University Physics, PHYS 227/232 Quiz 2 Fri Feb 7, 2014

Your name and table number: ______________________________

Please show your work, write neatly, write units, and box your answers.

1. A solid perfectly conducting sphere of radius $r = 0.50\,\text{m}$ is charged with Q=+55 μC of charge.

   (a) (2 points) What is the electric potential difference between a point on the sphere's surface and infinity?
   Solution: The electric potential of a point charge or outside a perfectly conducting sphere of charge Q between points r_A and r_B is

$$V_\text{AB} = kQ\left(\frac{1}{r_\text{A}} - \frac{1}{r_\text{B}}\right)$$

   Using $Q = +55\,\mu\text{C}$, $r_\text{A} = 0.5\,\text{m}$, and $r_\text{B} = \infty$, we get

$$V_\text{AB} = (9\times10^9\,\text{N m}^2/\text{C}^2)(5.5\times10^{-5}\,\text{C})\left(\frac{1}{0.5\,\text{m}}\right) = \boxed{0.99\,\text{MV} = V_\text{AB}}$$

   (b) (2 point) What is the electric potential difference between the center of the sphere and the surface of the sphere?
   Solution: Everywhere inside a solid conductor is an equipotential. So everwhere inside the sphere, the potential is the same as at the surface of the sphere, and thus here $\boxed{\Delta V = 0\,\text{V}}$ between the surface and center of the sphere.

   (c) (1 points) What is the electric field at the surface of the sphere (magnitude and direction)?
   Solution: The electric field on the surface of a conductor is perpendicular to the surface, so the electric field must point in the $\hat{r}$ direction. It's magnitude is given by Coulomb's law (or you can quickly rederive it using Gauss's law):

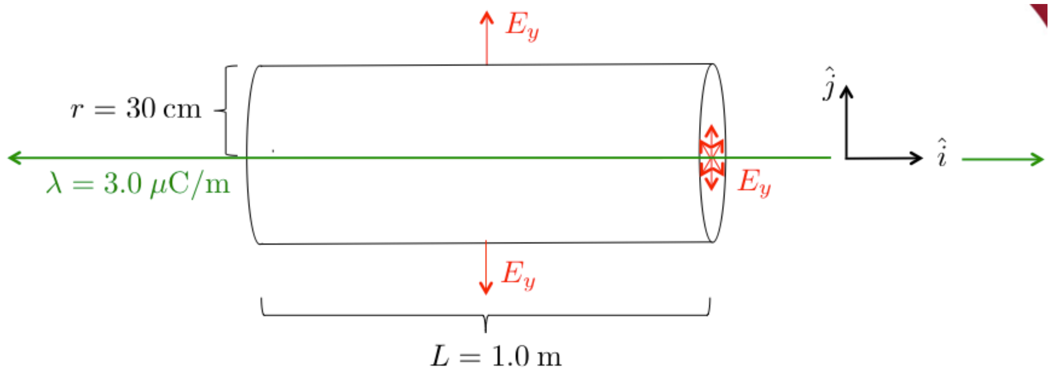
$$E = \frac{kQ}{r^2} = \frac{(9\times10^9\,\text{N m}^2/\text{C}^2)(5.5\times10^{-5}\,\text{C})}{(0.5\,\text{m})^2} = \boxed{2.0\,\text{N/C} = 2.0\,\text{V/m} = E}$$

2. Consider an infinitely long line of charge that has a constant charge density throughout of $\lambda = 3.0\,\mu\text{C/m}$. You want to calculate the electric field produced by this line of charge so you draw a cylinder of radius r=30 cm and length L=1.0 m around it as a Gaussian surface pictured in the figure below.

   (a) (1 point) What is the total electric flux through both ends of the cylinder (the circular portions at left and right in the figure), in $\text{N m}^2/\text{C}$?
   Solution: The electric field is perpendicular to the normal of the Gaussian surface on the ends of the cylinder, so $\boxed{\text{the total electric flux through the ends is zero.}}$

   (b) (2 points) What is the total electric flux through the side of the cylinder, in Nm^2/C?
   Solution: Since the flux through the ends is zero, you can use Gauss's law to figure out the flux through the side without needing to know the electric field:

$$\Phi_\text{total} = \Phi_\text{side} = 4\pi k q_\text{enclosed}$$



The charge enclosed here is

$$q_{\text{enclosed}} = L\lambda = (1.0\,\text{m})(3.0\,\mu\text{C/m}) = 3.0 \times 10^{-6}\,\text{C} = q_{\text{enclosed}}$$

So

$$\begin{aligned}
\Phi_{\text{total}} &= \Phi_{\text{side}} = 4\pi k q_{\text{enclosed}} \\
\Phi_{\text{side}} &= 4\pi(9\times10^{9}\,\text{N m}^2/\text{C}^2)(3.0\times10^{-6}\,\text{C}) = \boxed{3.4\times10^{5}\,\text{N m}^2/\text{C} = \Phi_{\text{side}}}
\end{aligned}$$

(c) (2 points) What is the electric field (magnitude and direction) at a point on the side of the cylinder, in N/C?
Solution: The electric field is constant everywhere on the side of the cylinder, and points parallel to the normal to the side of the cylinder. So the total flux is just EA where E is the electric field magnitude and $A = 2\pi r\,L$ is the area of the side of the cylinder. Using the result from part (b),

$$\begin{aligned}
\Phi_{\text{side}} &= EA = E(2\pi r\,L) = 3.4\times10^{5}\,\text{N m}^2/\text{C} \\
E &= \frac{(3.4\times10^{5}\,\text{N m}^2/\text{C})}{2\pi r\,L} = \frac{(3.4\times10^{5}\,\text{N m}^2/\text{C})}{2\pi(0.3\,\text{m})\,(1.0\,\text{m})} = \boxed{1.8\times10^{5}\,\text{N/C} = E}
\end{aligned}$$

This is consistent with the equation we derived in class:

$$E(\text{linecharge}) = \frac{2k\lambda}{r} = \frac{2(9\times10^{9}\,\text{N m}^2/\text{C}^2)(3.0\times10^{-6}\text{C/m})}{0.3\,\text{m}} = 1.8\times10^{5}\,\text{N/C}$$