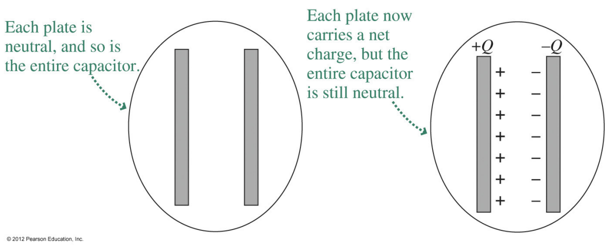

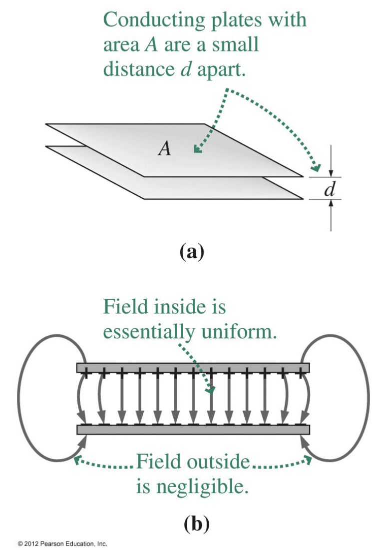

University Physics 227N/232N
Old Dominion University

Chapter 23, Capacitors

Lab deferred to Fri Feb 28

Happy Valentine's Day!

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Friday, February 14 2014

Happy Birthday to Tyler Clippard, Simon Pegg, Teller (of Penn and Teller), Terry Gross, and Charles Thomson Rees Wilson (Nobel Prize 1959)!

Review: Chapter 22: Electric Potential

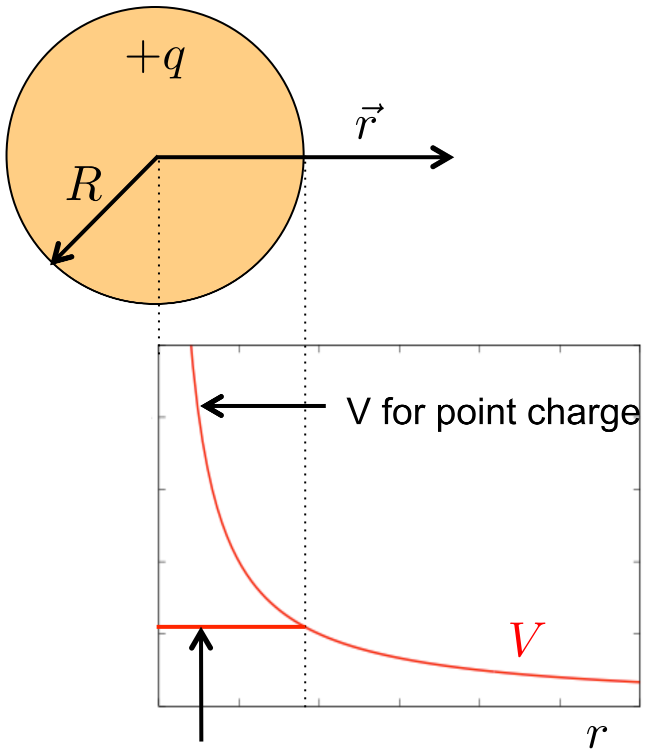
- **Electric potential difference** describes the work per unit charge involved in moving charge between two points in an electric field:

$$\Delta U_{\rm AB} = q\Delta V_{\rm AB} \qquad \Delta V_{\rm AB} = -\int_{\rm A}^{\rm B} \vec{E} \cdot d\vec{r}$$

  - The SI unit of electric potential is the volt (V), equal to 1 J/C.
  - Electric potential *always* involves two points;
    - To say "the potential at a point" is to assume a second reference point at which the potential is defined to be zero.
  - Electric potential differences of a point charge

$$V_r = \frac{kq}{r}$$

    - where the "other point" of potential is taken to be zero at infinity.



V *inside* spherical *conductor*: constant!

Electrostatic Energy

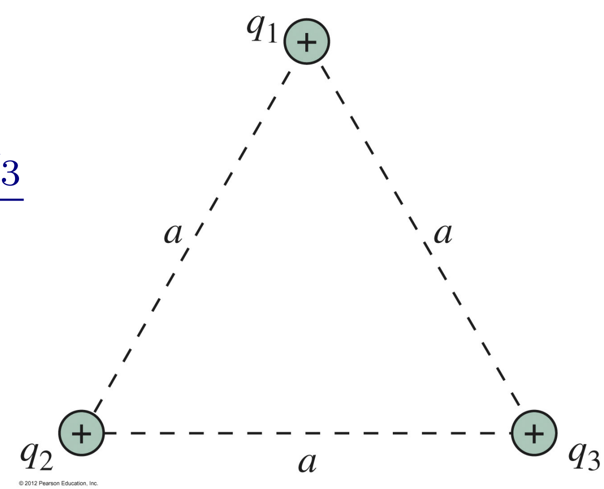
- So it takes work (energy) to assemble a distribution of electric charges.
  - This is **electrostatic energy** of the new configuration of charges.
    - If we put energy in, we effectively **store it** in the distribution of charges.
  - Each charge pair q_i, q_j contributes energy where r_{ij} is the distance between the charges in the final configuration.

$$U_{\mathrm{ij}} = \frac{k q_i q_j}{r_{ij}^2}$$

$$U_{\mathrm{total}} = \sum_{ij} U_{ij}$$

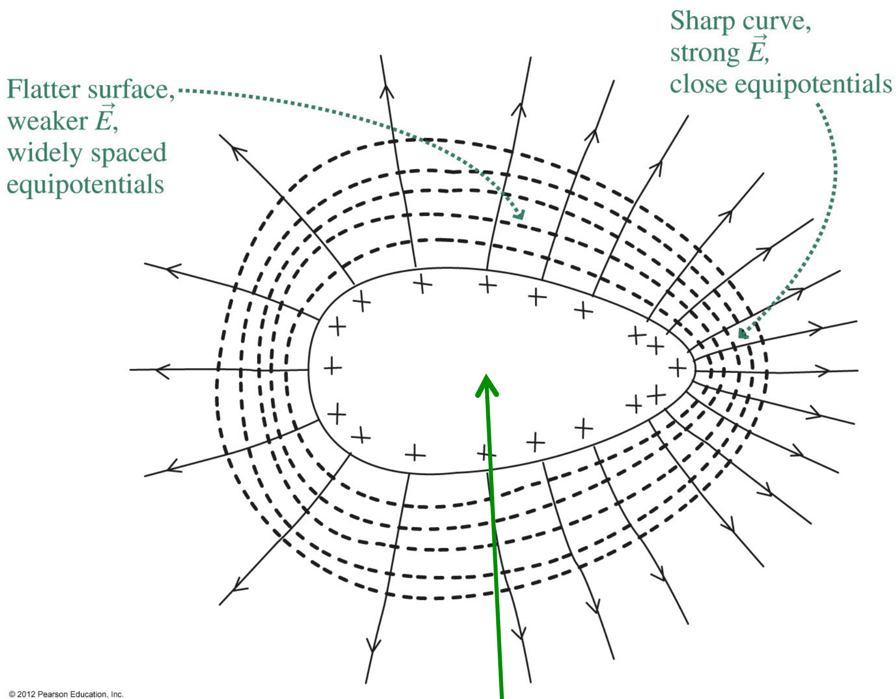
- Example: Three point charges assembled to form an equilateral triangle:

$$U_{\mathrm{electrostatic}} = \frac{k q_1 q_2}{a} + \frac{k q_1 q_3}{a} + \frac{k q_2 q_3}{a}$$



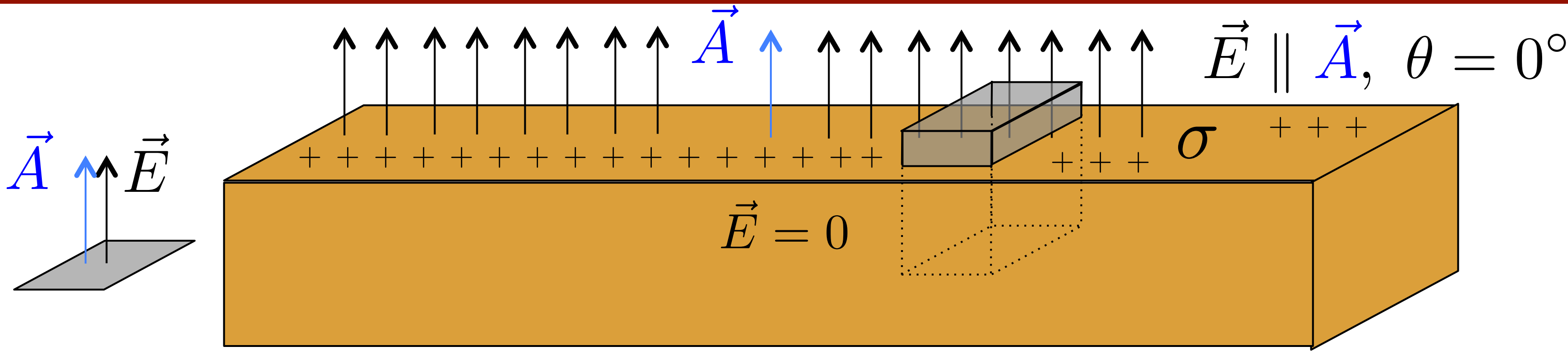
Review: Chapter 22: Equipotentials and Conductors

- **Equipotentials** are surfaces of constant electric potential.
  - It takes no work (energy) to move charges between points of equal electric potential.
  - A perfect conductor in equilibrium is an equipotential in the conductor
    - So we can move charges in perfect conductors while doing no work



Equipotential *inside* conductor

Review: Electric Field on Flat Surface of Conductor



- We had used Gauss's law to figure out

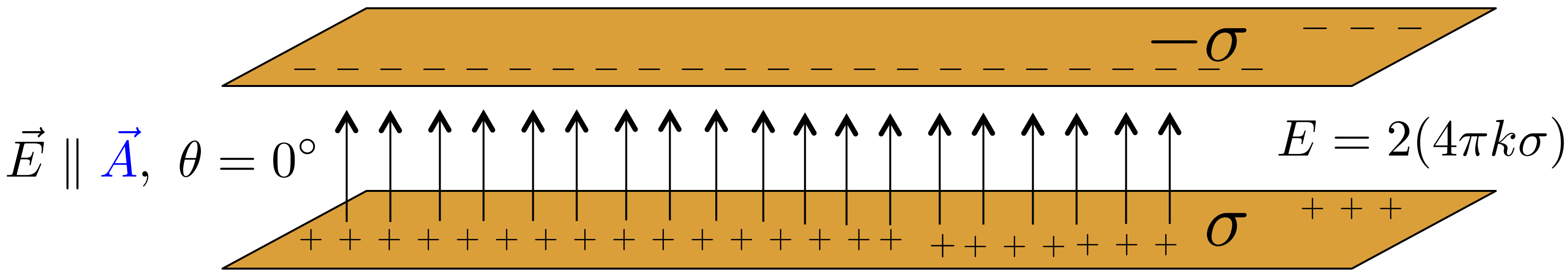
$$E = 4\pi\, k\, \sigma$$

Independent of A or distance from the (infinite flat) conductor!!

σ: charge per unit area of the surface (total charge divided by total area)

- What about two flat conductors with equal and opposite charges?

Two Flat Conductors



- What about two flat conductors (plates) with equal and opposite charges?
  - We can use this to store electrostatic energy
    - Lots of charges are separated by a small distance

$$U_{\rm ij} = \frac{kq_iq_j}{r_{ij}^2}$$

    - The electric potentials V of the two flat conductors are equal and opposite
  - If we connect the two potentials by a conductor, then charges will move (current will flow) until the plates and conductor are all at the same potential
    - We've created an electrostatic energy storage device: a battery

- A **capacitor** is a pair of conductors, insulated from each other, and used to **store charge and energy**.
  - The two conductors are given equal but opposite charges.
  - The work used in separating charge is stored as electrostatic energy in the capacitor.

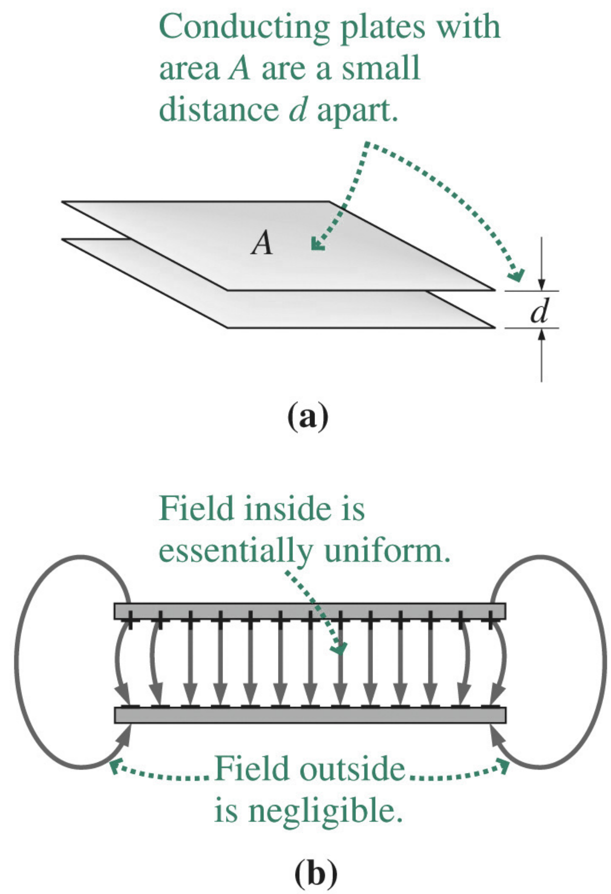
– **Capacitance** is the charge stored per unit potential difference: $C \equiv Q/V$ $\quad Q = CV$

- Its SI unit is the **farad** (F):

  1 F = 1 Coulomb/Volt $\quad (1\ \mu\text{F} = 1\ \mu\text{C}/V)$

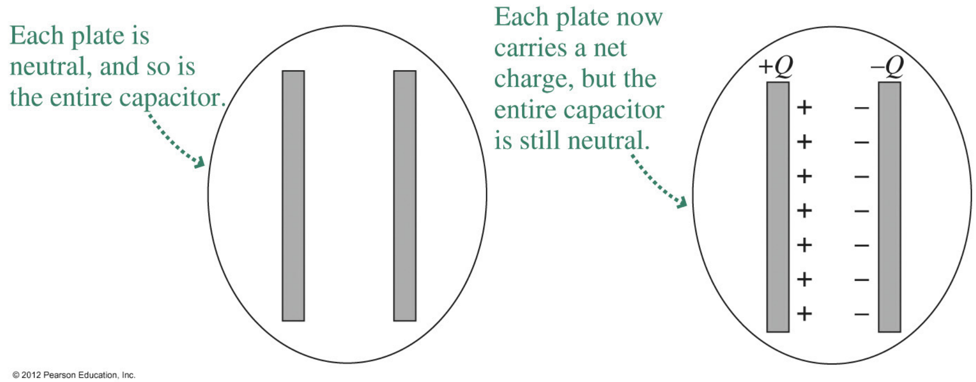
- The capacitance of a **parallel-plate capacitor** is

$$C_{\text{parallel plates}} = \frac{\epsilon_0 A}{d} = \frac{A}{4\pi k d}$$



Energy Stored in a Capacitor

- Charging a capacitor involves transferring charge between the initially neutral plates.
  - The whole capacitor remains neutral, but the individual plates become charged.



  - The work dW involved in moving charge dQ is $dW = V\,dQ$, where V is the potential difference between the plates.
    - For a capacitor, $Q = CV$, so $dW = CV\,dV$.
  - Then the work involved in building up a potential difference V between the plates is

$$W = \int dW = \int_0^V (C\,V)\,dV = \frac{1}{2}CV^2$$

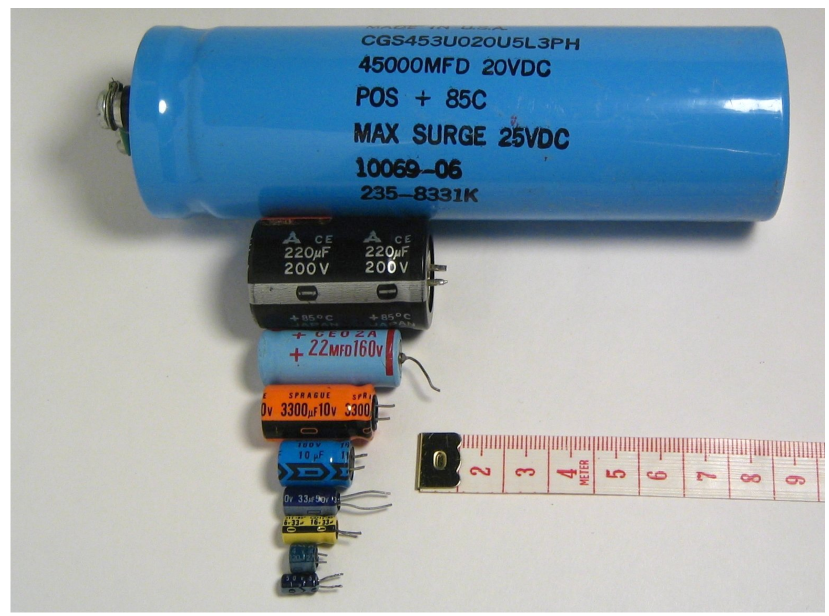
  - Therefore the electrostatic energy U stored in a capacitor is

$$U_{\text{stored in capacitor}} = \frac{1}{2}CV^2$$

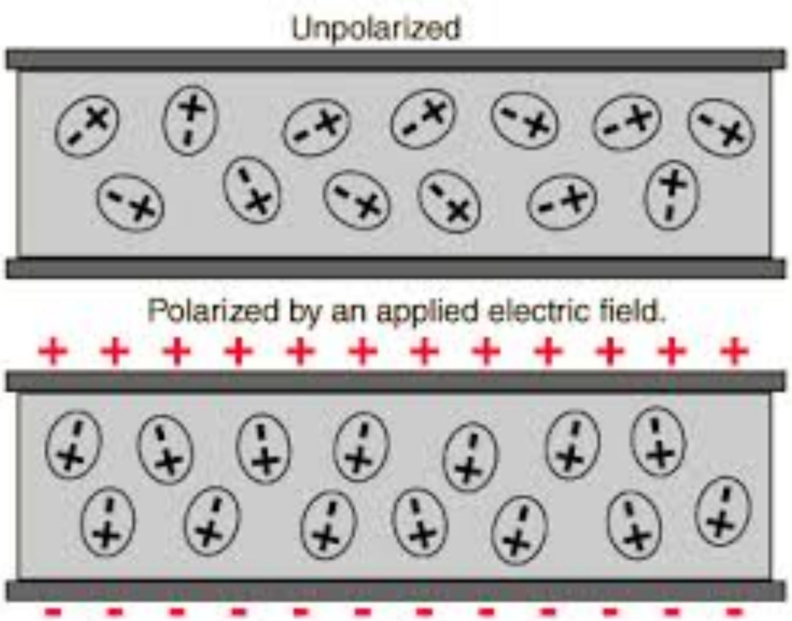
Practical Capacitors

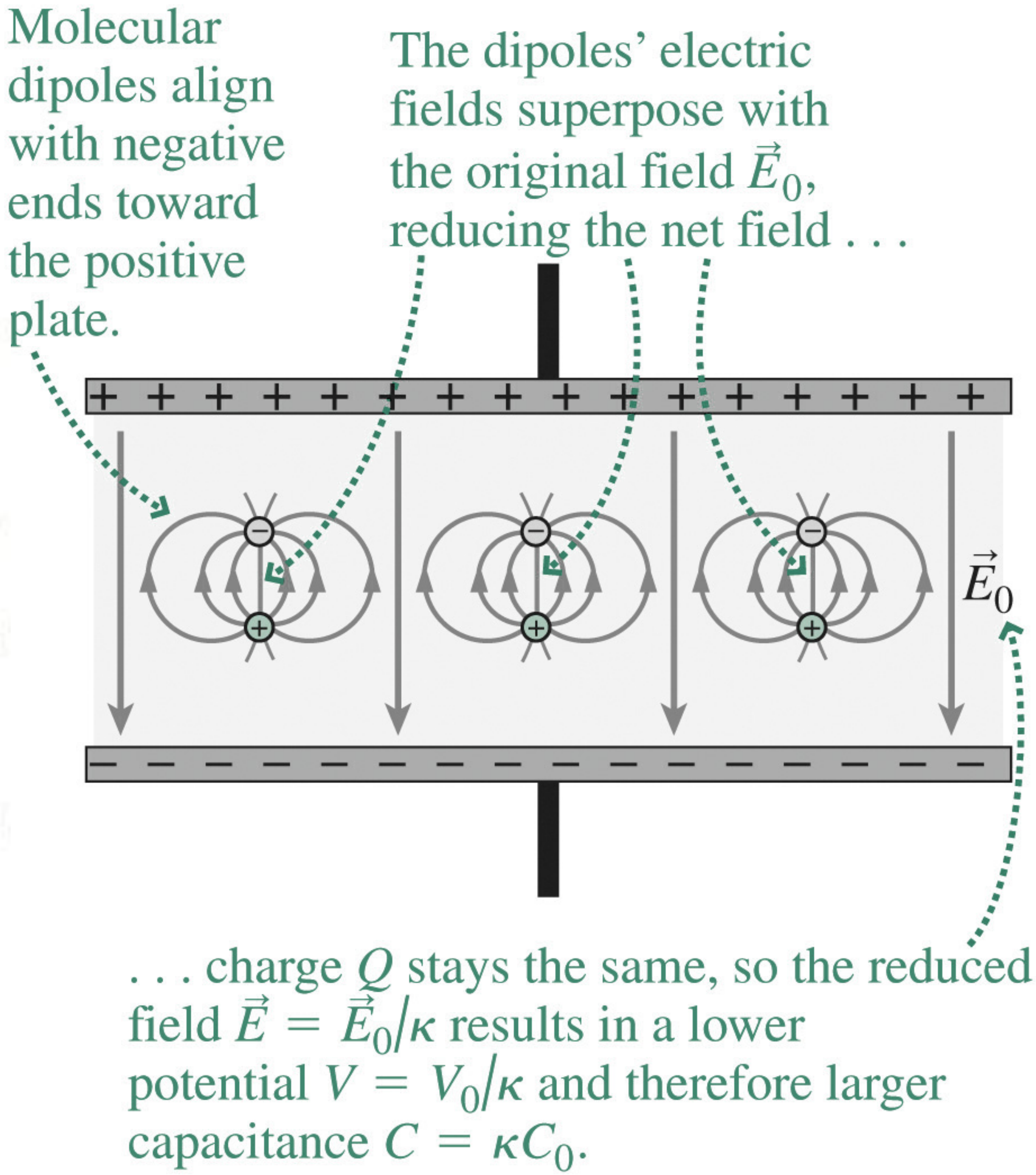
- Capacitors are manufactured using a variety of technologies, in capacitances ranging from picofarads (pF; 10^{-12} F) to several farads.
  - Most use a dielectric material (electric dipoles) between their plates.
  - The dielectric increases capacitance *C* by lowering the electric field and thus the potential difference required for a given charge on the capacitor.

Typical capacitors



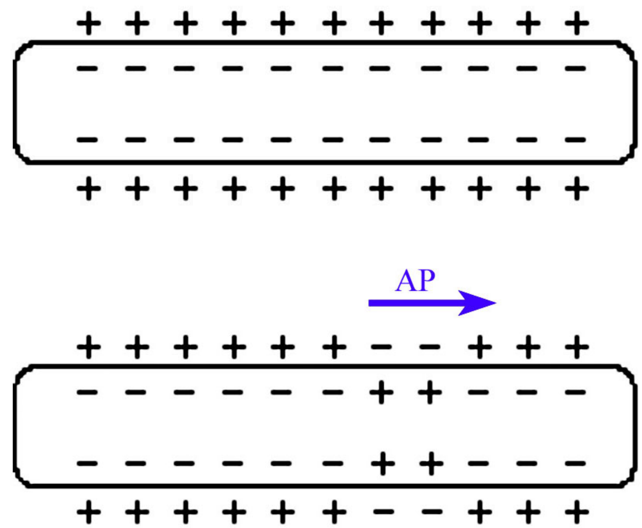
Microscopic View of a Dielectric

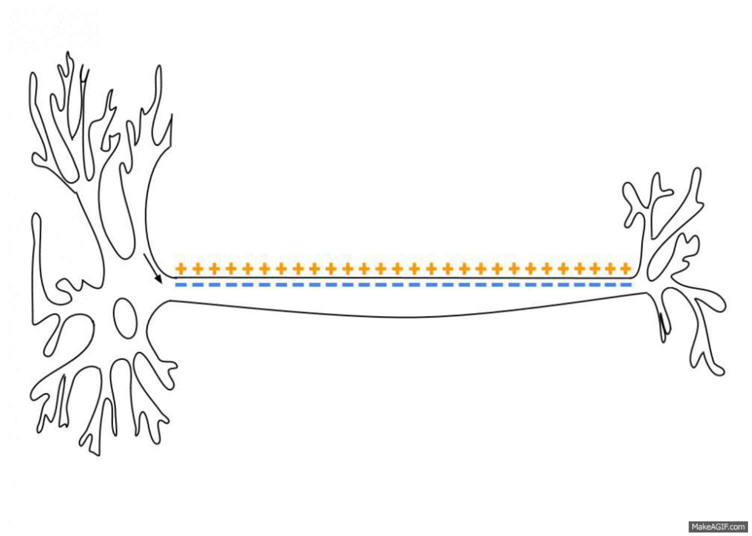




(Capacitance of Cell Membranes)

- The cell membranes of nerve cells are also capacitors
  - Nerve cells send signals by "discharging" this stored energy down the cell membrane
  - This results in a moving change of potential along the cell
    - "Action Potential"
  - Capacitance is about 7 mF/m^2
  - Dielectric constant $\kappa \approx 2$





Dielectric Constants

- The **dielectric constant**, κ, is a property of the dielectric material that gives the reduction in field and thus the increase in capacitance.
  - For a parallel-plate capacitor with a dielectric between its plates, the capacitance is

$$C = \kappa \frac{\epsilon_0 A}{d} = \kappa C_0 \qquad C_0 = \frac{\epsilon_0 A}{d} \qquad \kappa >= 1$$

Table 23.1 Properties of Some Common Dielectrics

Dielectric Material	Dielectric Constant	Breakdown Field (MV/m)
Air	1.0006	3
Aluminum oxide	8.4	670
Glass (Pyrex)	5.6	14
Paper	3.5	14
Plexiglas	3.4	40
Polyethylene	2.3	50
Polystyrene	2.6	25
Quartz	3.8	8
Tantalum oxide	26	500
Teflon	2.1	60
Water	80	depends on time and purity

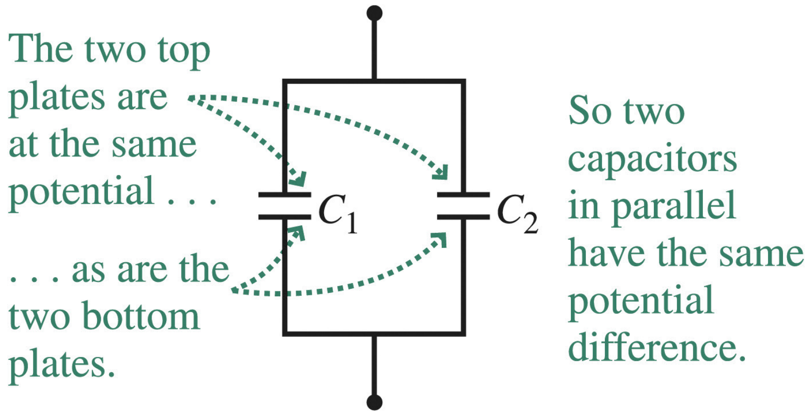
Titanium dioxide
κ=100!

Connecting Capacitors in Parallel

- Capacitors connected in **parallel** have their top plates connected together and their bottom plates connected together.
  - Therefore the potential difference across the two capacitors is the same.
  - The capacitance of the combination is the sum of the capacitances:

$$C_{\text{parallel}} = C_1 + C_2 + C_3 + \ldots$$

  - The maximum safe **working voltage** of the combination is that of the capacitor with the lowest voltage rating.

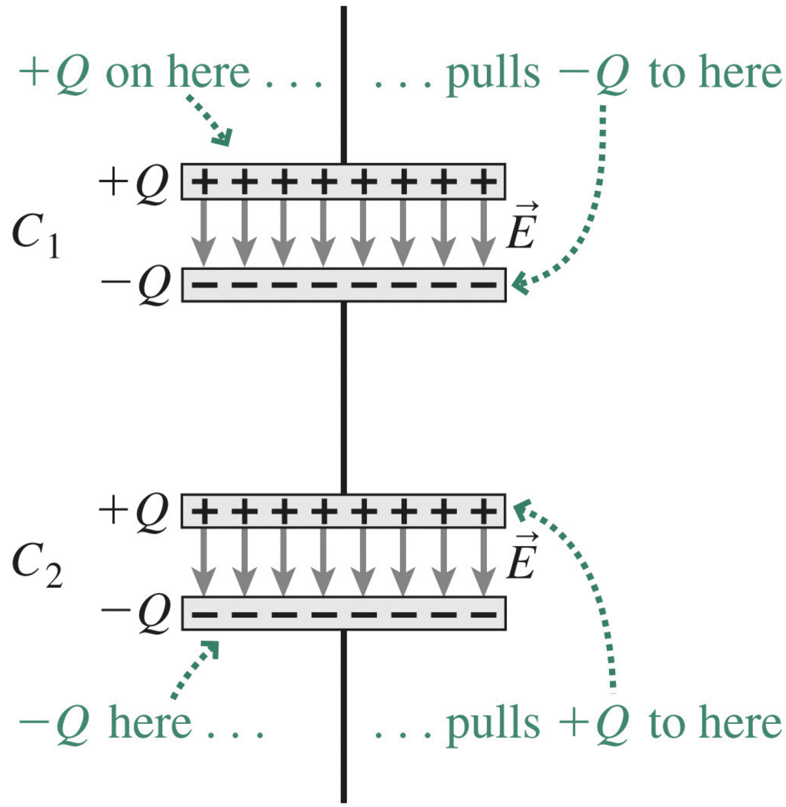


Connecting Capacitors in Series

- Capacitors connected in **series** are wired so that one capacitor follows the other.
  - The figure shows that this makes the charge on the two capacitors the same.
  - With series capacitors, capacitance adds reciprocally:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

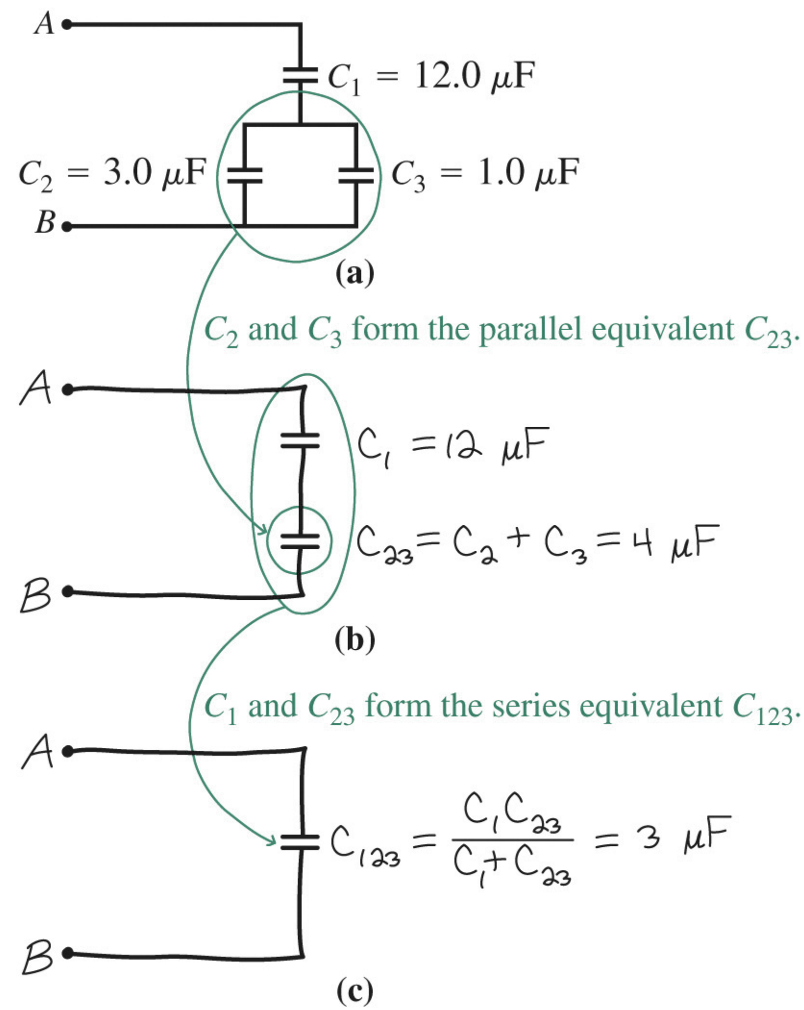
Thus the combined capacitance is lower than that of any individual capacitor.



  - The working voltage of the combination is higher than that of any individual capacitor.

Circuits with Parallel and Series Capacitors

- To analyze a circuit with several capacitors, look for series and parallel combinations.
  - Calculate the equivalent capacitances, and redraw the circuit in simpler form.
  - This technique will work later for more general electric circuits.



Energy in the Electric Field

- The electrostatic energy associated with a charge distribution is stored in the electric field of the charge distribution.
  - Considering the uniform field of the parallel-plate capacitor shows that the electric **energy density** is

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$

- This is a universal result:
  - *Every* electric field contains energy with this density.
  - Example: Shrinking a sphere of charge requires work, which ends up as stored electric energy.

$$U = \int u_E \, dV = \tfrac{1}{2}\varepsilon_0 \int E^2 \, dV = \int_{R_2}^{R_1} \left(\frac{kQ}{r^2}\right)^2 4\pi r^2 \, dr = \frac{kQ^2}{2}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$