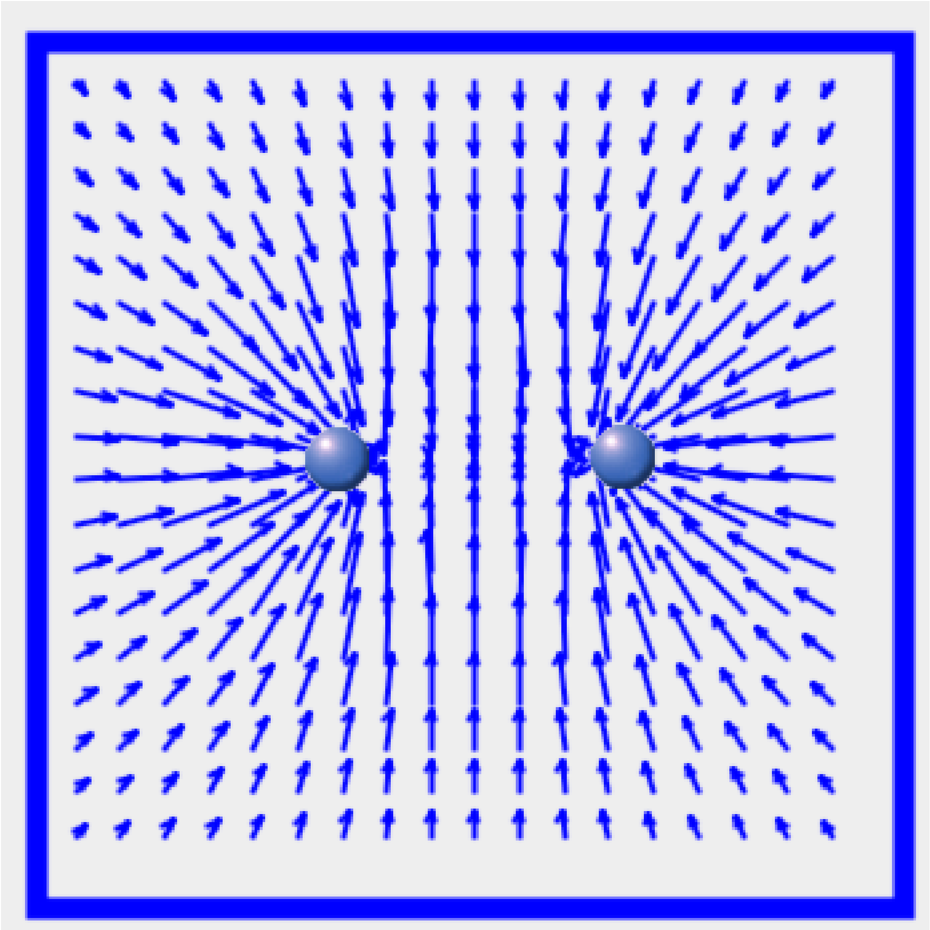
Electric field points **towards** charge.

Electric field points in the direction that a positive test charge would move.

So this charge is **negative**.

Electric Field (of Dreams): Two Equal Charges

http://phet.colorado.edu/en/simulation/efield



Two equal negative charges

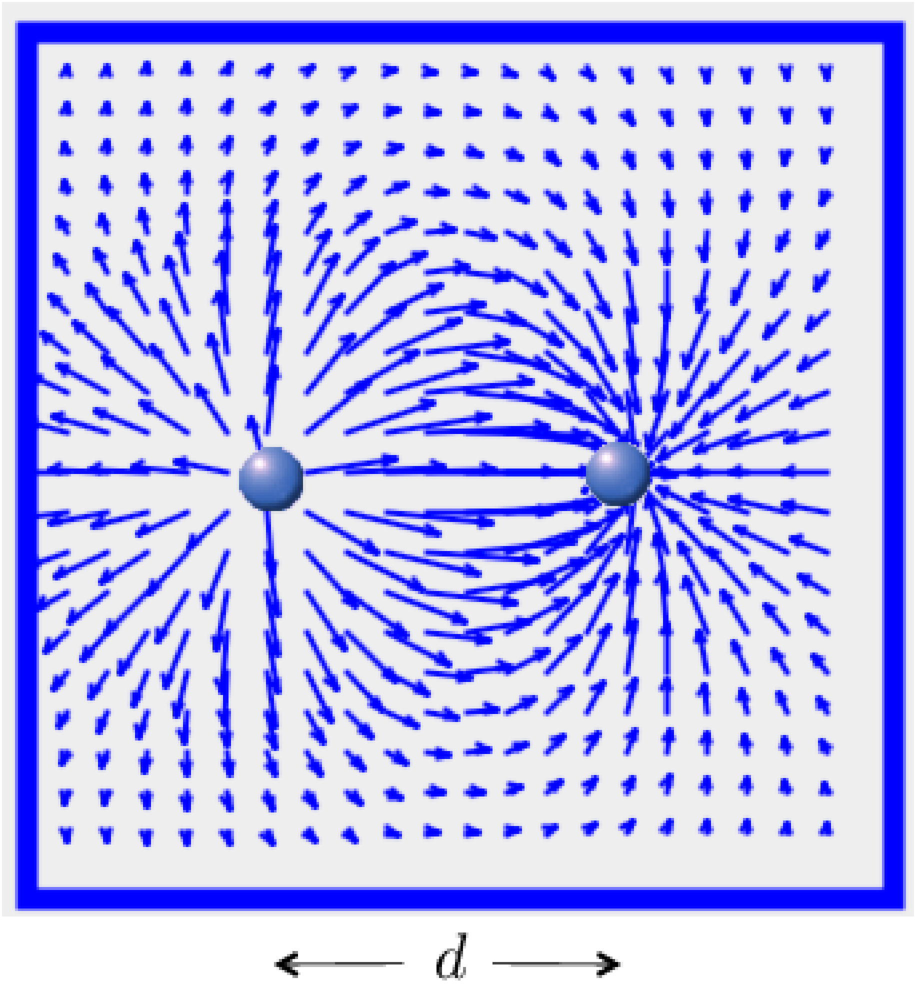
$$\vec{E}_{\text{total}} = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2} \hat{r}_i$$

Note the absence of **horizontal** electric field in the middle.

Horizontal electric field (and forces) exactly cancel out along vertical line of symmetry.

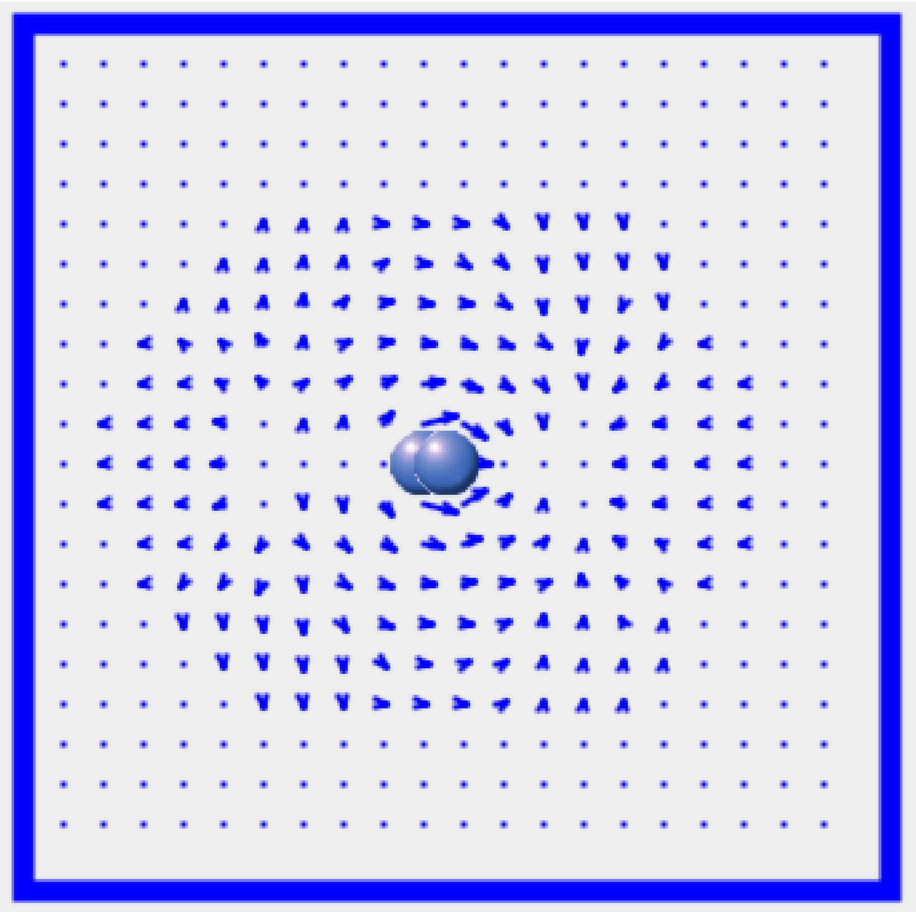
Electric field exactly between these two equal charges is zero.

Electric Dipole (Close Up, r $\approx$ d)



- Here is the field created by two **equal and opposite** charges
  - Like my hydrogen "atom" in Wed Jan 15 lecture
- Electric field lines point away from positive charge, towards negative charge
- But far away (compared to the distance between them), the electric fields cancel
  - (r is very nearly the same for + and – charges)
  - This is an **electric dipole**

Electric Dipole (Farther away, r >> d)

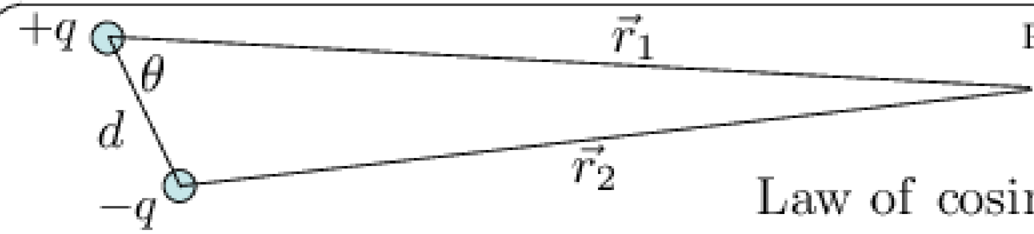


Electric field intensity E is proportional to $1/r^3$, not $1/r^2$

- Here is the field created by two equal and opposite charges
  - Like my hydrogen "atom" in Wed Jan 15 lecture
- Electric field lines point away from positive charge, towards negative charge
- But far away (compared to the distance between them), the electric fields cancel
  - (r is very nearly the same for + and – charges)
  - This is an **electric dipole**

(Advanced: Dipole Electric Field Scaling*)

- How does electric field change with r, far from an electric dipole made from charges +/-q separated by distance d?



Fields point very nearly in opposite directions

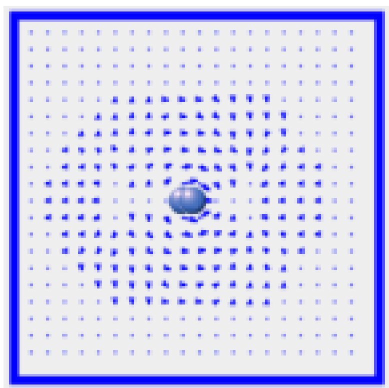
Law of cosines : $r_2^2 = r_1^2 + d^2 - 2r_1 d\cos\theta$

$$E = k\left(\frac{q}{r_1^2} - \frac{q}{r_2^2}\right) = kq\left(\frac{1}{r_1^2} - \frac{1}{r_1^2 + d^2 - 2r_1 d\cos\theta}\right)$$

$$E = kq\left(\frac{d^2 - 2r_1 d\cos\theta}{r_1^2(r_1^2 + d^2 - 2r_1 d\cos\theta)}\right) = kq\left(\frac{\cancel{d^2} - 2r_1 d\cos\theta}{r_1^2(r_1^2 + \cancel{d^2} - \cancel{2r_1 d\cos\theta})}\right)$$

when $r_1 \gg d$

$$E(\text{dipole}, r \gg d) \approx kq\left(\frac{-2d\cos\theta}{r_1^3}\right)$$

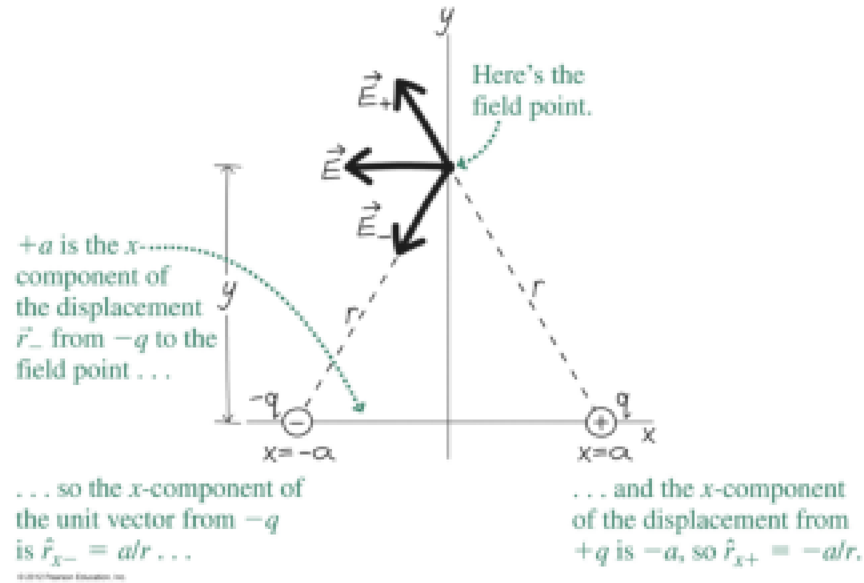


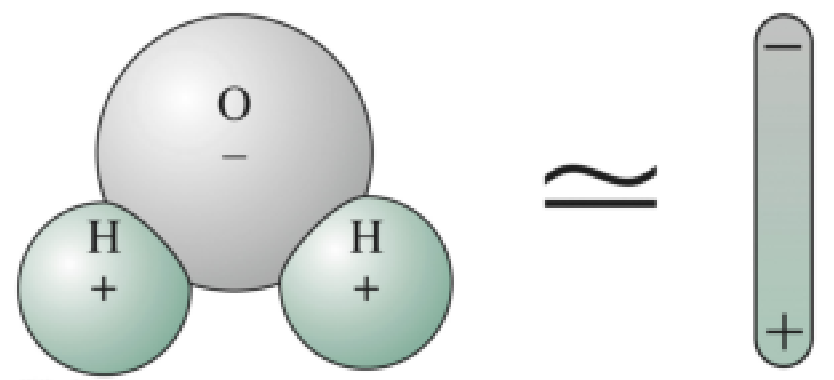
Dipole "moment": qd

Electric Dipole: an Important Charge Distribution

- An **electric dipole** consists of two point charges of equal magnitude but opposite signs, held a short distance apart.
  - The dipole is electrically neutral, but the separation of its charges results in an electric field.
  - Many charge distributions, especially molecules, behave like electric dipoles.
  - The product of the charge and separation is the **dipole moment**: *qd*
  - At distances r>>d

$$E \propto \frac{kqd}{r^3}$$





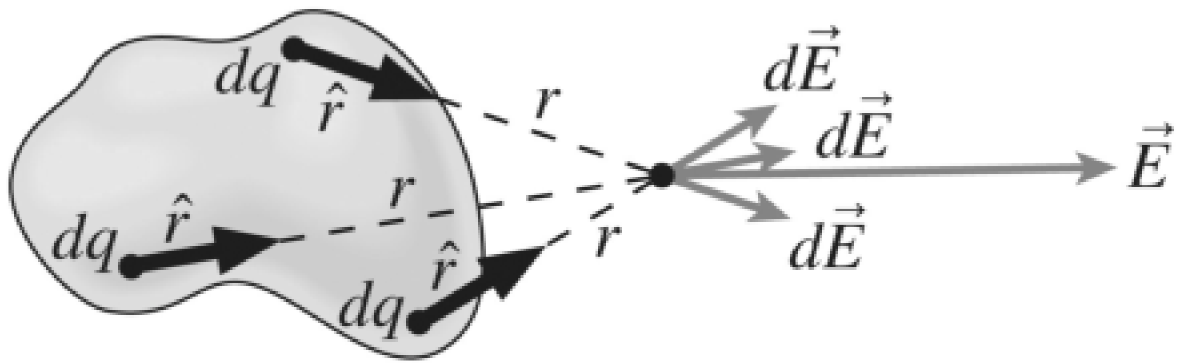
Dipoles, Quadrupoles, Many-Poles, Matter…

- Single charge: $E \propto 1/r^2$
- Electric dipole (+/- charges): $E \propto 1/r^3$
- Electric quadrupole (+/+/-/- charges): $E \propto 1/r^4$
- Macroscopic pieces of matter have > 10^{20} atoms/molecules
  - The "multipole" electric moment of this matter is **very** small
  - This is why we don't observe or experience electric forces in everyday normal neutral matter
  - All electric forces we do observe are a result of excess charge
    - Excess negative charge (extra electrons)
    - Excess positive charge (not enough electrons to cancel out positive charges of nuclei)

Continuous Charge Distributions

- Charge ultimately resides on individual particles, but it's often convenient to consider it distributed continuously on a line, over an area, or throughout space.
  - The electric field of a charge distribution follows by summing (or integrating) the small fields of infinitesimally small charge elements *dq*, each treated like a "point" charge:

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2(q)}\hat{r}(q)$$



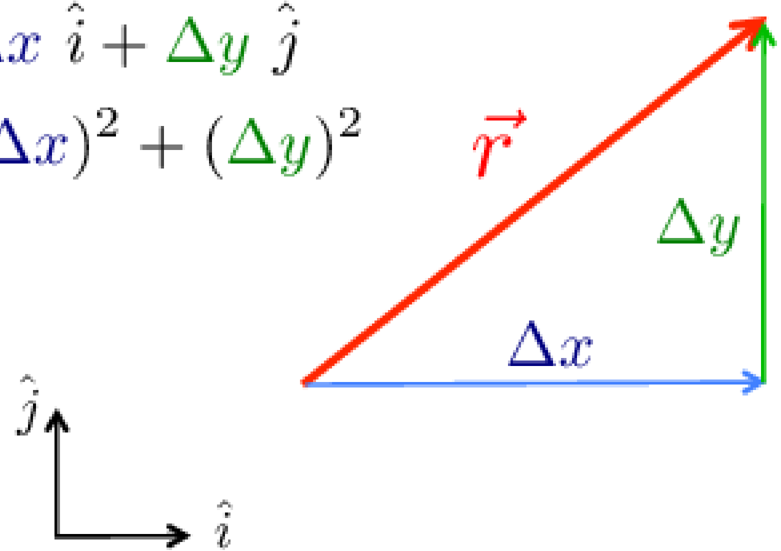
Charge distribution

Figuring Out $\hat{r}$

- $\hat{r}$ is the vector of length 1 that points in the $\vec{r}$ direction
  - It has **no units**! It only indicates a **direction**.
  - It can be calculated by taking components of the $\vec{r}$ vector and dividing those by the length of r

$$\vec{r} = \Delta x\ \hat{i} + \Delta y\ \hat{j}$$

$$r^2 = (\Delta x)^2 + (\Delta y)^2$$

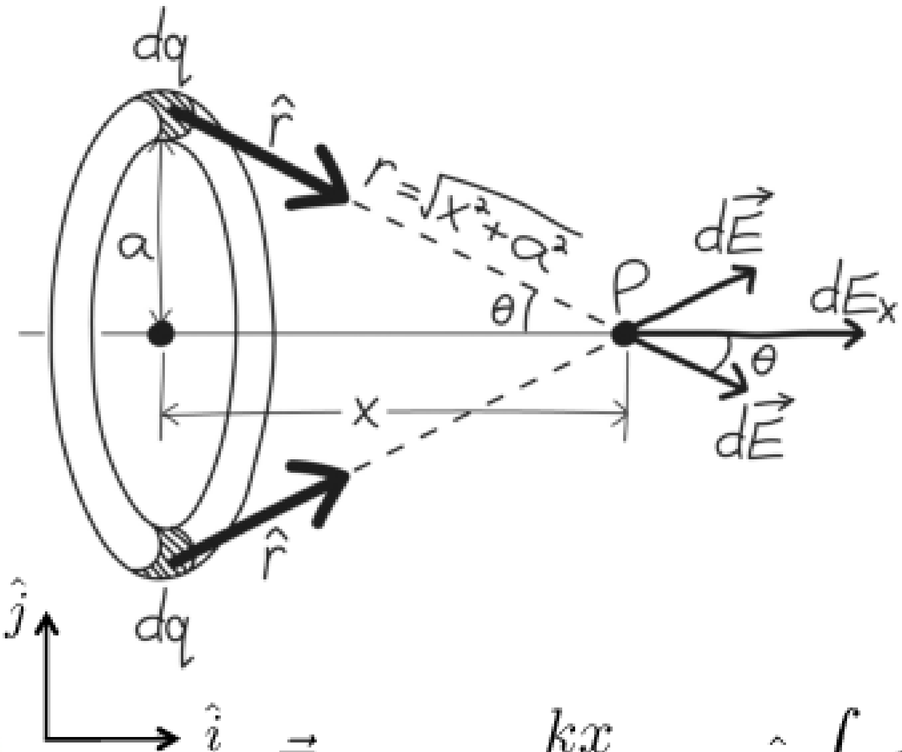


$$\hat{r} = \frac{\vec{r}}{r} = \frac{\Delta x}{r}\ \hat{i} + \frac{\Delta y}{r}\ \hat{j}$$

(when you draw a diagram in a coordinate system, always draw and label your axes)

Continuous Electric Field Example: Ring Charge

- What is the electric field on the axis of an evenly distributed ring of charge of radius a and total charge Q?



$$\vec{E} = k\int \frac{dq}{r^2(q)}\hat{r}(q)$$

$$r^2 = x^2 + a^2$$

only $\hat{\mathrm{i}}$ components add up

$$\hat{r}_x = \frac{x}{r}\hat{i}$$

$$\vec{E} = k\int \frac{dq}{(x^2+a^2)}\frac{x}{r}\hat{i}$$

But x, a, and r are constant for all dq!

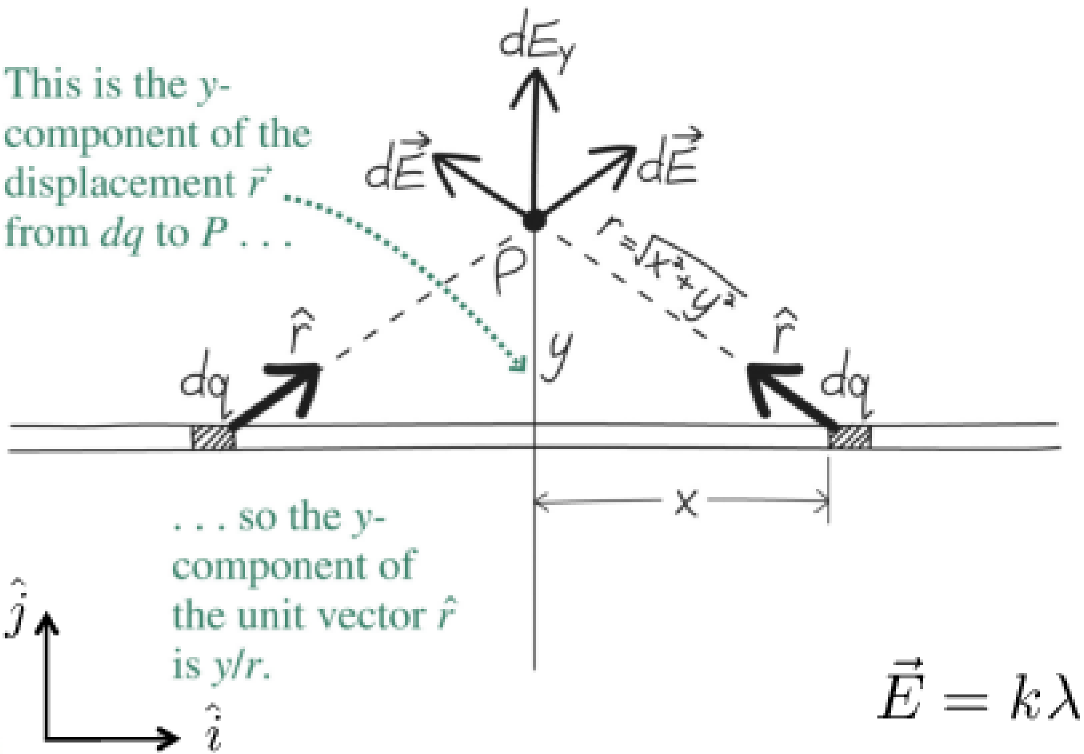
$$\vec{E} = \frac{kx}{(x^2+a^2)^{3/2}}\hat{i}\int dq$$

$$\boxed{\vec{E} = \frac{kQx}{(x^2+a^2)^{3/2}}\hat{i}}$$

Does this make sense for "simple" cases like a=0 m and x=0 m?

Continuous Electric Field Example: Line Charge

- What is the electric field created by an infinitely long straight line of charge, with charge per unit length λ ?



$$\vec{E} = k\int \frac{dq}{r^2(q)}\hat{r}(q)$$

$$dq = \lambda\, dx$$

$$r^2(q) = x^2 + y^2$$

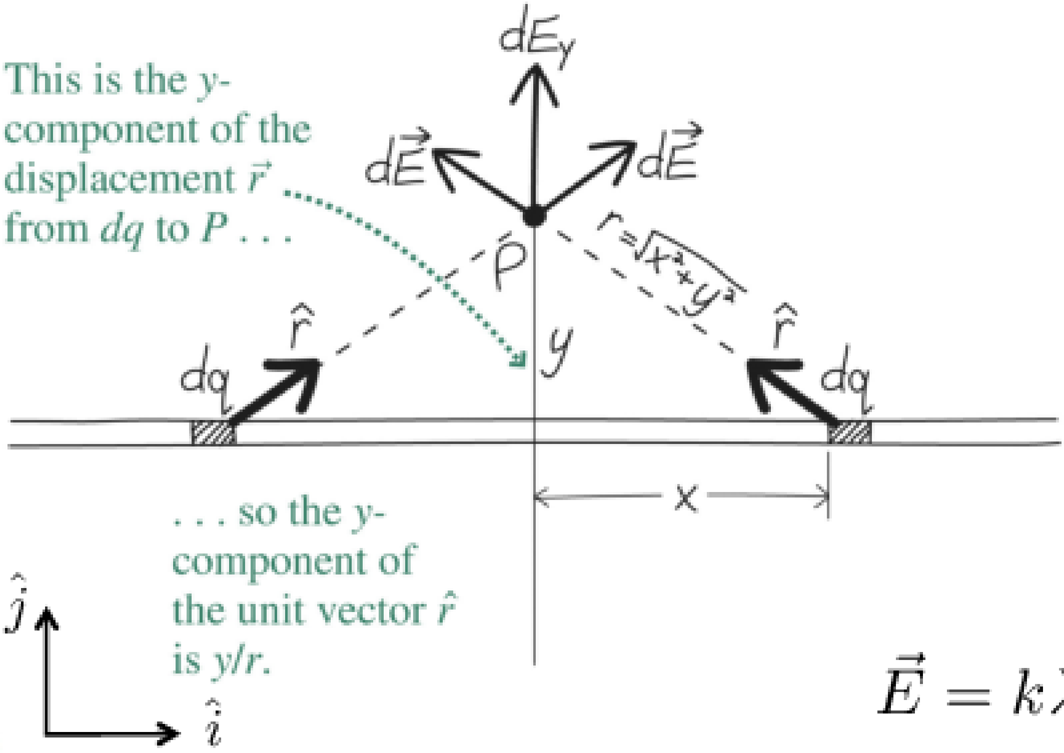
$$\hat{r} = \frac{x}{r}\,\hat{i} + \frac{y}{r}\,\hat{j}$$

$$\vec{E} = k\lambda \int_{-\infty}^{\infty} \frac{dx}{x^2+y^2}\left[\frac{x}{r}\,\hat{i} + \frac{y}{r}\,\hat{j}\right]$$

- The $\hat{i}$ component is an odd function of x, so it's zero
  - (horizontal electric field components from +/-x dq charges cancel)

Continuous Electric Field Example: Line Charge

- What is the electric field created by an infinitely long straight line of charge, with charge per unit length λ ?



$$\vec{E} = k \int \frac{dq}{r^2(q)} \hat{r}(q)$$

$$dq = \lambda\, dx$$

$$r^2(q) = x^2 + y^2$$

$$\hat{r} = \frac{x}{r}\,\hat{i} + \frac{y}{r}\,\hat{j}$$

$$\vec{E} = k\lambda \int_{-\infty}^{\infty} \frac{dx}{x^2+y^2} \left[\frac{y}{r}\,\hat{j}\right]$$

$$\vec{E} = k\lambda y \hat{j} \int_{-\infty}^{\infty} \frac{dx}{(x^2+y^2)^{3/2}}$$

Full credit happens here!

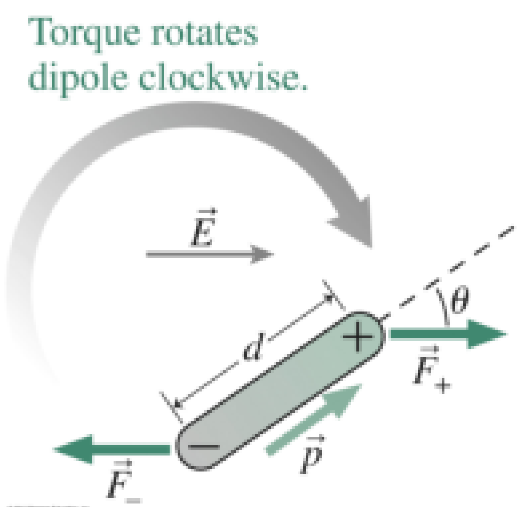
$$\vec{E} = \frac{2k\lambda}{y}\,\hat{j}$$

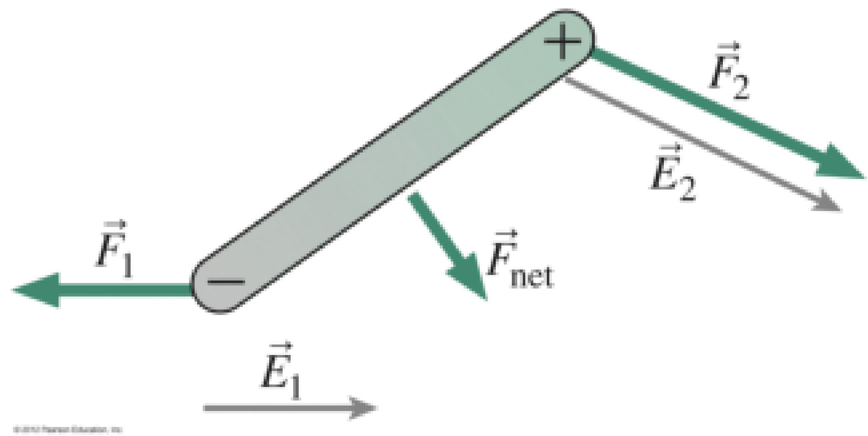
We'll learn an easier way to find this next week

Integral table / Mathematica / Magic

Matter in Electric Fields

- For a point charge *q* in an electric field $\vec{E}$, Newton's second law and the electric force combine to give acceleration: $\vec{a} = q\vec{E}/m$
- A dipole in an electric field experiences a torque that tends to align the dipole moment with the field: $\vec{\tau} = q\vec{d} \times \vec{E}$
- If the field is not uniform, the dipole also experiences a net force.
- The work required to rotate the dipole is $W = -qdE(\cos\theta - \cos\theta_0)$ where *q* is the angle between the dipole and the field.
- A dipole in an electric field has a potential energy $U = -q\vec{d} \cdot \vec{E}$





Conductors, Insulators, and Dielectrics

- Materials in which charge is free to move are **conductors.**
- Materials in which charge isn't free to move are **insulators.**
  - Insulators generally contain molecular dipoles, which experience torques and forces in electric fields.
    - Such materials are called **dielectrics.**

– Even if molecules aren't intrinsically dipoles, they acquire induced dipole moments as a result of electric forces stretching the molecule.

– Alignment of molecular dipoles reduces an externally applied field.