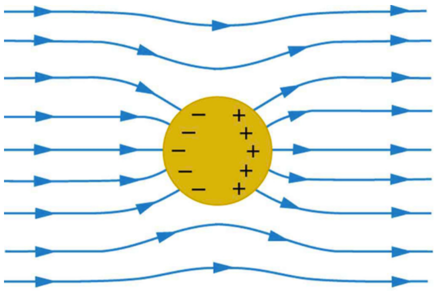

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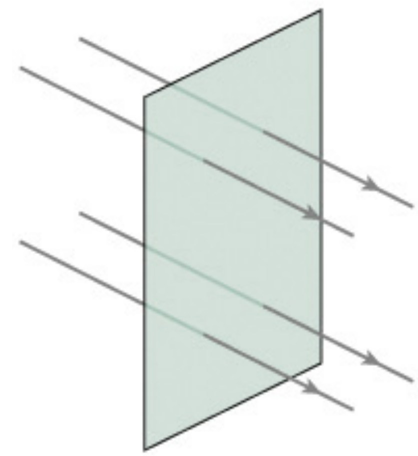

Conductors, Electric Flux
Introduction to Gauss's Law

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Monday, January 27 2014

Happy Birthday to Sam Ting, Rosamund Pike, Patton Oswalt,
Lewis Carroll, and Beatrice Tinsley!

Announcements

- Evening Problem Solving Sessions
  - An opportunity to develop your problem-solving skills!
  - Run by Eric Stacy, estac003@odu.edu, Physics Learning Center, Mondays and Tuesdays 7-9 PM.
  - Similar sessions held 12:30-1:30 on Tuesdays(tomorrow!) in the SCALE-UP classroom (by your valiant TA, Fred Miller).

- Public Talk: The Physics of Football
  - North Cafeteria, Webb Center, Tuesday Jan 28 7 PM (tomorrow!)
  - Tim Gay, University of Nebraska, Lincoln
    - Has consulted with NFL Films, ESPN, New York Times, and others
  - RSVP to 757-683-3116 or http://www.odu.edu/univevents (TGL14) if you plan to attend.

Happy Birthday to Sam Ting, Rosamund Pike, Patton Oswalt, Lewis Carroll, and Beatrice Tinsley!

Homework Sanity Check

- Have covered material / Today / Wednesday

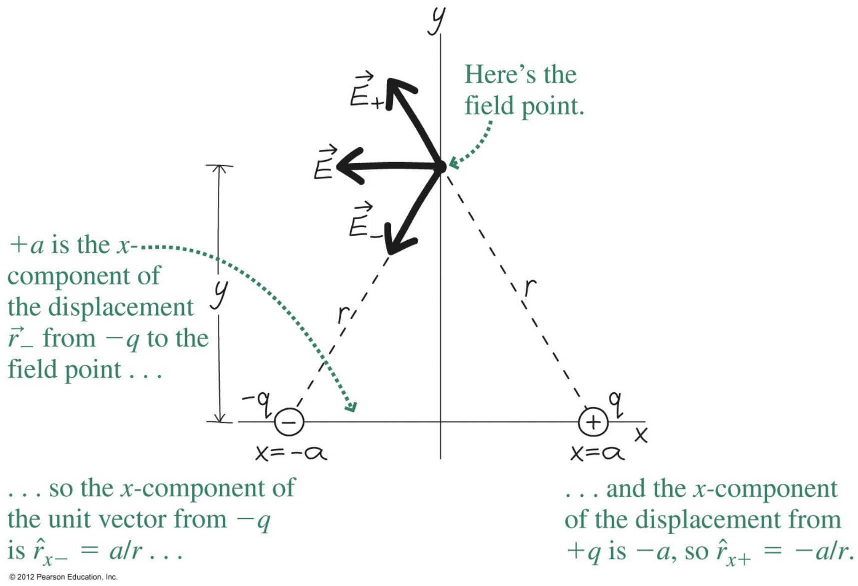
- Homework Electrostatics II due Thu Jan 31 2014 11:59 PM
  - 20.23: Electric field and force (review)
  - 20.26: Electric field of a proton
  - 20.27: Electric field of two charges, vectors
  - 20.29: Electric field from a wire with constant line density
  - 21.20: Flux
  - 21.23: Flux (sphere)
  - 21.25: Gauss's law (basic)
  - 21.27: Gauss's law (a little more advanced)
  - Tutorial: Field from surface charge on conductor

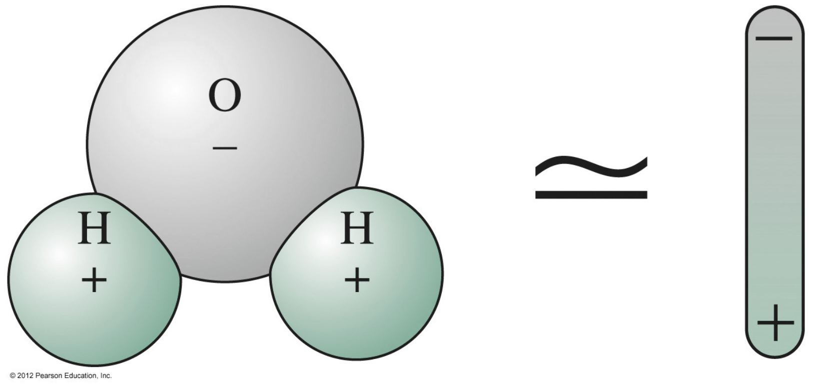
- There **will** be a quiz this Friday

Review: Electric Dipole

- An **electric dipole** consists of two point charges of equal magnitude but opposite signs, held a short distance apart.
  - The dipole is electrically neutral, but the separation of its charges results in an electric field.
  - Many charge distributions, especially molecules, behave like electric dipoles.
  - The product of the charge and separation is the **dipole moment**: *qd*
  - At distances r>>d

$$E \propto \frac{kqd}{r^3}$$

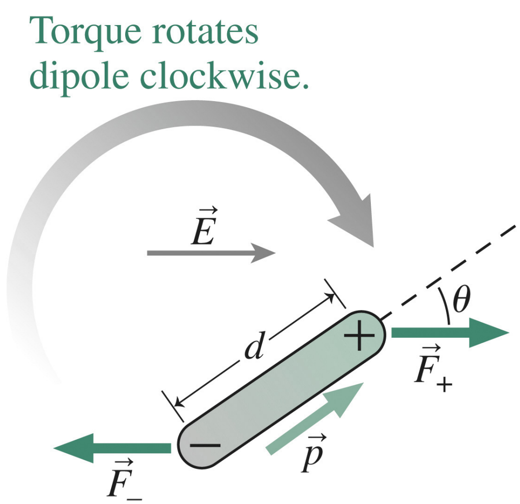


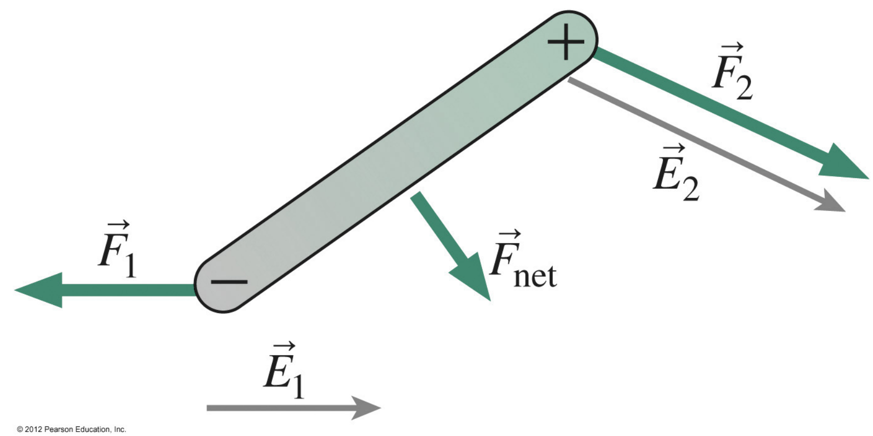


(Review: Matter in Electric Fields)

- For a point charge *q* in an electric field $\vec{E}$, Newton's second law and the electric force combine to give acceleration: $\vec{a} = q\vec{E}/m$

- A dipole in an electric field experiences a torque that tends to align the dipole moment with the field: $\vec{\tau} = q\vec{d} \times \vec{E}$
- If the field is not uniform, the dipole also experiences a net force.
- The work required to rotate the dipole is $W = -qdE(\cos\theta - \cos\theta_0)$ where *q* is the angle between the dipole and the field.
- A dipole in an electric field has a potential energy $U = -q\vec{d} \cdot \vec{E}$



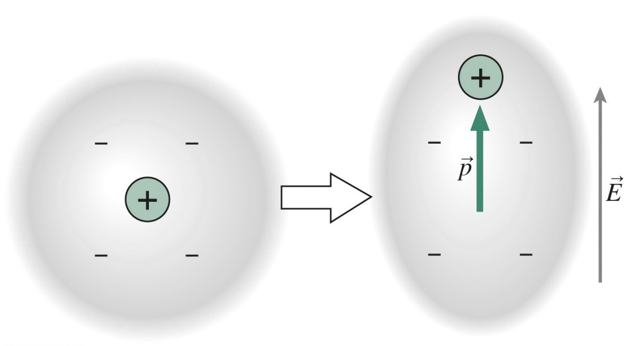


Review: Insulators and Dielectrics

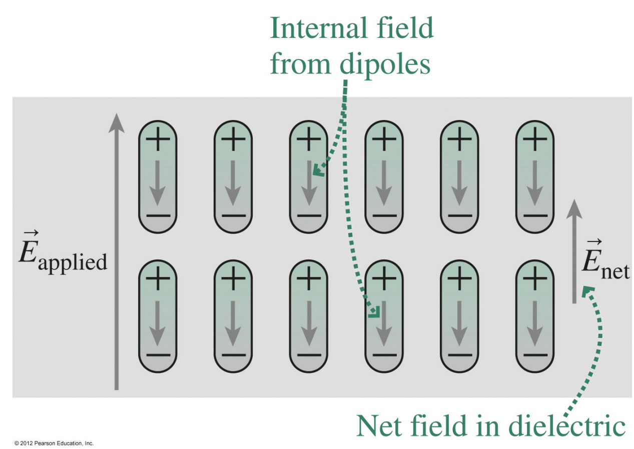
- Materials in which charge isn't free to move are **insulators**
  - Some insulators contain molecular dipoles, which (as we've seen) experience torques and forces in electric fields.
    - Such materials are called **dielectrics.**

– Even if molecules aren't intrinsically dipoles, they acquire induced dipole moments as a result of electric forces stretching the molecule.

– Alignment of molecular dipoles reduces an externally applied field.



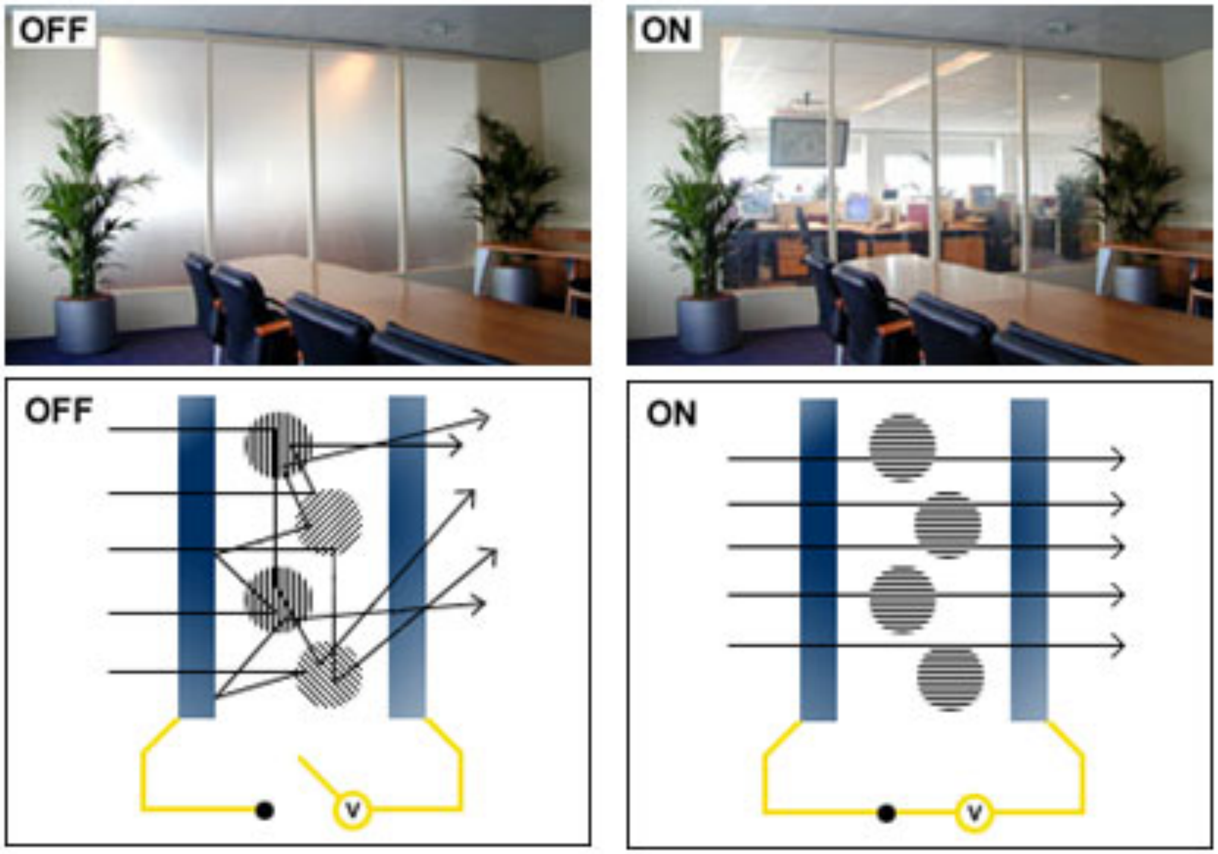
External field "stretches" atom



Dielectric Example: Smart Glass

- A modern example of a dielectric application is **smart glass**
  - Glass sandwich with electric dipole polymer filling
  - Adjust transparency by changing external voltage (electric field)

No external electric field
Dipoles in random orientation
No light transmitted



Applied external electric field
Dipoles all aligned in same direction
Light transmitted!

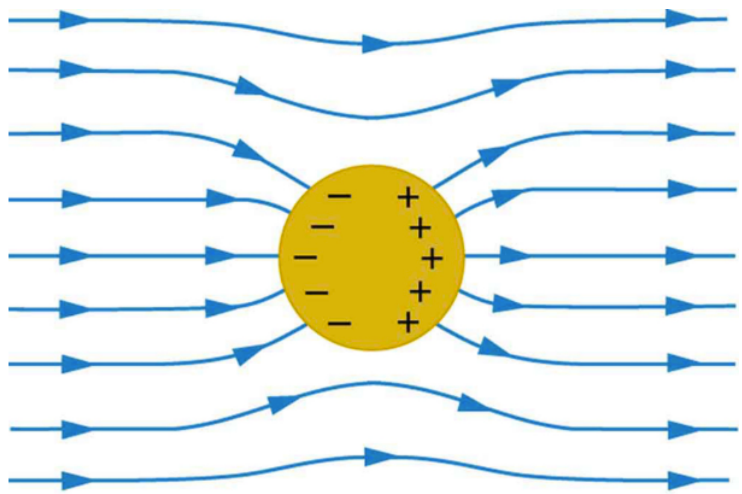
http://en.wikipedia.org/wiki/Smart_glass

Conductors

- Materials in which charge is free to move are **conductors**
- In this class, we assume perfect conductors
  - **Zero** resistance to motion of electrons
  - Many conductors are very good approximations (e.g. Cu, Ag)
  - Superconductors are **real** examples of **perfect** conductors
    - We will learn more about them when we learn about magnetism
- Electrons move freely in conductors until **electrical forces balance**
  - Excess electrons distribute on the **surface** of a conductor
    - (Excess positive charge "distributes" on the surface of a conductor too)
    - More charge (and higher field) around points

Conductors and Electric Fields

- Electrons move freely in conductors until **electrical forces balance**
  - The electric field **inside** a perfect conductor is **always zero**
    - Any nonzero field moves electrons until the overall electric field is zero
  - The electric field on the **surface** of a perfect conductor is always only **perpendicular** to the surface
    - Any tangential field moves electrons until the tangential field is zero

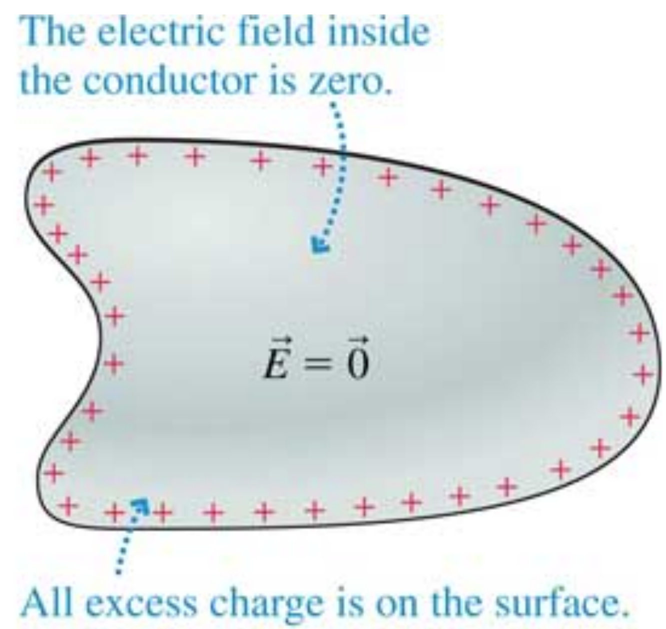


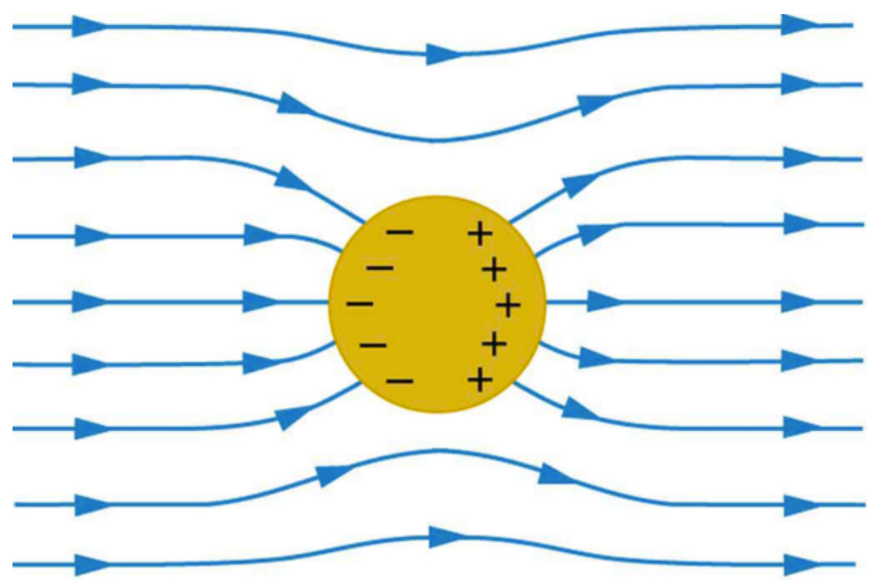
Electric field at surface of conductor perpendicular to surface

Electric field inside conductor: E=0 N/m

Onwards to Chapter 21: Gauss's Law

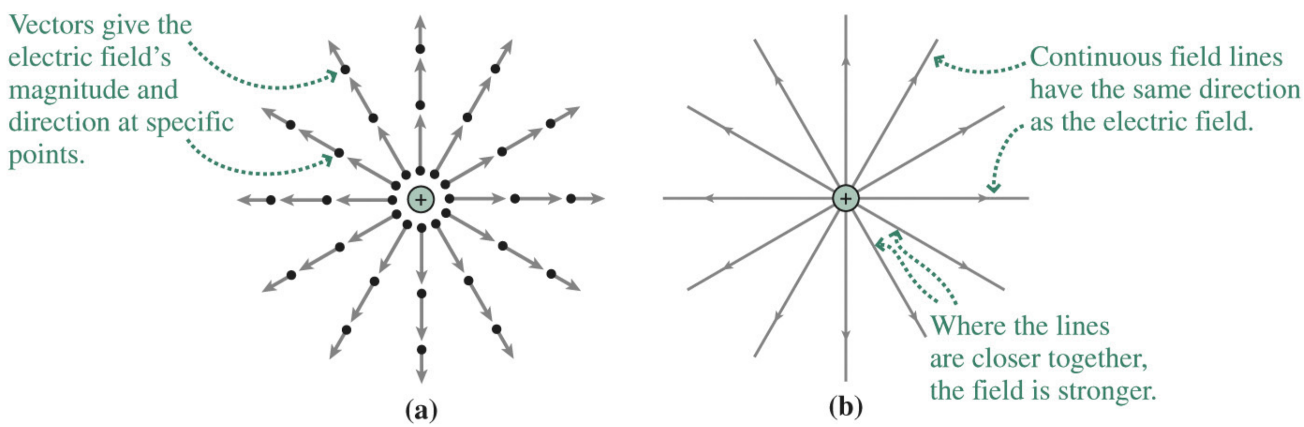
- Represent electric fields using field-line diagrams
- Understand Gauss's law and how it relates to Coulomb's law (charges as "sinks" and "sources")
- Calculate the electric fields for **symmetric** charge distributions with Gauss's law
- Describe the behavior of charge on conductors in "electrostatic equilibrium"



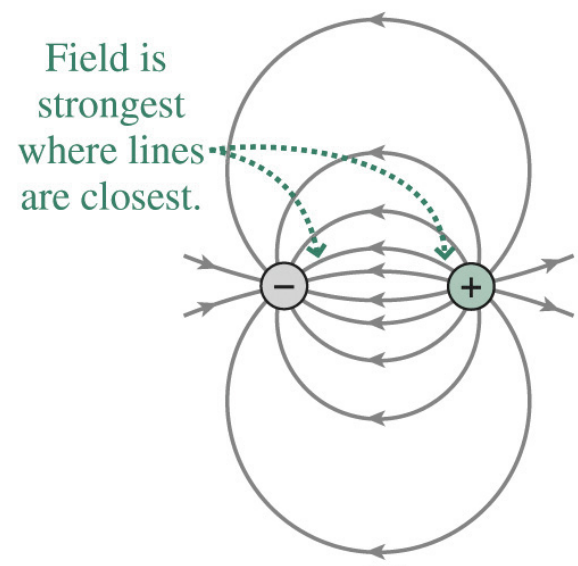


Electric Field Lines (or Curves)

- **Electric field lines** provide a convenient way to generally draw electric fields
  - Each curve's direction at any point is the E field direction
  - Spacing of field lines describes the magnitude of the field
    - Where lines are closer, the field is stronger
    - (Some artistic license is taken)



Vector and field-line diagrams of a point-charge field



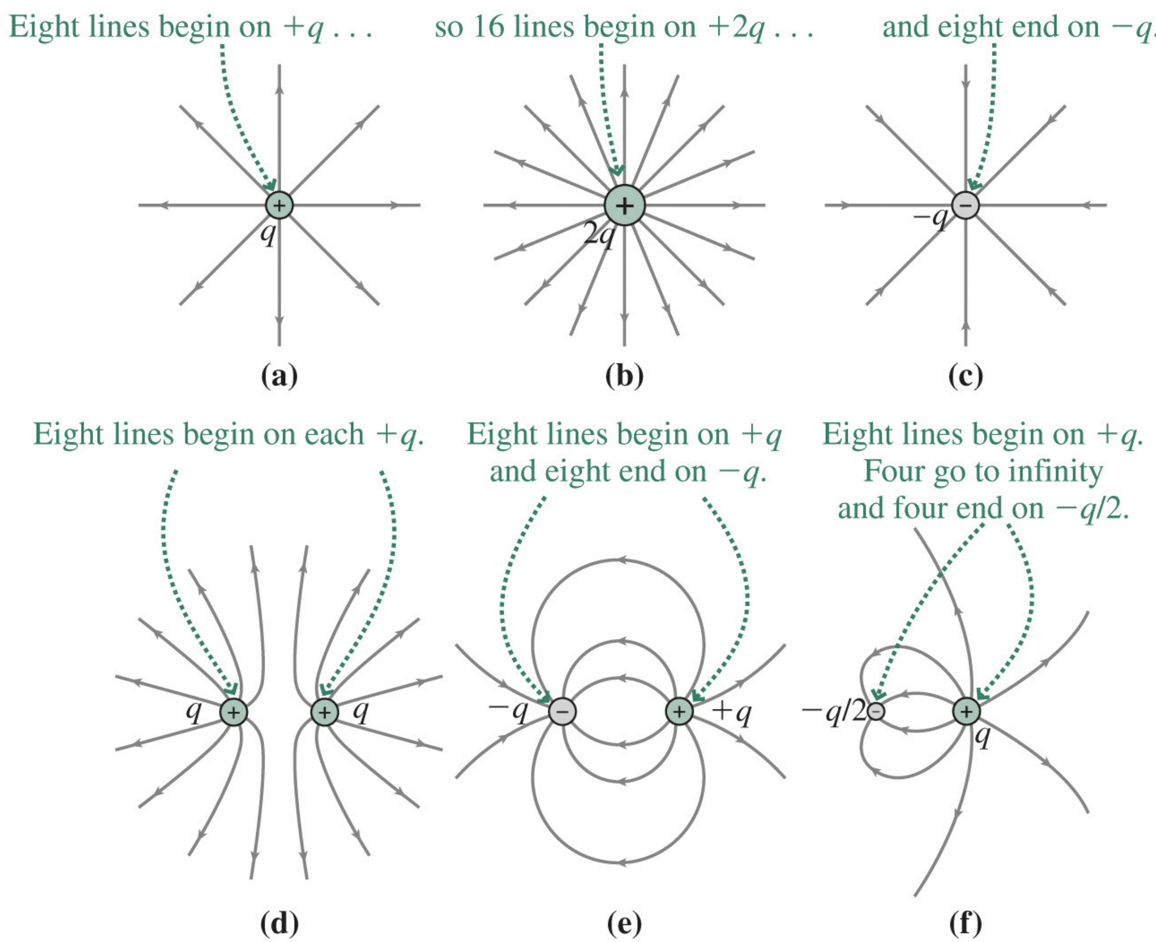
Field lines of an electric dipole

Field Lines for Simple Charge Distributions

- There are field lines everywhere, so every charge distribution has infinitely many field lines

- We associate a certain finite number of field lines with a charge of a given magnitude.
- In the diagrams shown, 8 lines are associated with a charge *q.*
- Field lines of static charge distributions always begin and end on charges, or extend to infinity.

Positive: "sources"
Negative: "sinks"

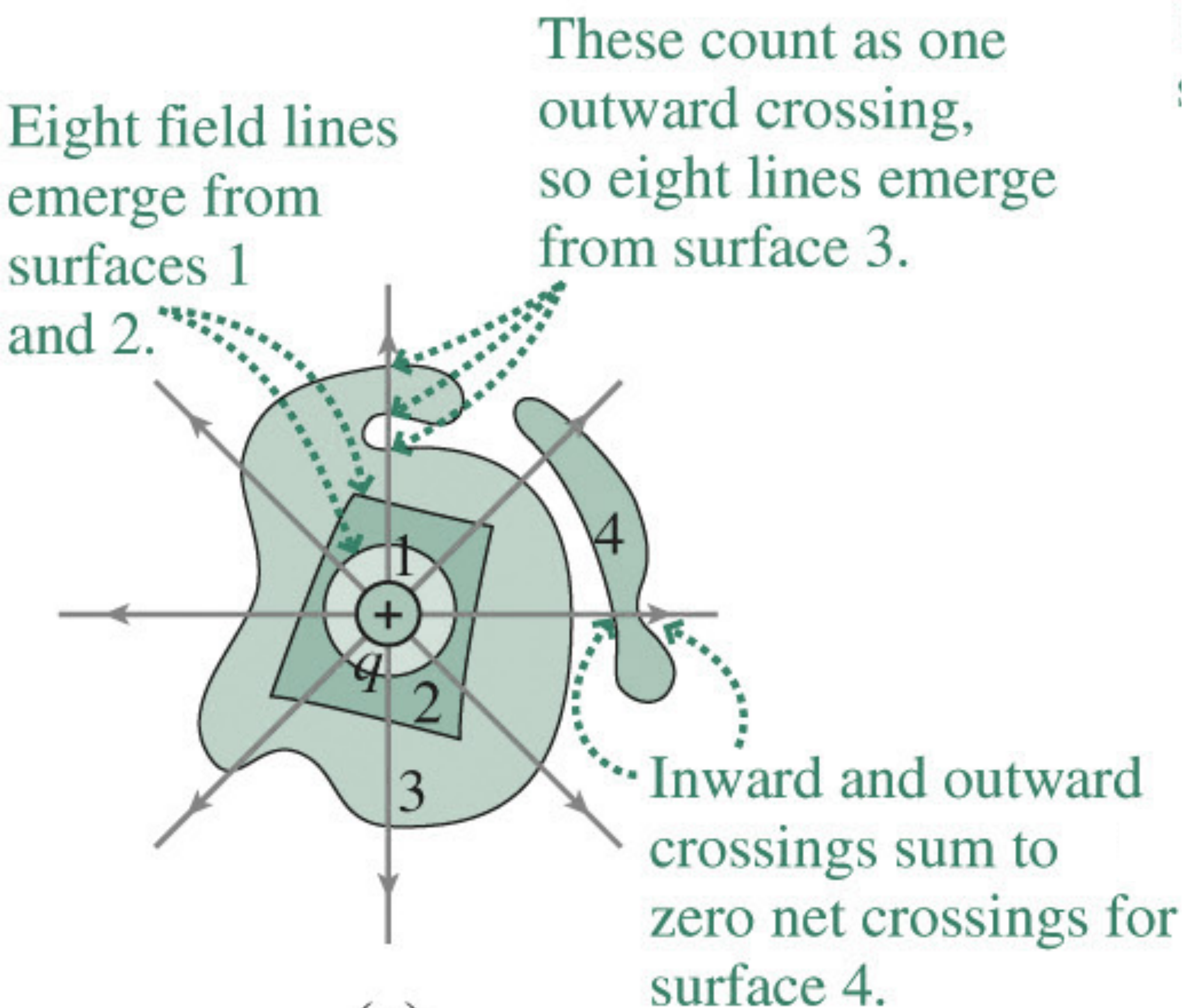


Counting Field Lines

- How many field lines emerge from any closed surface?
  - Think of + and – charges like sources and sinks of electric field lines
  - These are like water (or incompressible) fluid flow
  - Any surface that doesn't enclose a source or a sink
    - must have net flow through it of zero
    - Everything going in also comes out
  - Any surface that encloses only a source
    - must have net flow through it that is outward (positive)
  - Any surface that encloses only a sink
    - must have a net flow through it that is inward (negative)

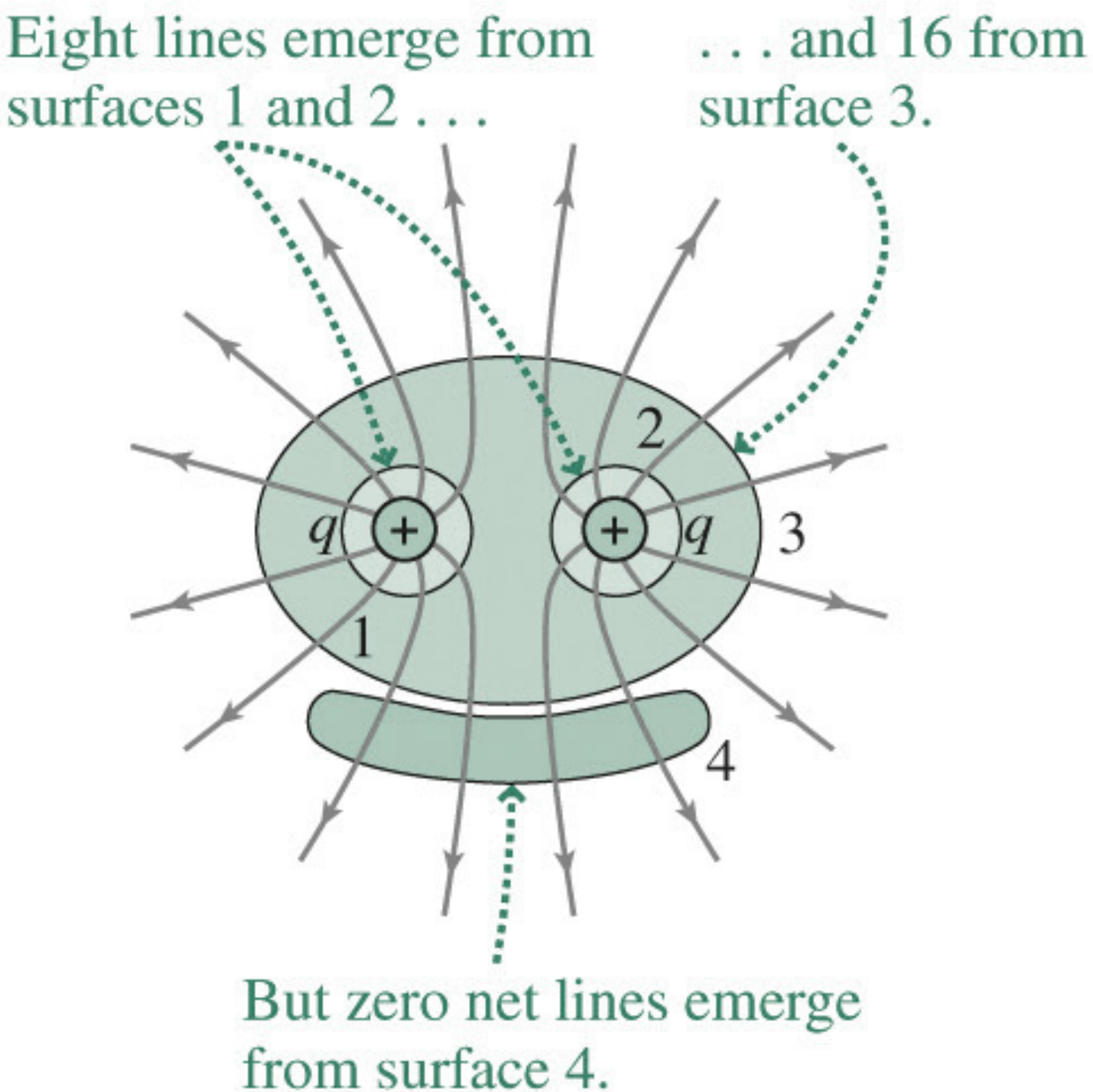
- Count each field line crossing going outward as +1, each inward crossing as –1.
- We find that **the number of field lines crossing any closed surface is proportional to the net charge** (sum of charges, or sources and sinks) **enclosed.**

Counting Field Lines



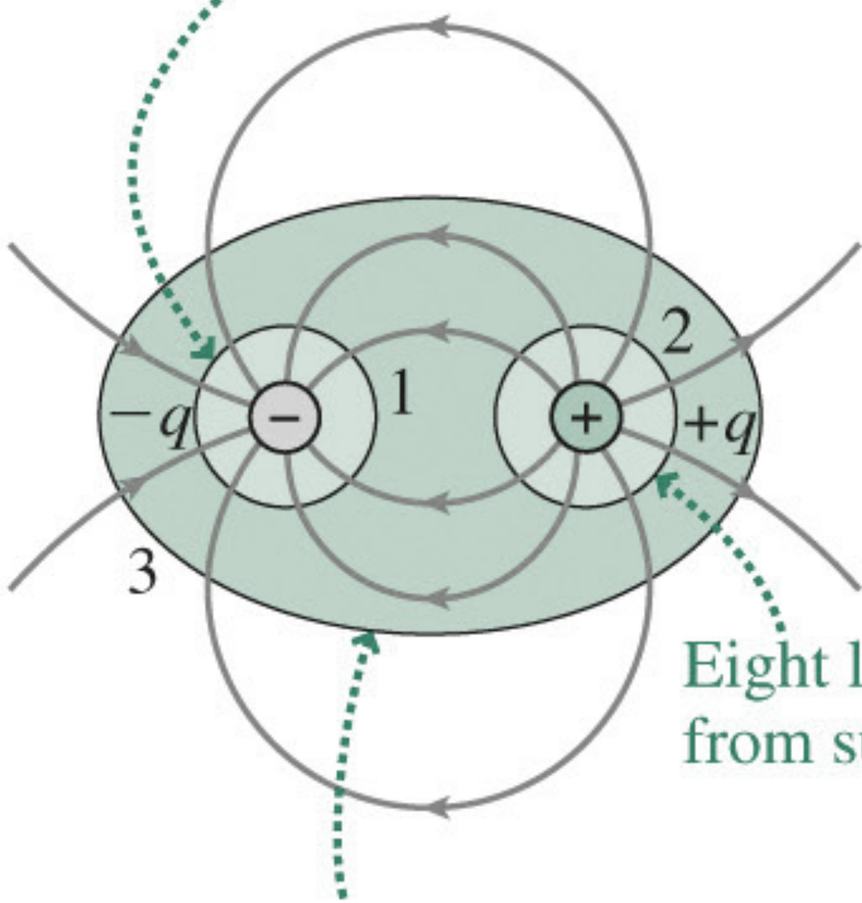
(a)

Counting Field Lines



Counting Field Lines

Going inward, −8 lines emerge from surface 1.

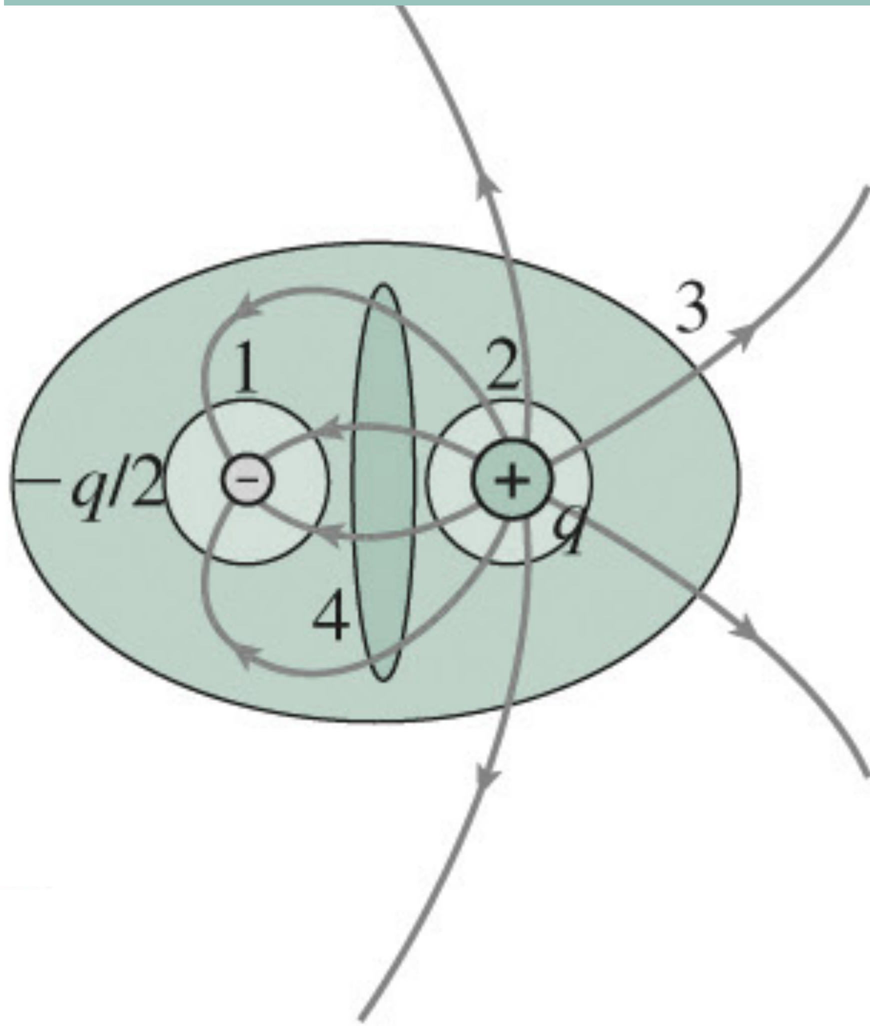


Eight lines emerge from surface 2.

But zero net lines emerge from surface 3.

Activity: Counting Field Lines

Left charge is –q/2, right charge is +q



Count these field lines!

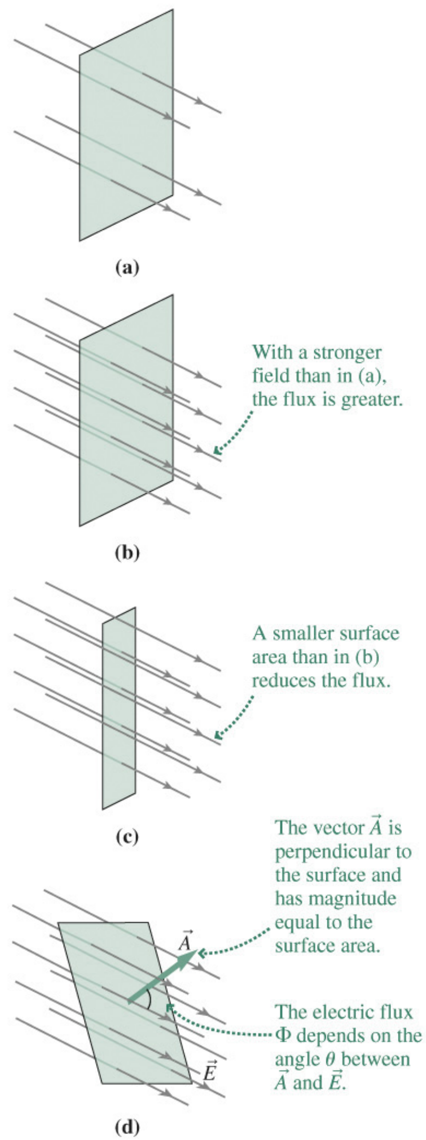
Surface	N(out)	N(in)	N(tot)	Q(enc)
1				
2				
3				
4				

Electric Flux

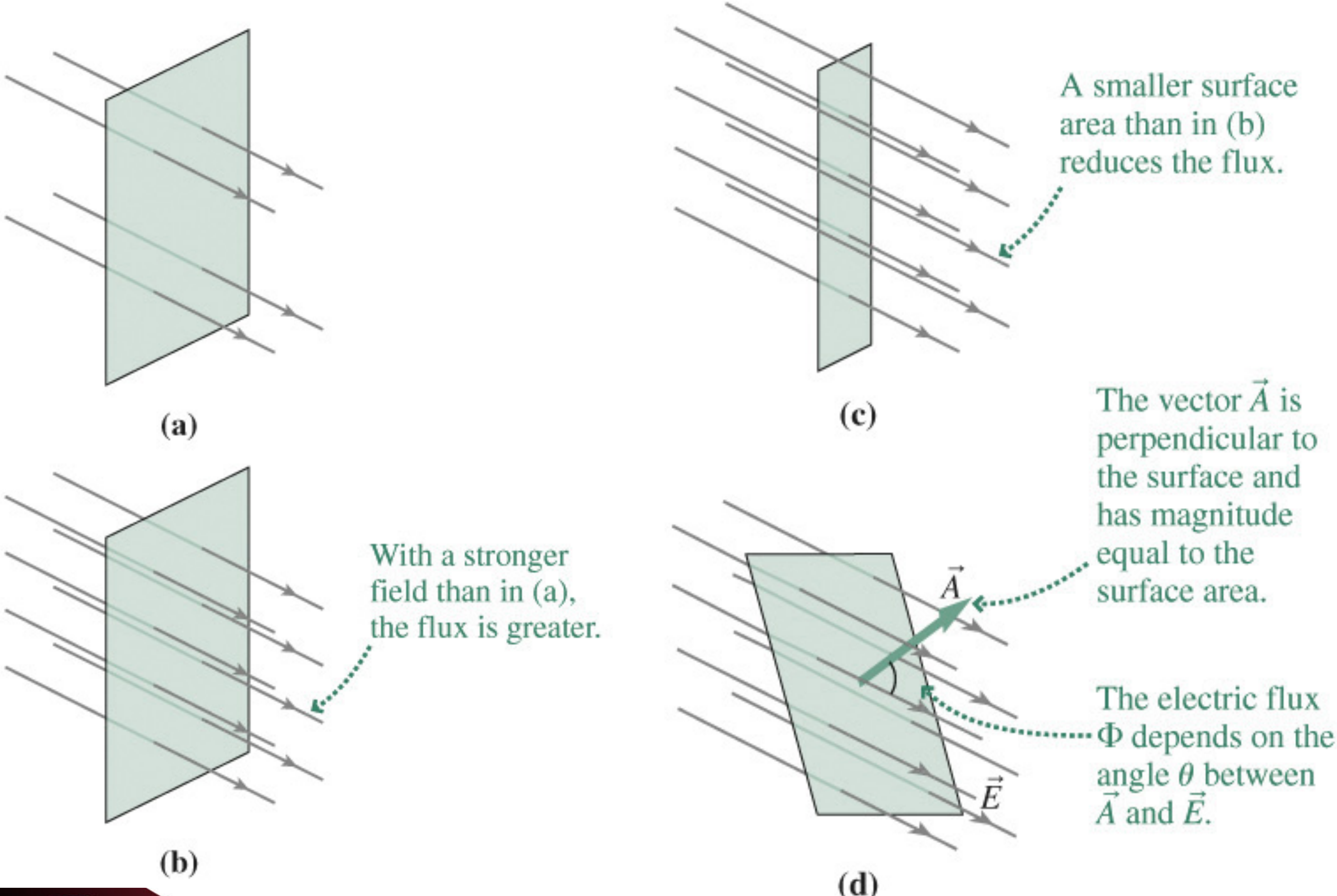
- **Electric flux** is a precise mathematical version of "number of lines crossing through the surface"
- The **electric flux** Φ through a flat surface in a uniform electric field depends on the field strength *E*, the surface area *A*, and the angle θ between the field and the normal to the surface.
- Mathematically, the flux over a surface where the electric field intensity is constant is given by

$$\Phi = EA\cos\theta = \vec{E}\cdot\vec{A}$$

  - Here $\vec{A}$ is "the" vector whose magnitude is the surface area *A* and whose orientation is normal (perpendicular) to the surface.



Electric Flux: Flat Surface, Constant E



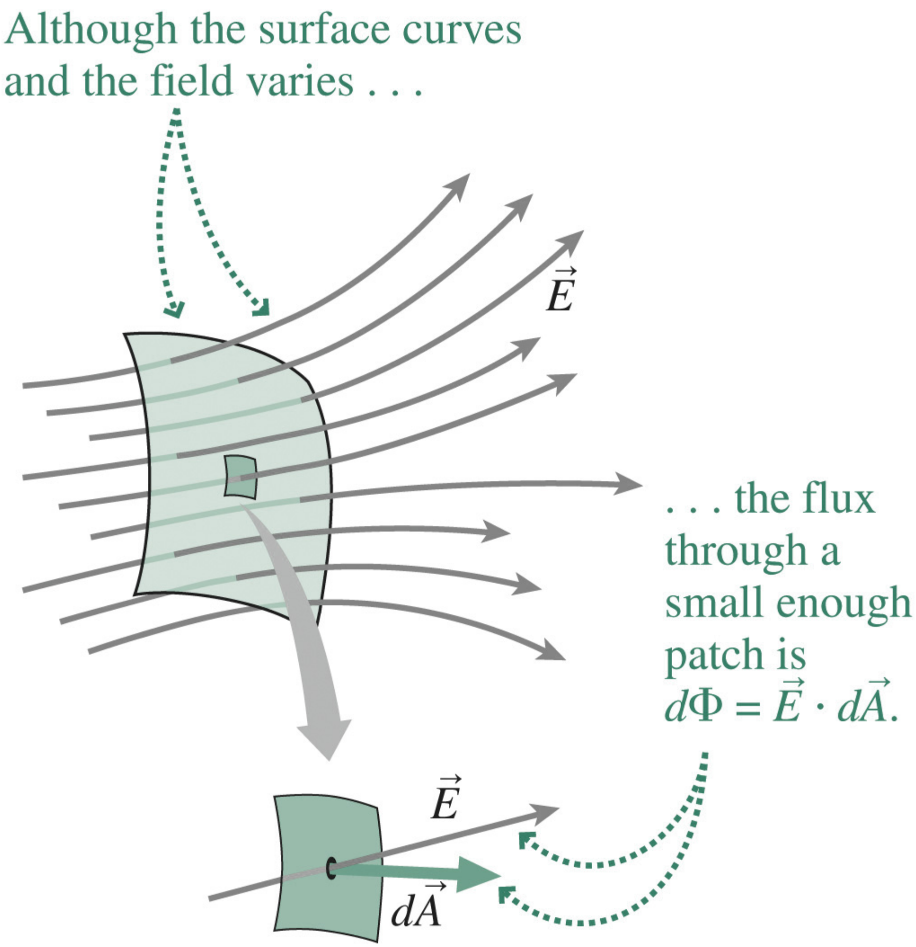
Electric Flux: Curved Surfaces, Changing E

- When the surface is curved and/or the electric field intensity is not uniform, we can still calculate **flux** Φ
  - Divide surface into small patches $d\vec{A}$, so small that each patch is (approximately) flat and the field is (approximately) uniform over each

$$d\Phi = \vec{E} \cdot d\vec{A}$$

- Then, as before, we can add up the pieces to get the total flux
  - (The sum becomes an integral)

$$\Phi = \int \vec{E} \cdot d\vec{A}$$



Gauss's Law

- We've seen that the electrical flux through a closed surface depends on how much charge is "inside" the surface
  - Makes sense when considering charges as sources and sinks of electrical field (or electrical field lines)

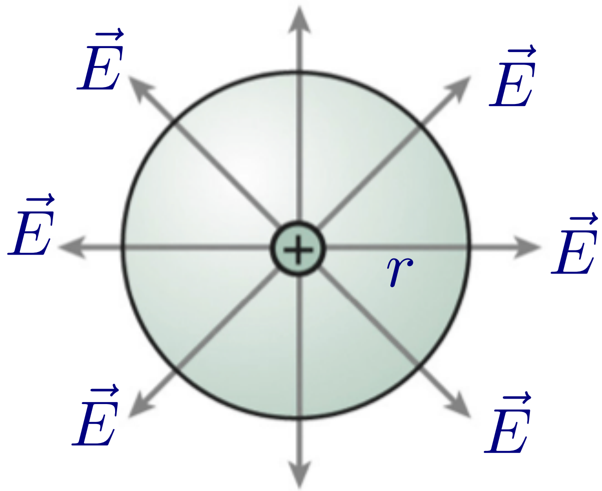
  - Can we write this down mathematically? Yes, **the total flux through any surface is proportional to the total charge enclosed** $q_{\mathrm{tot,enclosed}}$

$$\Phi = \int_{\mathrm{surface}} \vec{E} \cdot d\vec{A} = 4\pi k \, q_{\mathrm{tot,enclosed}} = \frac{q_{\mathrm{tot,enclosed}}}{\epsilon_0}$$

  - This is true for **any surface** and **any distribution of charges**
  - For lots of symmetric problems, we can pick a surface that has the same symmetry, with constant E over the surface area.
  - This will make the integral become just a multiplication

Gauss's Law: Example (Point Charge)

- The electric field from a point charge points radially outward
- A sphere of radius r centered on the charge has a surface that's always perpendicular to the electric field



$$\vec{E} \cdot d\vec{A} = E\, dA\, \cos\theta$$

$$\vec{E} \perp d\vec{A} \quad \Rightarrow \quad \cos\theta = 1$$

$$\vec{E} \cdot d\vec{A} = E\, dA \text{ over}$$
$$\text{entire surface of sphere}$$

$$\Phi = \int_{\text{surface}} \vec{E} \cdot d\vec{A} = 4\pi k\, q_{\text{tot,enclosed}} = \frac{q_{\text{tot,enclosed}}}{\epsilon_0}$$

$$\Phi = E\, A = E\left(4\pi r^2\right) = 4\pi k q \qquad E = \frac{kq}{r^2}$$