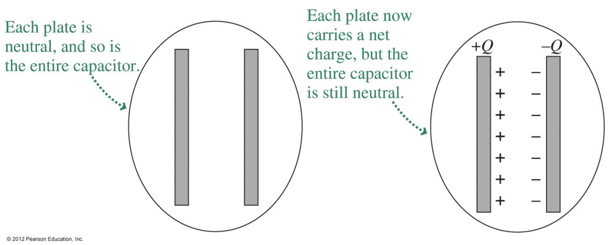
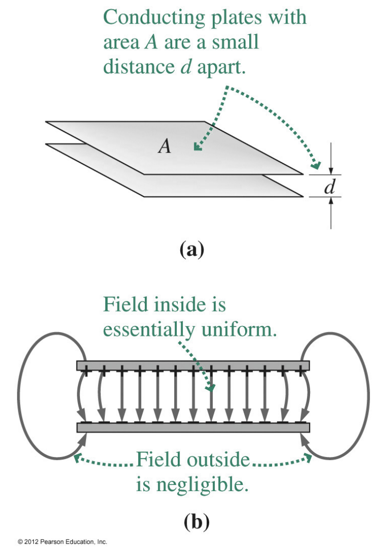




University Physics 227N/232N
Old Dominion University

(More) Chapter 23, Capacitors

Lab deferred to Fri Feb 28

Exam Solutions will be posted Tuesday PM

QUIZ this Fri (Feb 21), Fred lectures Mon (Feb 24)

Dr. Todd Satogata (ODU/Jefferson Lab)

satogata@jlab.org

http://www.toddsatogata.net/2014-ODU

Monday, February 17 2014

Happy Birthday to Bonnie Wright, Ed Sheeran, Paris Hilton, Joseph Gordon-Levitt, Michael Jordan, and Otto Stern (Nobel Prize 1943)!

Review: Chapter 22: Electric Potential

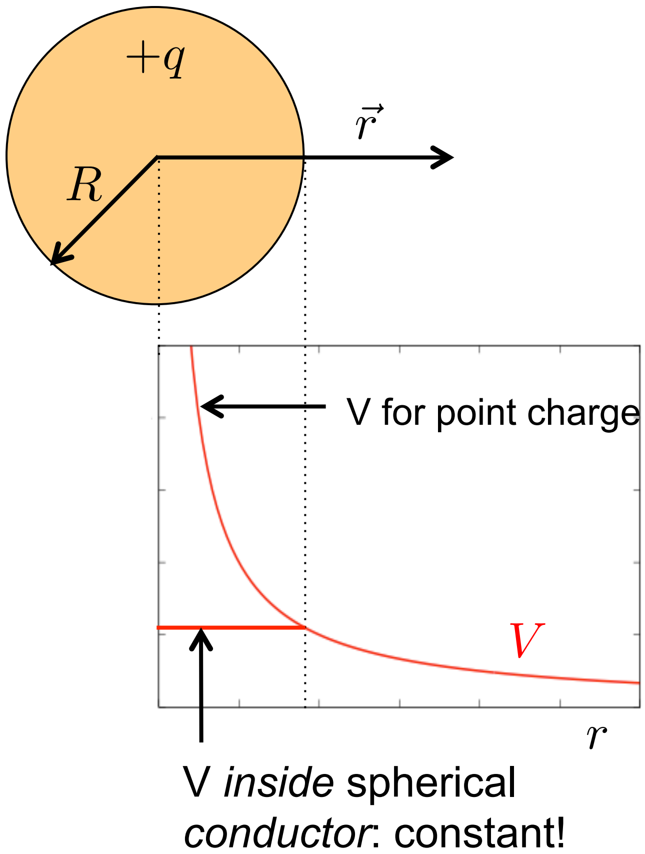
- **Electric potential difference** describes the work per unit charge involved in moving charge between two points in an electric field:

$$\Delta U_{\mathrm{AB}} = q\Delta V_{\mathrm{AB}} \qquad \Delta V_{\mathrm{AB}} = -\int_{\mathrm{A}}^{\mathrm{B}} \vec{E} \cdot d\vec{r}$$

  - The SI unit of electric potential is the volt (V), equal to 1 J/C.
  - Electric potential *always* involves two points;
    - To say "the potential at a point" is to assume a second reference point at which the potential is defined to be zero.
  - Electric potential differences of a point charge

$$V_r = \frac{kq}{r}$$

    - where the "other point" of potential is taken to be zero at infinity.



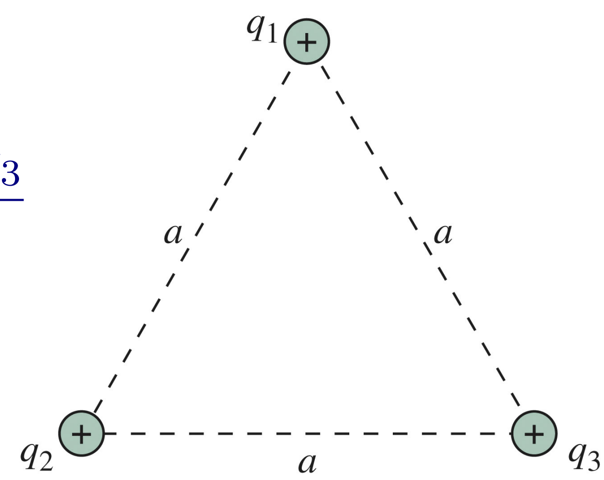
Electrostatic Energy

- So it takes work (energy) to assemble a distribution of electric charges.
  - This is **electrostatic energy** of the new configuration of charges.
    - If we put energy in, we effectively **store it** in the distribution of charges.
  - Each charge pair q_i, q_j contributes energy where r_{ij} is the distance between the charges in the final configuration.

$$U_{\mathrm{ij}} = \frac{k q_i q_j}{r_{ij}^2}$$

$$U_{\mathrm{total}} = \sum_{ij} U_{ij}$$

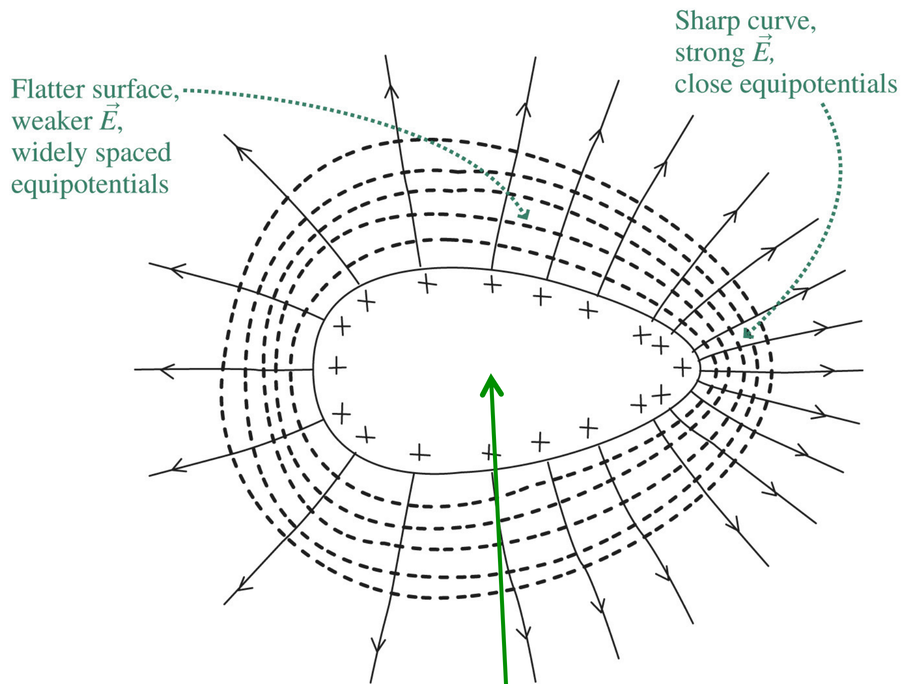
- Example: Three point charges assembled to form an equilateral triangle:



$$U_{\mathrm{electrostatic}} = \frac{k q_1 q_2}{a} + \frac{k q_1 q_3}{a} + \frac{k q_2 q_3}{a}$$

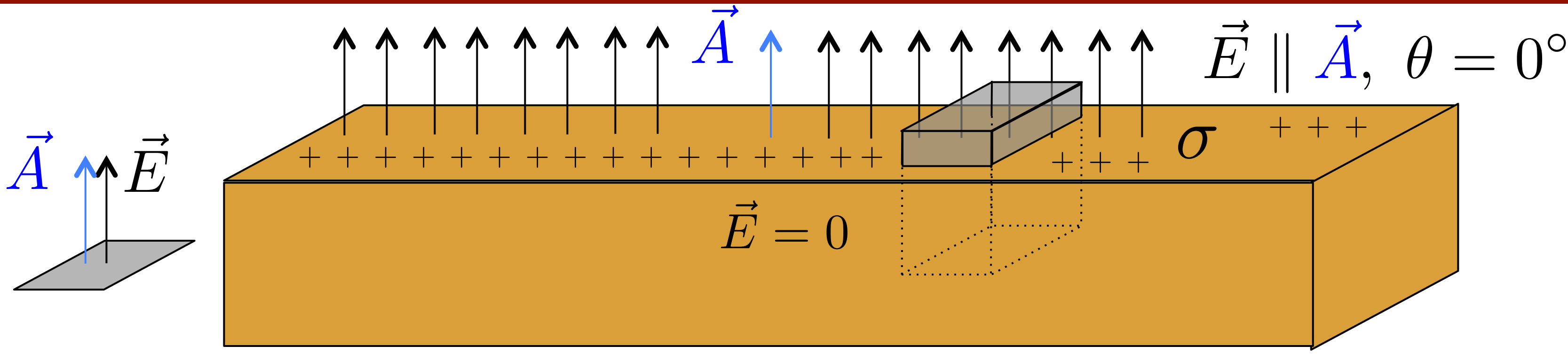
Review: Chapter 22: Equipotentials and Conductors

- **Equipotentials** are surfaces of constant electric potential.
  - It takes no work (energy) to move charges between points of equal electric potential.
  - A perfect conductor in equilibrium is an equipotential in the conductor
    - So we can move charges in perfect conductors while doing no work



Equipotential *inside* conductor

Review: Electric Field on Flat Surface of Conductor



- We had used Gauss's law to figure out

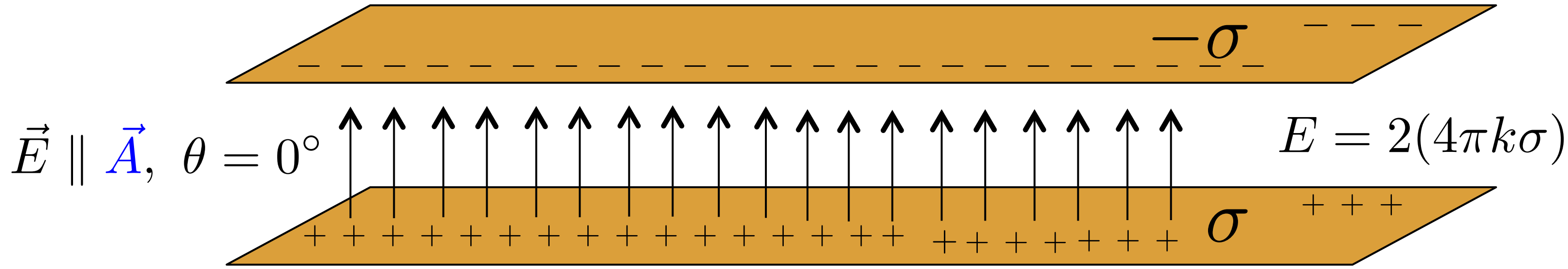
$$E = 4\pi\, k\, \sigma$$

Independent of A or distance from the (infinite flat) conductor!!

σ: charge per unit area of the surface (total charge divided by total area)

- What about two flat conductors with equal and opposite charges?

Two Flat Conductors



- What about two flat conductors (plates) with equal and opposite charges?
  - We can use this to store electrostatic energy
    - Lots of charges are separated by a small distance

$$U_{\text{ij}} = \frac{kq_iq_j}{r_{ij}^2}$$

    - The electric potentials V of the two flat conductors are equal and opposite
  - If we connect the two potentials by a conductor, then charges will move (current will flow) until the plates and conductor are all at the same potential
    - We've created an electrostatic energy storage device: a battery

Chapter 23: Capacitors

- A **capacitor** is a pair of conductors, insulated from each other, and used to **store charge and energy**.
  - The two conductors are given equal but opposite charges.
  - The work used in separating charge is stored as electrostatic energy in the capacitor.

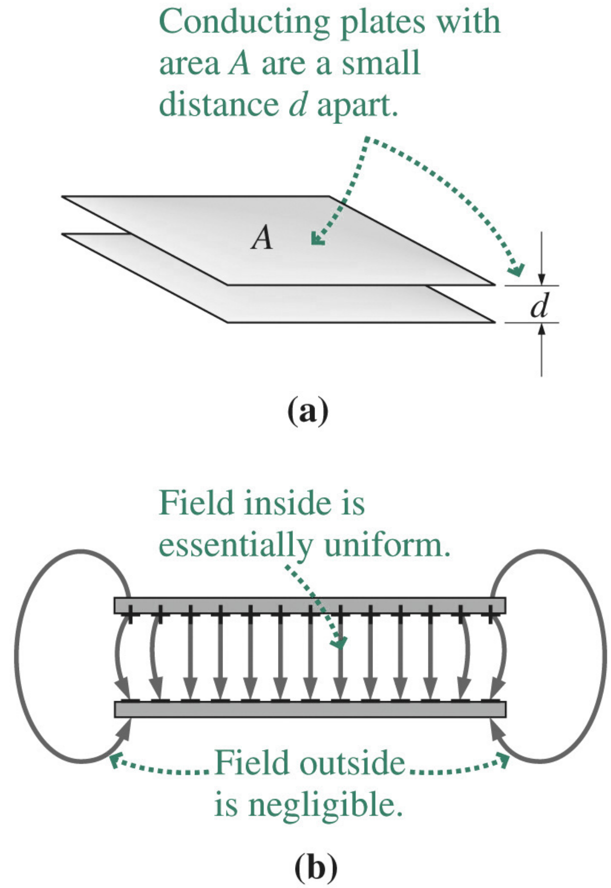
– **Capacitance** is the charge stored per unit potential difference: $C \equiv Q/V$ $\quad Q = CV$

- Its SI unit is the **farad** (F):

  1 F = 1 Coulomb/Volt $\quad (1\ \mu\text{F} = 1\ \mu\text{C}/V)$

- The capacitance of a **vacuum parallel-plate capacitor** is

$$C_{\text{parallel plates}} = \frac{\epsilon_0 A}{d} = \frac{A}{4\pi k d}$$



Voltage and Potential in Circuits

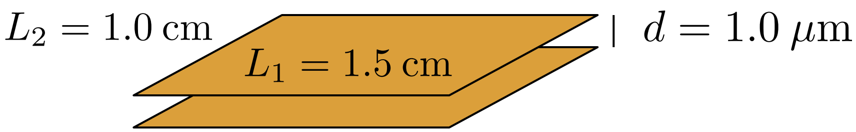
$$C \equiv Q/V$$

- You have been taught about electric potential, and learned how to calculate it for point charges etc.
- We are moving into the realm of **electric circuits**
  - All sorts of elements connected by (good) conducting wires
  - Good conductors in equilibrium are equipotentials
    - That's why the "A conductor is an equipotential" thing is important
  - So whenever you see a wire in an electrical circuit, think
    - Same potential V everywhere on the wire
  - These potentials are created by many charges moving around in the circuits
    - So don't get confused and try to calculate this V using the equation for potential from point charges
    - We'll learn about electric currents possibly later this week

Capacitor Example

$$C \equiv Q/V \qquad Q = CV$$

$$C_{\text{parallel plates}} = \frac{\epsilon_0 A}{d} = \frac{A}{4\pi k d}$$

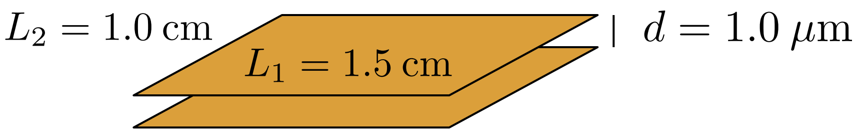


- A (vacuum) capacitor is made of two (parallel) plates of sides 1.5 cm and 1.0 cm separated by 1.0 μm.
  - What is its capacitance?
  - If it is rated at 1 kV, how much charge can it store?

Capacitor Example

$$\boxed{C \equiv Q/V} \qquad Q = CV$$

$$\boxed{C_{\text{parallel plates}} = \frac{\epsilon_0 A}{d} = \frac{A}{4\pi k d}}$$



- A (vacuum) capacitor is made of two (parallel) plates of sides 1.5 cm and 1.0 cm separated by 1.0 μm.
  - What is its capacitance?
  - If it is rated at 1 kV, how much charge can it store?

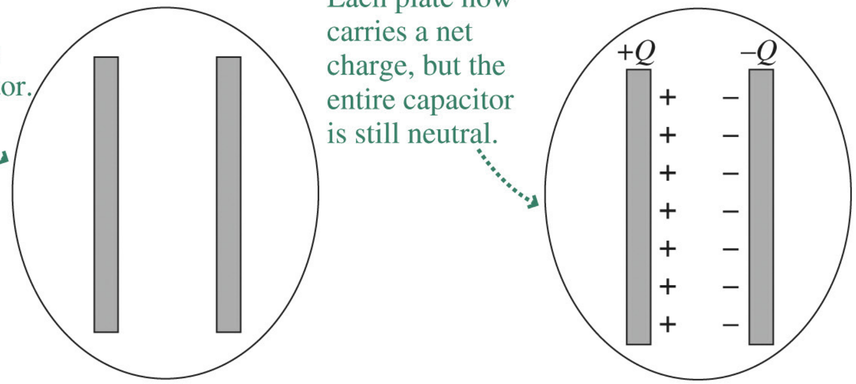
$$C = \frac{A}{4\pi k d} = \frac{1.5\times 10^{-4}\ \text{m}^2}{4\pi(9\times 10^9\ \text{N m}^2/\text{C}^2)(10^{-6}\ \text{m})} = \boxed{1.3\ \text{nF} = C}$$

$$Q = CV = (1.3\times 10^{-9}\ \text{F})(10^3\ \text{V}) = \boxed{1.3\ \mu\text{C} = Q}$$

- If we wanted to raise the capacitance, we would need to *increase* the surface area *A* or *decrease* the separation *d*
  - Or change the material between the plates

Work Done Charging a Capacitor

- Charging a capacitor involves transferring charge between the initially neutral plates.
  - The whole capacitor remains **neutral**, but the individual plates become **charged**.



  - The work ΔW involved in moving the charge by ΔQ over a (fairly constant) voltage V is

$$W = QV \qquad \Rightarrow \qquad \Delta W = V \Delta Q$$

  - For a capacitor, $Q=CV$ so $\Delta Q=C\Delta V$

$$\Delta W = V\Delta Q = CV\Delta V$$

$$W = \sum CV\Delta V = \int CV\, dV = C\int V\, dV = \boxed{\frac{1}{2}CV^2 = W}$$

Energy Stored in a Capacitor

- Work is energy!
- So capacitors can store energy
  - This is really stored in the electric field built up between the charges
  - This is stored energy, so it's potential energy *U*

$$U_{\text{stored in capacitor}} = \frac{1}{2} C V^2 \qquad C \equiv Q/V$$

- This is very much like a spring storing mechanical energy

$$U_{\text{stored in spring}} = \frac{1}{2} k x^2$$

k is a spring constant here!!!

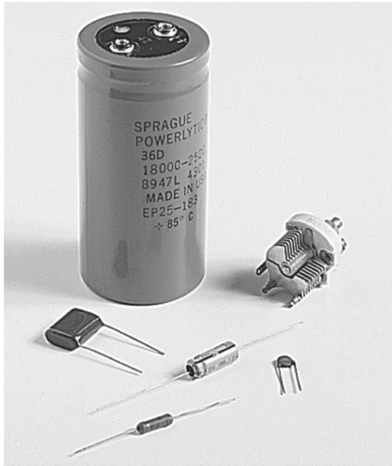
- We can get other relationships using definition of capacitance *C*

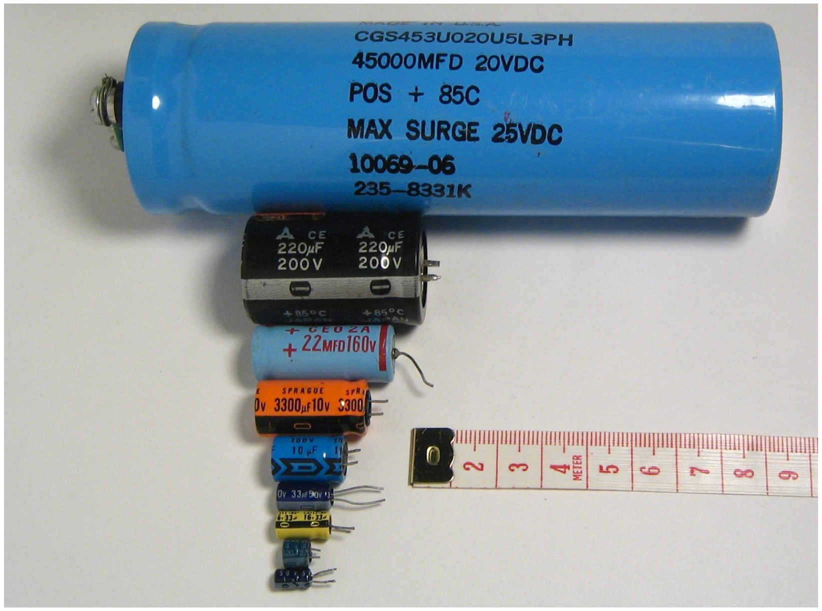
$$U_{\text{stored in capacitor}} = \frac{1}{2} C V^2 = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C}$$

Review: Practical Capacitors

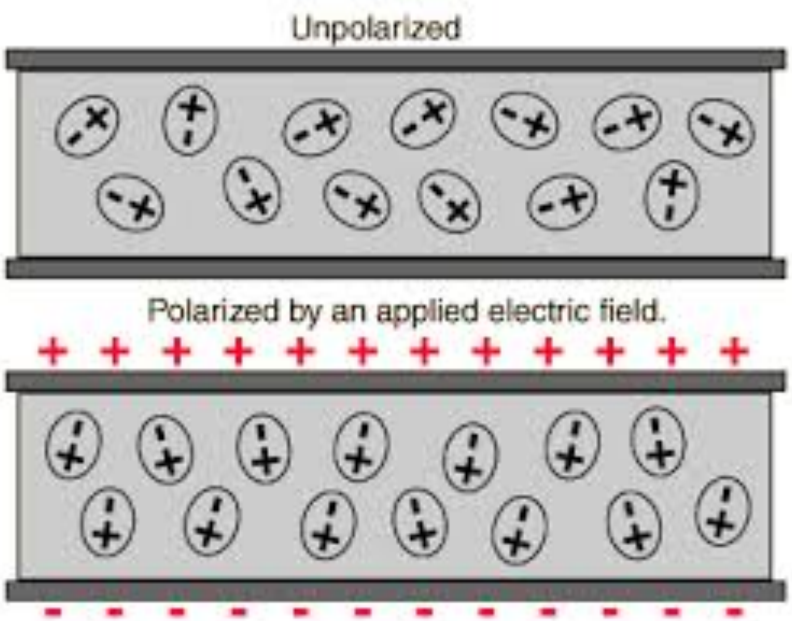
- Capacitors are manufactured using a variety of technologies, in capacitances ranging from picofarads (pF; 10^{-12} F) to several farads.
  - Most use a dielectric material (electric dipoles) between their plates.
  - The dielectric increases capacitance *C* by lowering the electric field and thus the potential difference required for a given charge on the capacitor.

Typical capacitors



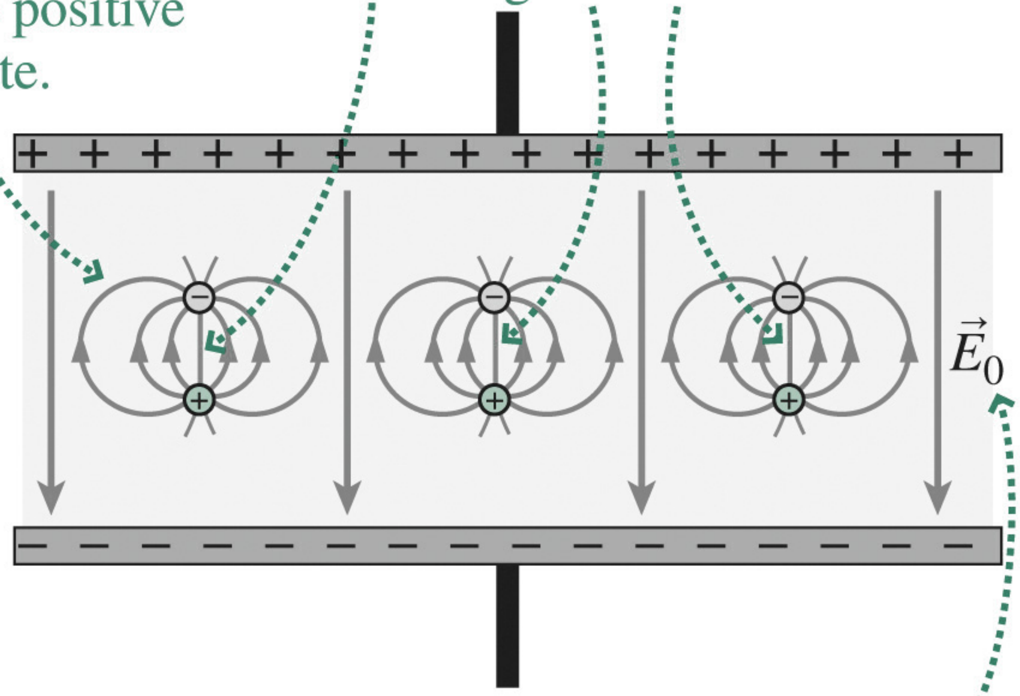


Review: Microscopic View of a Dielectric



Molecular dipoles align with negative ends toward the positive plate.

The dipoles' electric fields superpose with the original field $\vec{E}_0$, reducing the net field . . .



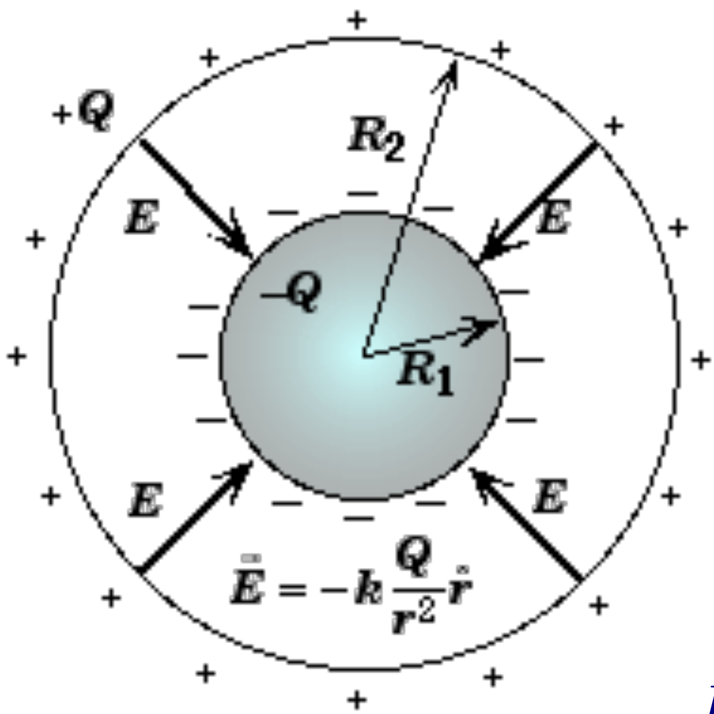
. . . charge Q stays the same, so the reduced field $\vec{E} = \vec{E}_0/\kappa$ results in a lower potential $V = V_0/\kappa$ and therefore larger capacitance $C = \kappa C_0$.

So What Limits Capacitance?

- Why use a dielectric in a capacitor?
- What process limits how much I can charge a capacitor?
  - Recall our Van de Graaff generators and lightning bolts
  - Large voltage (electric potential differences) can lead to atomic-level breakdown and ionization
    - Electrons can move, conductance goes up: lightning bolts!

(They're Not Really Lightning Bolts, Are They?)

- **Yes**, they are. The earth is a **spherical capacitor**.
- The earth and upper atmosphere (ionosphere) are two pretty good conductors for these types of voltages



- We know enough to calculate the earth's capacitance
- Assume the ionisphere is very far away (so $R_2 >> R_1$)

$$C = \frac{Q}{V} = \frac{Q}{kQ\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{R_1 R_2}{k(R_2 - R_1)}$$

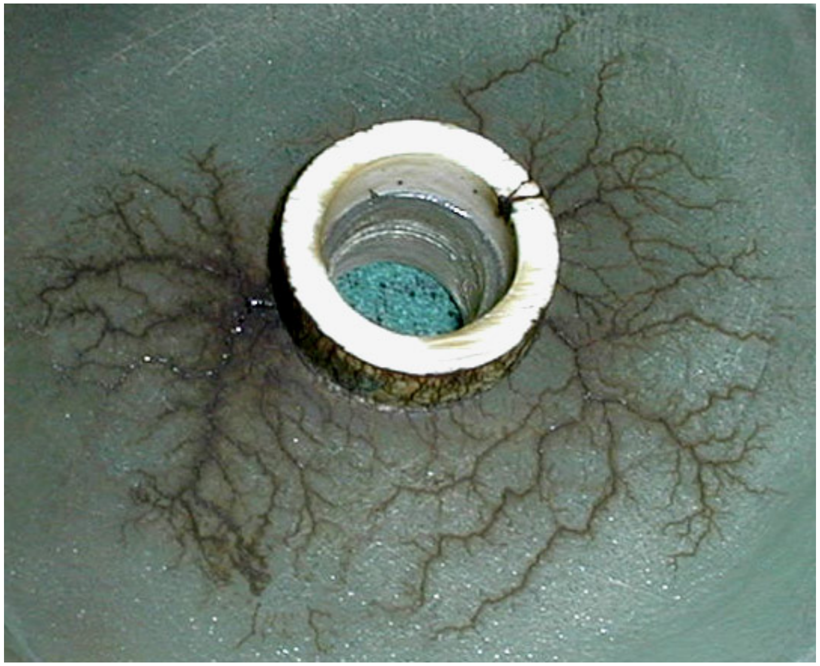
$$C \approx \frac{R_1 R_2}{k R_2} \approx \frac{R_{\mathrm{Earth}}}{k} = \frac{6.378 \times 10^6 \mathrm{~m}}{9 \times 10^9 \mathrm{~N~m^2/C^2}} \approx 700~\mu\mathrm{F}$$

What Happens During Dielectric Breakdown?

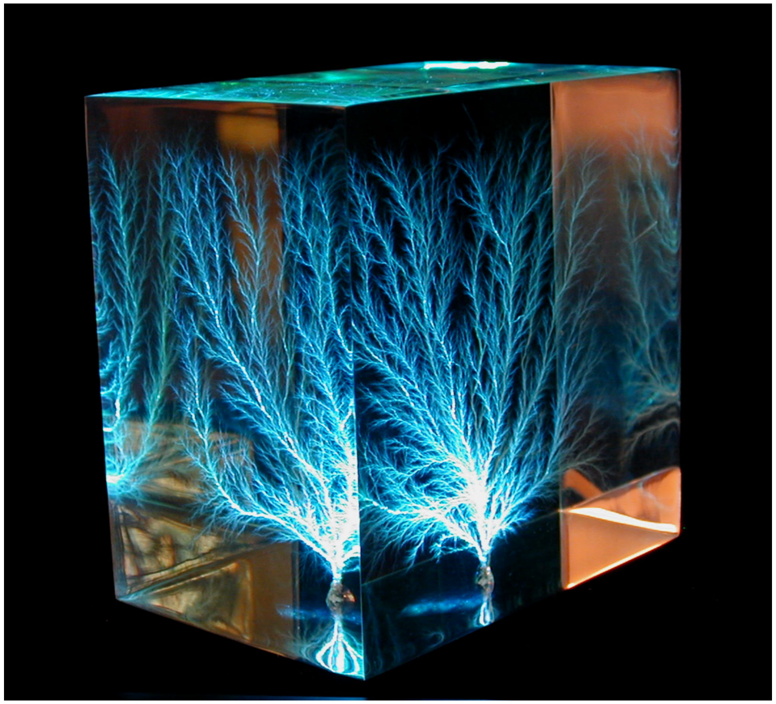
http://www.youtube.com/watch?v=dukkO7c2eUE

Dielectric Breakdown: Lichtenberg Figures

- These dielectric breakdowns can be very slow or very fast
  - They can be the bane of a high voltage system designer
  - They can also be used to create art



Polycarbonate insulator breakdown and conductive carbon trace development (develops over months/years)



Acrylic block irradiated by electron beam to create high voltage breakdown (develops over microseconds)

Dielectric Breakdown and Capacitors

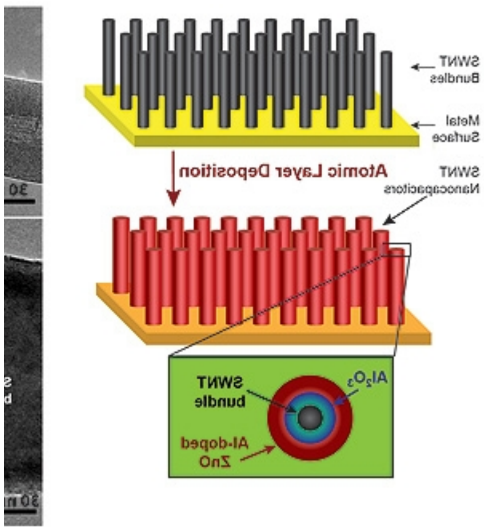
- There are various YouTube videos that show what happens when you overvoltage a capacitor
  - It's not nearly as pretty as Lichtenberg figures
  - In extreme cases, capacitors can explode
  - Or it can simply be a matter of "letting the blue smoke out"
  - http://www.youtube.com/watch?v=Ubw3cHM4YxU
  - In many cylindrical capacitors, the dielectric is an electrolyte

Nanocapacitors

- An active area of research is nanocapacitors
  - Energy storage and nanoelectronics applications

$$C_{\text{parallel plates}} = \frac{\epsilon_0 A}{d} = \frac{A}{4\pi k d}$$

  - Making a huge surface area *A* with very small distances *d* can make the capacitance of a small object *very* large



Dielectric Constants

- The **dielectric constant**, κ, is a property of the dielectric material that gives the reduction in field and thus the increase in capacitance.
  - For a parallel-plate capacitor with a dielectric between its plates, the capacitance is

$$C = \kappa \frac{\epsilon_0 A}{d} = \kappa C_0 \qquad C_0 = \frac{\epsilon_0 A}{d} \qquad \kappa >= 1$$

Table 23.1 Properties of Some Common Dielectrics

Dielectric Material	Dielectric Constant	Breakdown Field (MV/m)
Air	1.0006	3
Aluminum oxide	8.4	670
Glass (Pyrex)	5.6	14
Paper	3.5	14
Plexiglas	3.4	40
Polyethylene	2.3	50
Polystyrene	2.6	25
Quartz	3.8	8
Tantalum oxide	26	500
Teflon	2.1	60
Water	80	depends on time and purity

Titanium dioxide κ=100!

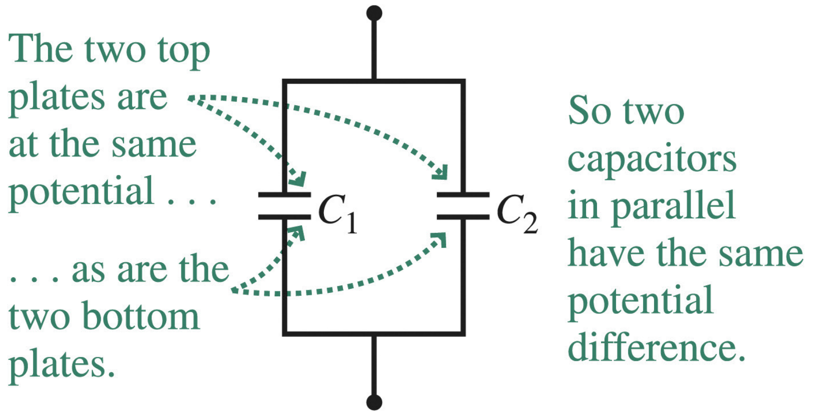
But breakdown fields
Only up to about 50 MV/m

Connecting Capacitors in Parallel

- Capacitors connected in **parallel** have their top plates connected together and their bottom plates connected together.
  - Therefore the potential difference across the two capacitors is the same.
  - The capacitance of the combination is the sum of the capacitances:

$$C_{\text{parallel}} = C_1 + C_2 + C_3 + \ldots$$

  - The maximum safe **working voltage** of the combination is that of the capacitor with the lowest voltage rating.

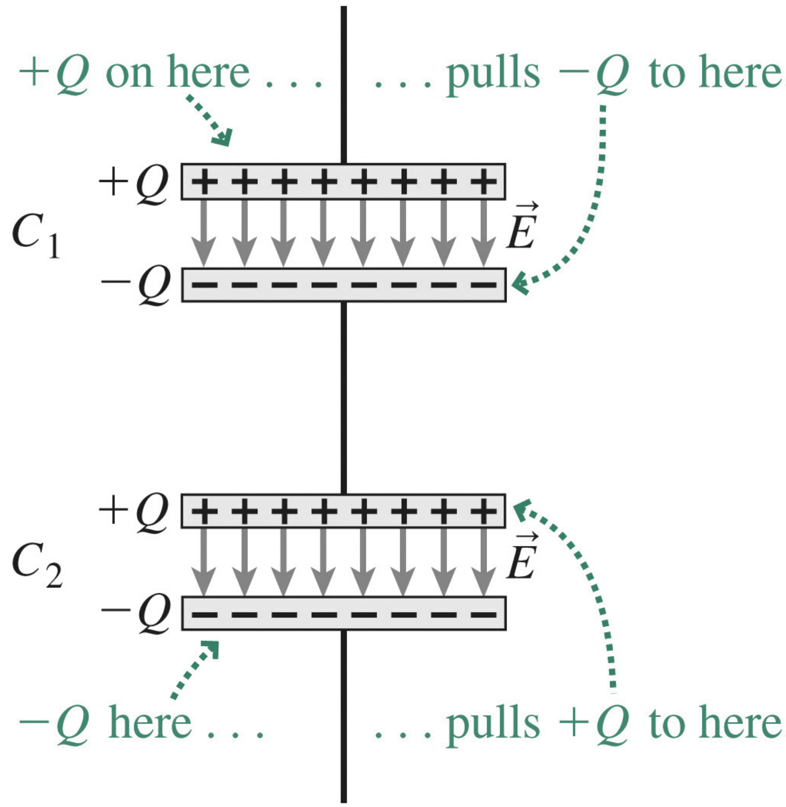


Connecting Capacitors in Series

- Capacitors connected in **series** are wired so that one capacitor follows the other.
  - The figure shows that this makes the charge on the two capacitors the same.
  - With series capacitors, capacitance adds reciprocally:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

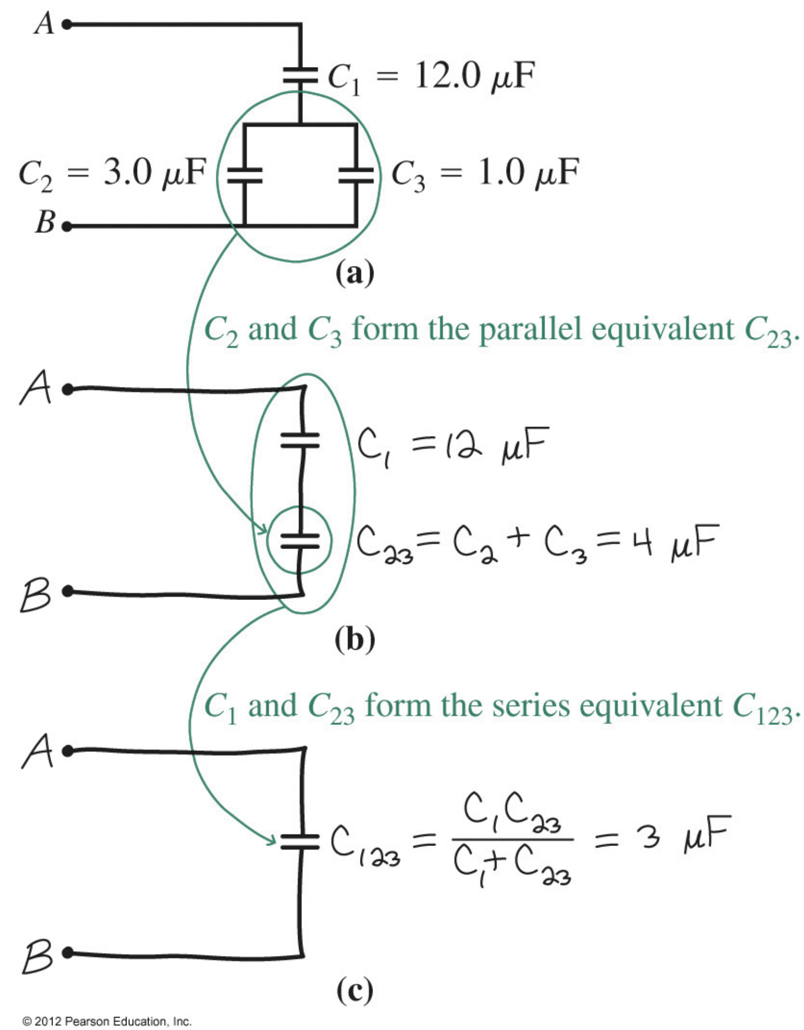
    Thus the combined capacitance is lower than that of any individual capacitor.



  - The working voltage of the combination is higher than that of any individual capacitor.

Circuits with Parallel and Series Capacitors

- To analyze a circuit with several capacitors, look for series and parallel combinations.
  - Calculate the equivalent capacitances, and redraw the circuit in simpler form.
  - This technique will work later for more general electric circuits.



Energy in the Electric Field

- The electrostatic energy associated with a charge distribution is stored in the electric field of the charge distribution.
  - Considering the uniform field of the parallel-plate capacitor shows that the electric **energy density** is

$$u_{\mathrm{E}} = \frac{1}{2}\epsilon_0 E^2$$

Energy per unit volume!

- This is a universal result:
  - *Every* electric field contains energy with this density.
  - Example: Shrinking a sphere of charge requires work, which ends up as stored electric energy.