University Physics, PHYS 227/232   Sample Midterm 1   Sun Feb 9, 2014

Useful Equations

Electric (Coulomb) force $\vec{F}$ between two point charges: $\vec{F} = \frac{kq_1q_2}{r^2}\hat{r}$

Electric field $\vec{E}$ of a point charge or charged sphere: $\vec{E} = \frac{kq}{r^2}\hat{r}$

Electric potential V of a point charge or charged sphere (relative to zero potential at $r = \infty$): $V = \frac{kq}{r}$

Electric potential energy U of a point charge at electric potential V: $U = qV$

Flux Φ and Gauss's Law: $\Phi = \int \vec{E} \cdot d\vec{A} = \int E dA \cos\theta$ $\Phi_{closed\,surface} = 4\pi k q_{\text{enclosed}}$

Area of circle: $A = \pi r^2$

Surface area of sphere: $A = 4\pi r^2$

Surface area of cylinder: $A = 2\pi r^2$ (ends) $+ 2\pi r L$ (side)

Volume of sphere: $\text{Vol} = \frac{4}{3}\pi r^3$

Volume of cylinder: $\text{Vol} = \pi r^2 L$

Useful Constants

$k = 9.00 \times 10^9$ N m^2/C^2

$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{12}$ C^2/N m^2

1 nC $= 10^{-9}$ C   1 μC $= 10^{-6}$ C

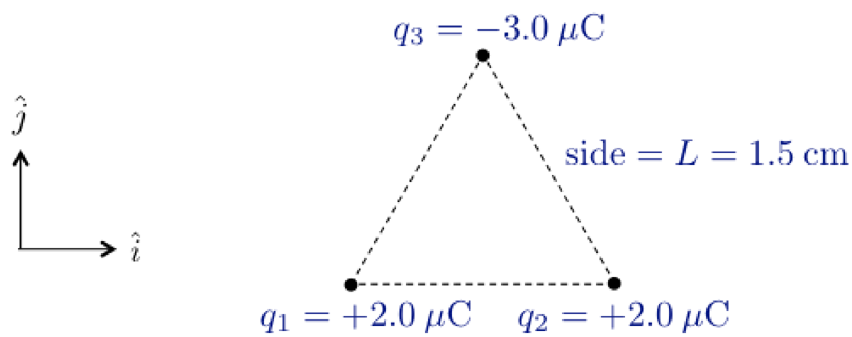
Electron charge: $e = -1.6 \times 10^{-19}$ C

Electron mass: $m_{\text{e}} = 9.11 \times 10^{-31}$ kg

Proton mass: $m_{\text{p}} = 1.67 \times 10^{-27}$ kg

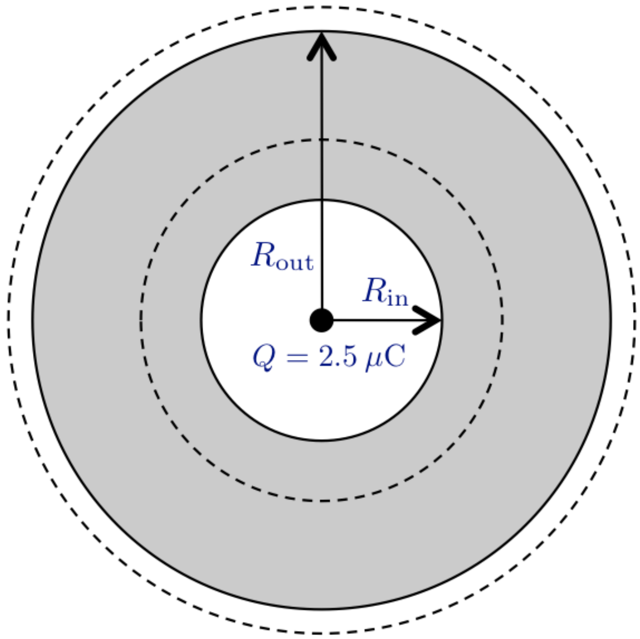
1. Two equal and opposite charges of $q_1 = +e = +1.6 \times 10^{-19}$ C and $q_2 = -e = -1.6 \times 10^{-19}$ C are separated by a distance of $L = 10.0$ cm.
   (a) What is the force (magnitude only, in N) on charge q_1 from charge q_2?
   (b) What is the force (magnitude only, in N) on charge q_2 from charge q_1?
   (c) What directions are the forces on each charge?
   (d) If q_1 does not move, and q_2 is an electron ($m_2 = m_e = 9.1 \times 10^{-31}$ kg), what is the acceleration of the electron (in m/s^2) from this electric force?
   (e) Assuming the electric potential at a point very far away is zero, how much energy does it take to move one of these charges from this distance from the other charge to a point very far away?

2. Three electric charges are arranged in the form of an equilateral triangle of side L =1.5cm shown below. I've also indicated the directions of the coordinate system that we're using.



   (a) What is the net force (magnitude and direction) on charge q_1 from charges q_2 and q_3, in N?
   (b) What is the net force (magnitude and direction) on charge q_3 from charges q_1 and q_2, in N?
   (c) What is the electric field (magnitude and direction) at the location of charge q_3 from charges q_1 and q_2, in N/C?
   (d) Assume the electric potential very far from the charges is zero. What is the electric potential at the center of the triangle (in V), relative to a point very far away?
   (e) (Bonus) What is the equation for the electric field from all three charges at a location $\vec{r}$, very far from the charges where $r \gg 1.5$ cm?

3. A perfectly conducting sphere of radius $R_{\text{out}} = 1.2\,\text{m}$ has a concentric hollow area inside it of radius $R_{\text{in}} = 0.5$ m. The hollowed-out sphere has zero net charge. A charge of $Q = 2.5\,\mu\text{C}$ is placed at the center of the sphere.



(a) You draw a Gaussian surface of a sphere of radius $R_1 = 0.7$ m centered on the charge; this Gaussian surface is sketched in the figure as the *smaller* of the two dotted circles. What is the flux through this Gaussian surface?

(b) Use Gauss's Law to calculate what the net charge must be on the inside surface of the conductor.

(c) You draw a Gaussian surface of a sphere of radius $R_1 = 1.3$ m centered on the charge; this Gaussian surface is sketched in the figure as the *larger* of the two dotted circles. What is the flux through this Gaussian surface?

(d) Use Gauss's Law to calculate what the net charge must be on the outside surface of the conductor.

(e) What is the electric field (magnitude and direction) at any point on the *larger* of the two dotted circles?

(f) (Bonus) Calculate the electric field on the outer surface of the conductor two ways: Using Gauss's Law with a Gaussian surface of a sphere, and $q_{\text{enclosed}} = Q$, and Using Gauss's Law wiht a Gaussian surface of a very small rectangular box very close to the surface of the sphere, where the surface looks like a plane. Are they the same?

(g) How do the answers of parts (a-d) change if another charge of $Q = 2.5\,\mu\text{C}$ is placed 1.5 m away from the charge at the center of the sphere? Why?