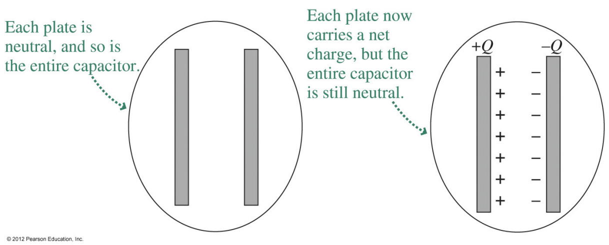

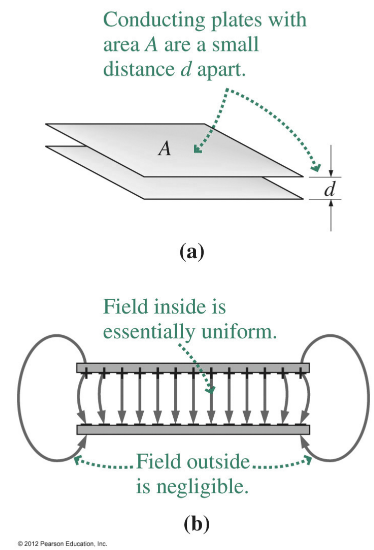

University Physics 227N/232N

Capacitors, Field Energy, Current and Ohm's Law

Lab deferred to Fri Feb 28

QUIZ this Friday! (Feb 21) Fred lectures Monday! (Feb 24)

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Wednesday, February 19 2014

Happy Birthday to Victoria Justice, Immortal Technique, Benicio del Toro, Copernicus, and David Gross (2004 Nobel Prize)

Capacitor Review

- A **capacitor** is a pair of conductors, insulated from each other, and used to **store charge and energy**.
  - The two conductors are given equal but opposite charges $\pm Q$
  - Definition of capacitance: $\boxed{C \equiv Q/V}$ $\quad Q = CV$
  - Capacitance is a **physical property** of the capacitor.
- A **parallel plate capacitor** has two parallel conductors of equal area *A* separated by distance *d*, possibly a **dielectric**

$$\boxed{C_{\text{parallel plate}} = \kappa \frac{A}{4\pi k d} = \kappa C_0} \quad C_0 = \frac{A}{4\pi k d} \quad \kappa >= 1$$

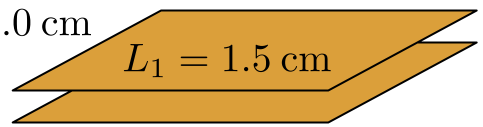
  - The dielectric constant κ for a vacuum is 1
- Energy stored in a capacitor

$$\boxed{U_{\text{stored in capacitor}} = \frac{1}{2} C V^2} \quad = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C}$$

Capacitor Example

$$C \equiv Q/V \qquad Q = CV$$

$$C_{\text{parallel plates}} = \frac{\epsilon_0 A}{d} = \frac{A}{4\pi k d}$$



- A (vacuum) capacitor is made of two (parallel) plates of sides 1.5 cm and 1.0 cm separated by 1.0 μm.
  - What is its capacitance?
  - If it is rated at 1 kV, how much charge can it store?

$$C = \frac{A}{4\pi k d} = \frac{1.5 \times 10^{-4}\ \text{m}^2}{4\pi(9 \times 10^9\ \text{N m}^2/\text{C}^2)(10^{-6}\ \text{m})} = \boxed{1.3\ \text{nF} = C}$$

$$Q = CV = (1.3 \times 10^{-9}\ \text{F})(10^3\ \text{V}) = \boxed{1.3\ \mu\text{C} = Q}$$

  - If we wanted to raise the capacitance, we would need to *increase* the surface area *A* or *decrease* the separation *d*
    - Or change the material between the plates

Dielectric Constants

- The **dielectric constant**, κ, is a property of the dielectric material that gives the reduction in field and thus the increase in capacitance.
  - For a parallel-plate capacitor with a dielectric between its plates, the capacitance is

$$C = \kappa \frac{\epsilon_0 A}{d} = \kappa C_0 \qquad C_0 = \frac{\epsilon_0 A}{d} \qquad \kappa >= 1$$

Table 23.1 Properties of Some Common Dielectrics

Dielectric Material	Dielectric Constant	Breakdown Field (MV/m)
Air	1.0006	3
Aluminum oxide	8.4	670
Glass (Pyrex)	5.6	14
Paper	3.5	14
Plexiglas	3.4	40
Polyethylene	2.3	50
Polystyrene	2.6	25
Quartz	3.8	8
Tantalum oxide	26	500
Teflon	2.1	60
Water	80	depends on time and purity

Titanium dioxide κ=100!

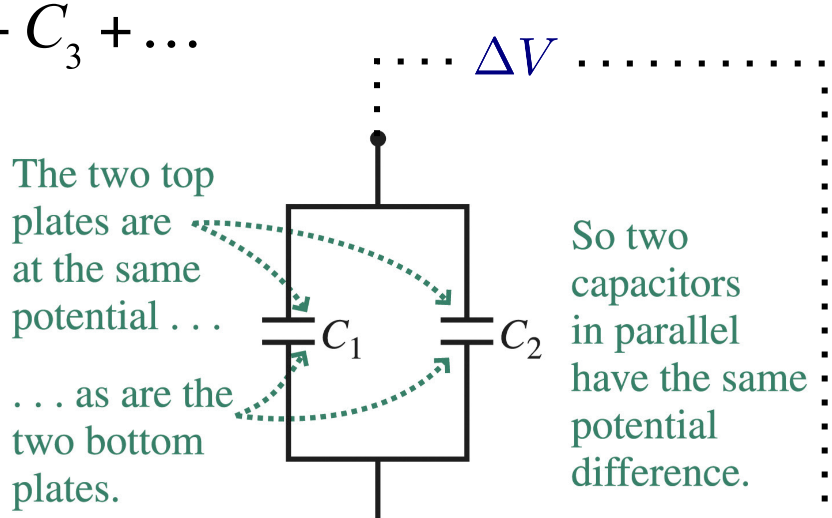
But breakdown fields
Only up to about 50 MV/m

Connecting Capacitors in Parallel

- Capacitors connected in **parallel** have their top plates connected together and their bottom plates connected together.
  - Therefore the potential difference ΔV across the two capacitors (between the conductive wires on either side) is the same.
  - The capacitance of the combination is the sum of the capacitances:

$$C_{\text{parallel}} = C_1 + C_2 + C_3 + \ldots$$

  - The maximum safe **working voltage** of the combination is that of the capacitor with the lowest voltage rating.

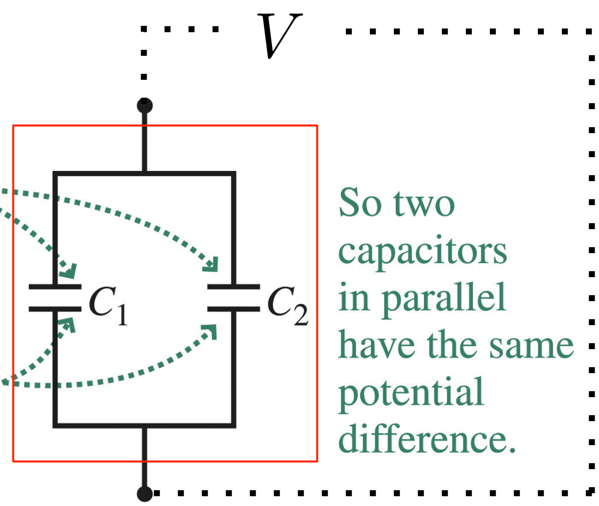


Connecting Capacitors in Parallel

We usually just write the voltage difference as *V* even though it's a difference!
Here the capacitors have the **same potential difference *V*.**

The two top plates are at the same potential . . .

. . . as are the two bottom plates.



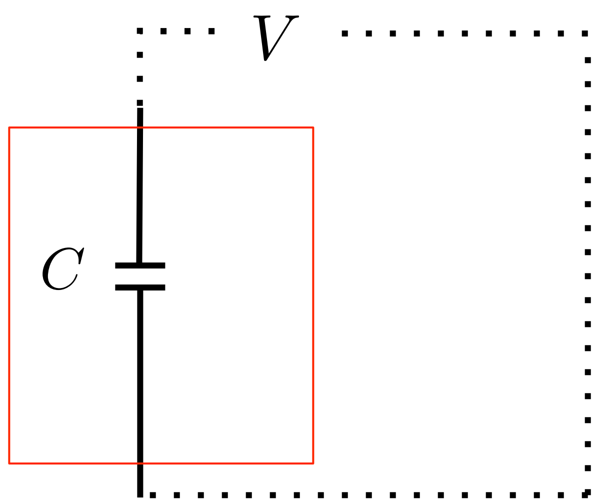
So two capacitors in parallel have the same potential difference.

$$C_1 = \frac{Q_1}{V}$$

$$C_2 = \frac{Q_2}{V}$$

$$Q = Q_1 + Q_2$$

Equivalent capacitance



$$C = \frac{Q}{V}$$

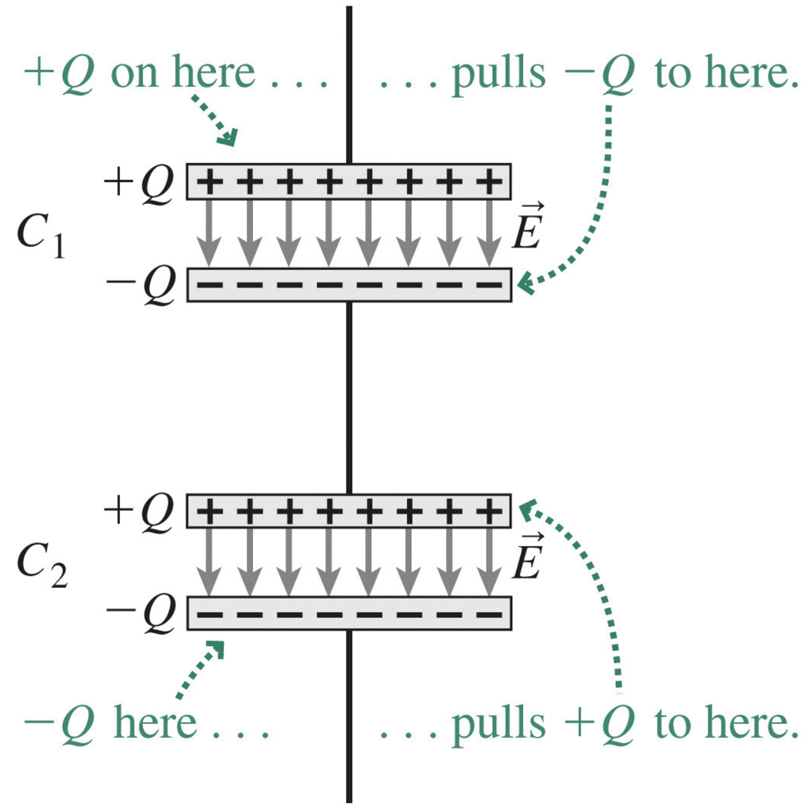
$$C = C_1 + C_2$$

Connecting Capacitors in Series

- Capacitors connected in **series** are wired so that one capacitor follows the other.
  - The figure shows that this makes the charge on the two capacitors the same.
  - With series capacitors, capacitance adds reciprocally:

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

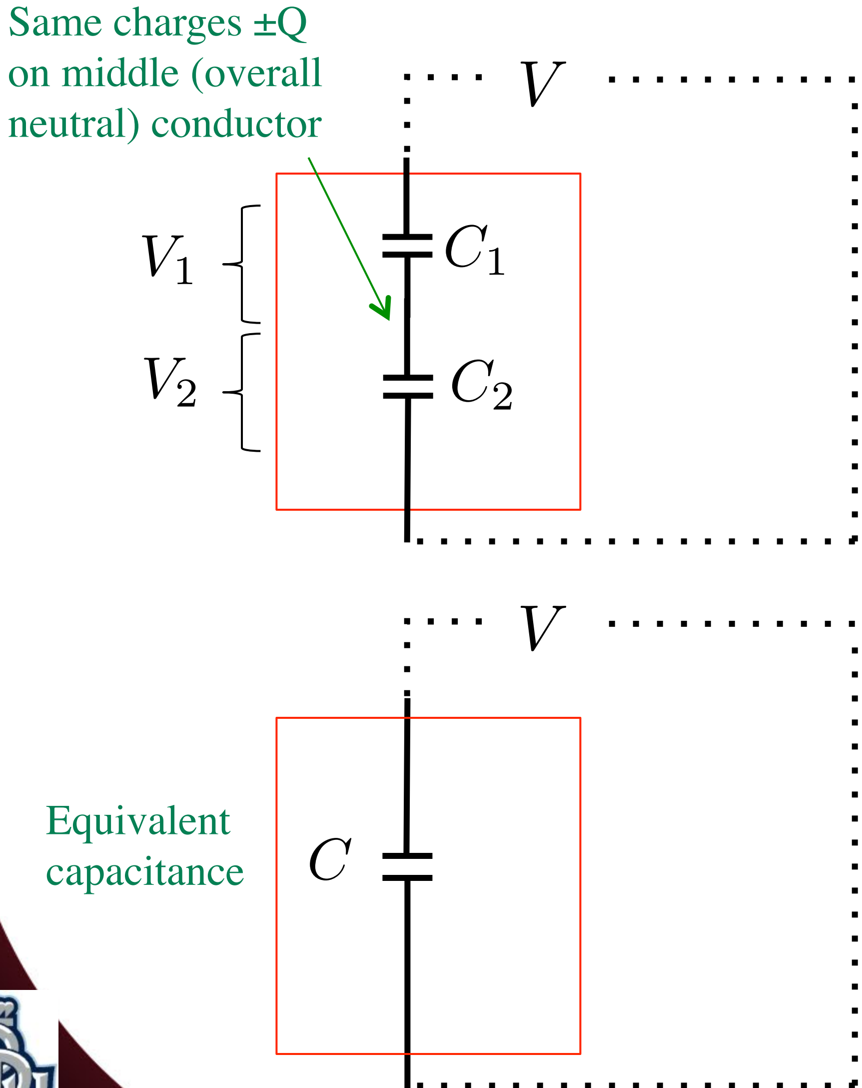
    Thus the combined capacitance is lower than that of any individual capacitor.



  - The working voltage of the combination is higher than that of any individual capacitor.

Connecting Capacitors in Series

Here the capacitors have the **same charge *Q***.



$$C_1 = \frac{Q}{V_1} \qquad V_1 = \frac{Q}{C_1}$$

$$C_2 = \frac{Q}{V_2} \qquad V_2 = \frac{Q}{C_2}$$

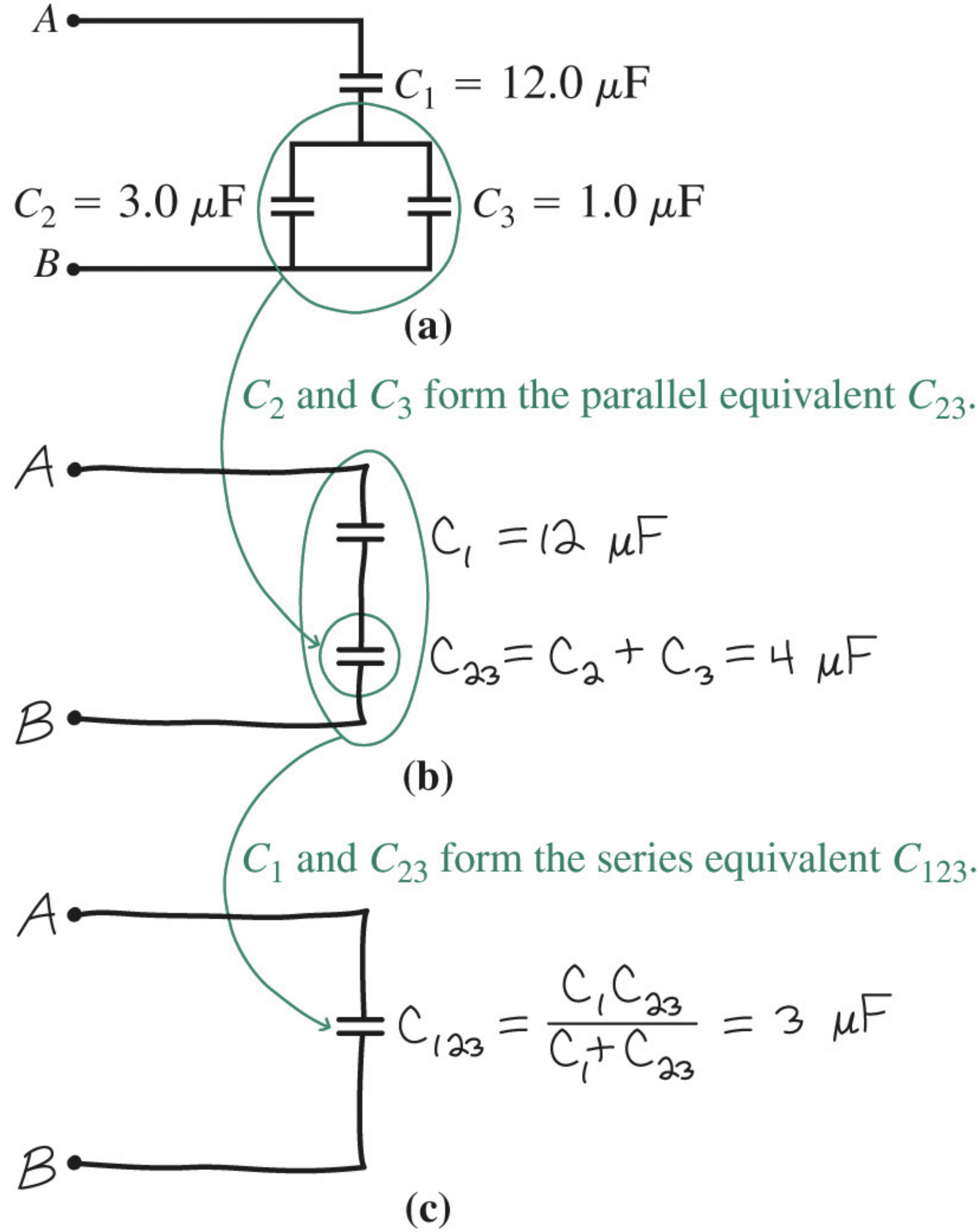
$$V = V_1 + V_2$$

$$C = \frac{Q}{V} \qquad V = \frac{Q}{C}$$

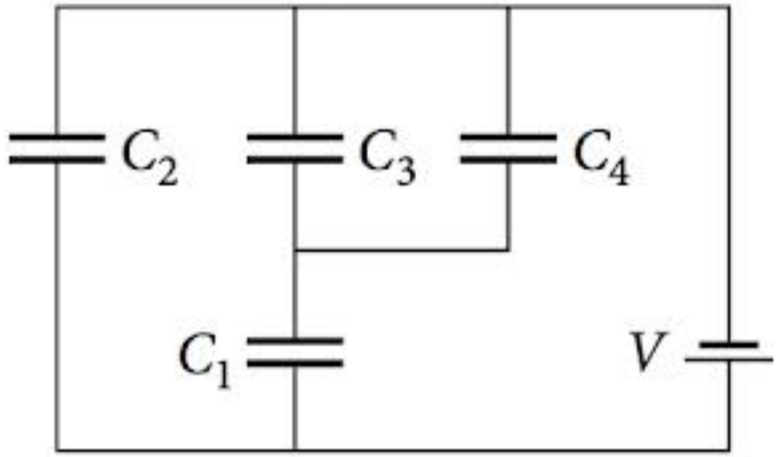
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Circuits with Parallel and Series Capacitors

- To analyze a circuit with several capacitors, look for series and parallel combinations.
  - Calculate the equivalent capacitances, and redraw the circuit in simpler form.
  - This technique will work later for more general electric circuits.
  - You don't have to draw every single equivalent circuit as long as it's clear to you what you're doing.



Let's Try It Out

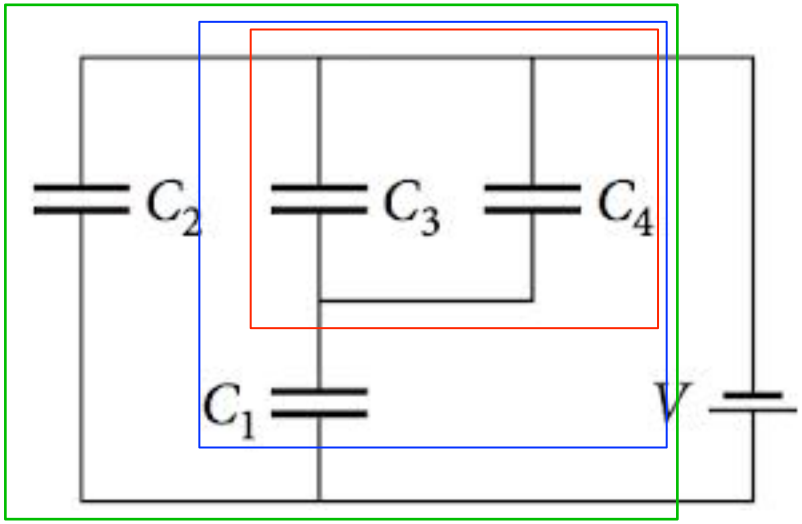


$$C_1 = C_2 = C_3 = C_4 = 4\ \mu\text{F}$$

$$V = 30\ \text{V}$$

- Find the equivalent capacitance of the capacitors
- Find the charge on each capacitor

Let's Try It Out: Hint



$$C_1 = C_2 = C_3 = C_4 = 4\ \mu\text{F}$$

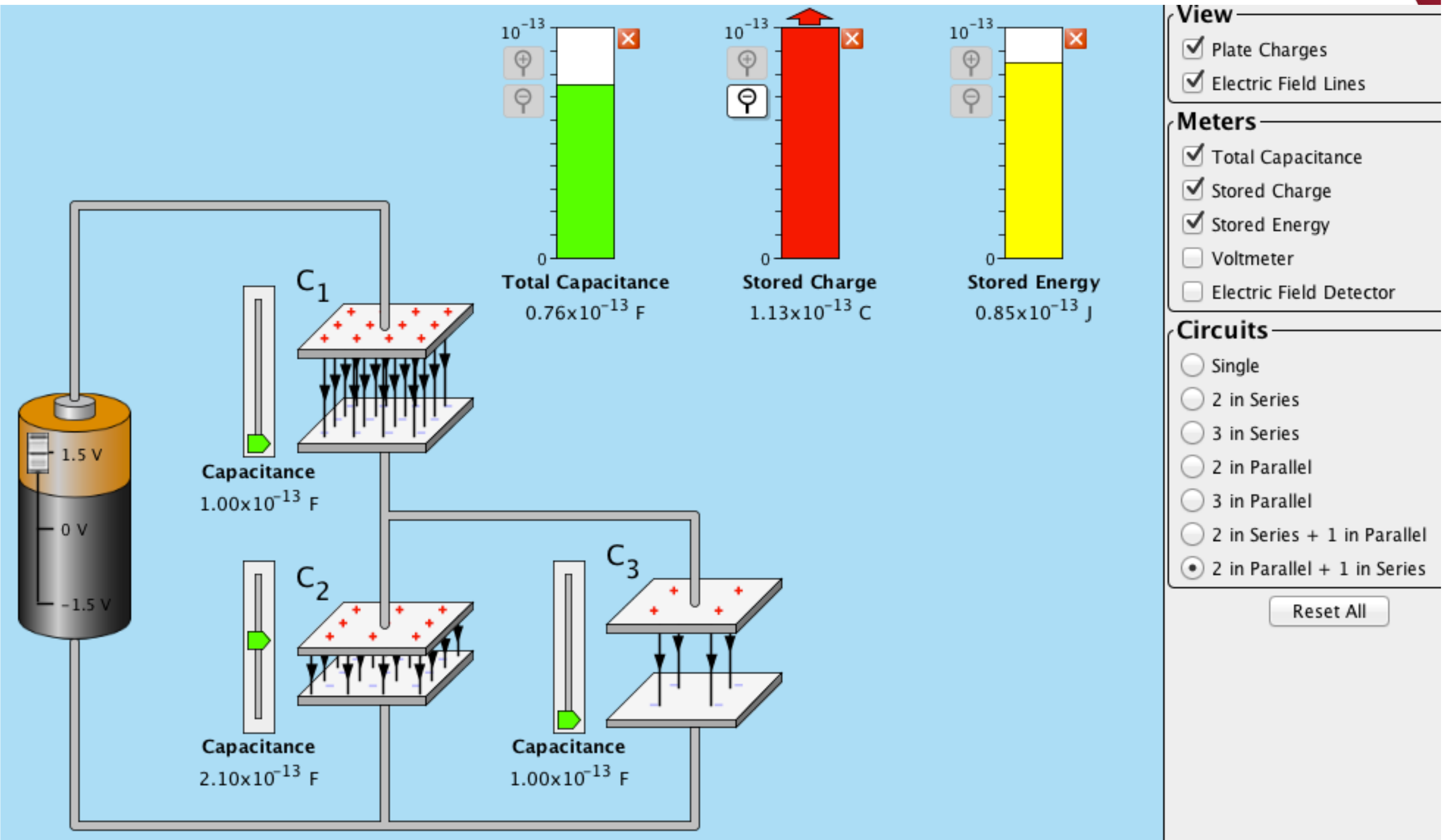
$$V = 30\ \text{V}$$

- Find the equivalent capacitance of the capacitors
- Find the charge on each capacitor

- "Unwrap" the circuit from the inside out
  - Red: two capacitors in parallel
  - Blue: two capacitors (using the above) in series
  - Green: two capacitors (using the above) in parallel

It's Time For Java App Wednesday™

http://phet.colorado.edu/en/simulation/capacitor-lab



Energy in the Electric Field

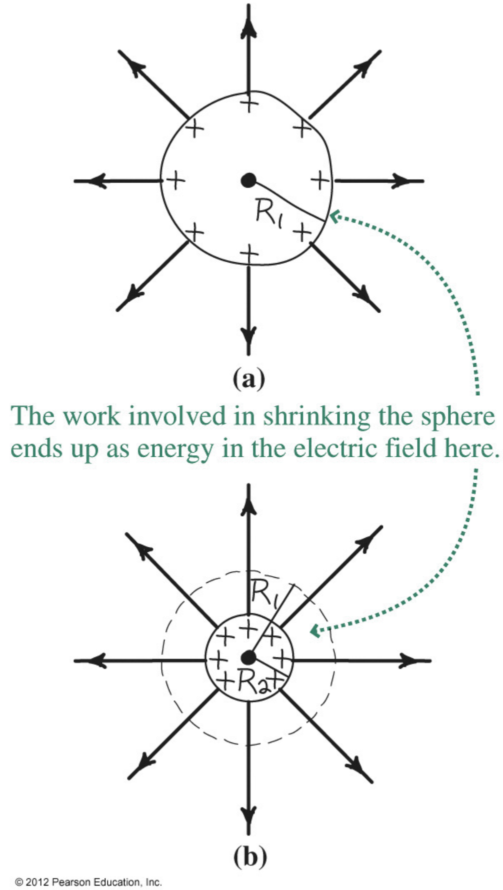
- The electrostatic energy associated with a charge distribution is stored in the electric field of the charge distribution.
  - Considering the uniform field of the parallel-plate capacitor shows that the electric **energy density** is

$$u_{\mathrm{E}} = \frac{E^2}{8\pi k}$$

Energy per unit volume!

- This is a universal result:
  – *Every* electric field contains energy with this density.

$$U_{\mathrm{E\ in\ volume\ V}} = \frac{E^2 V}{8\pi k}$$



What's The Electric Field of Sunlight?

- Energy/area at the Earth is about 1350 W/m^2
- Let's say sunlight travels at the speed of light, c=3x10^8 m/s

- One second of sunlight over one square meter therefore contains about 3x10^8 m^3 of sunlight and 1350 J of energy.

$$u = \frac{1350\ \mathrm{J}}{3 \times 10^8\ \mathrm{m}^3} = 4.5 \times 10^{-6}\ \mathrm{J/m^3}$$

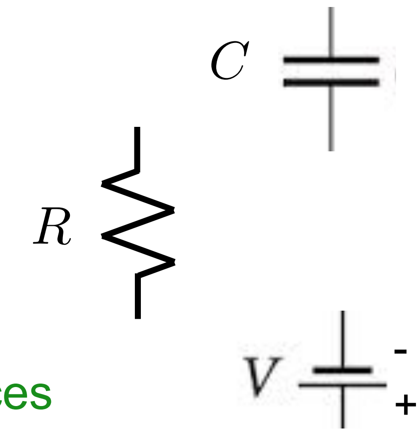
$$u_{\mathrm{E}} = \frac{E^2}{8\pi k}$$

$$E = \sqrt{8\pi k (4.5 \times 10^{-6}\ \mathrm{J/m^3})} = 1\ \mathrm{kV/m}$$

This is pretty close, but a little high for reasons we'll get to next week.
(The actual answer is still about 800 V/m!)

Chapter 24: Current, Resistors, and Ohm's Law

- "Classical" analog electronic circuits are made from four types of elements
  - Capacitors
    - Electrical energy storage: "springs"
  - Resistors
    - Electrical energy dissipation: "friction"
  - Voltage sources (EMF)
    - Electrical energy (potential difference) sources
  - Conductive wires
    - Treated as perfectly conductive
    - (But we know they really have some small resistance too)

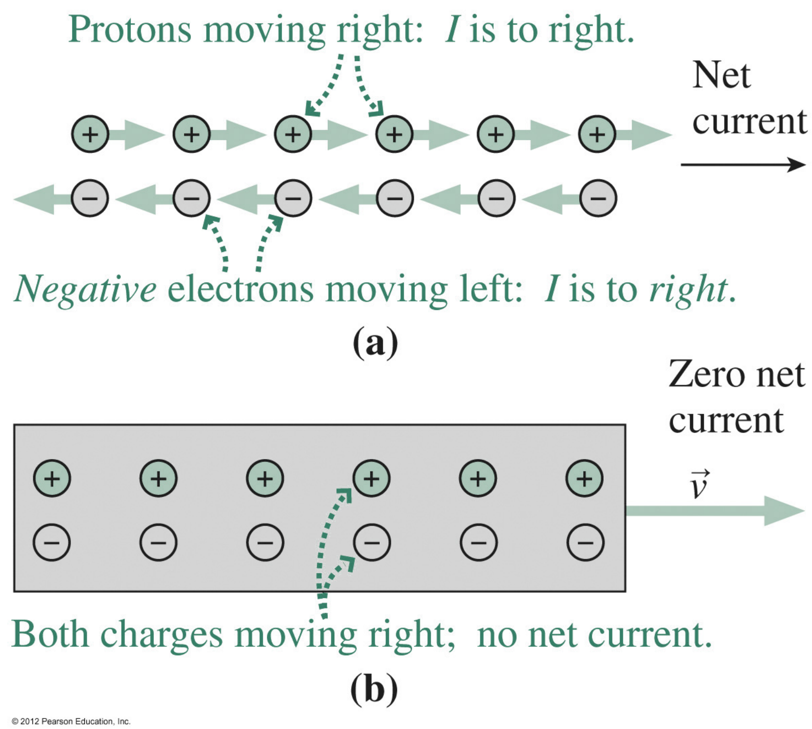


- We characterize electrical circuits with
  - **Voltage** (potential) differences *V* between various points
  - **Current** (electron) flow *I* between various points
  - **DC**: Constant current (including zero) **AC**: time-varying current

Electric Current

- **Electric current** is a net flow of electric charge.
  - Quantitatively, current is the rate at which charge crosses a given area.
    - For steady current, $I = \dfrac{\Delta Q}{\Delta t}$
    - When current varies with time, its instantaneous value is given by

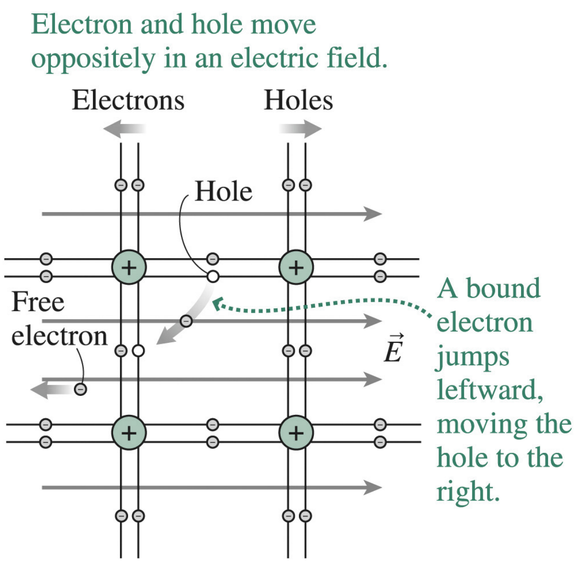
$$I = \frac{dQ}{dt} \qquad \text{Ampere} \equiv \frac{\text{Coulomb}}{\text{sec}}$$



    - The direction of the current corresponds to the direction of flow of the **positive** charges.
    - Current has a **direction**

Conduction Mechanisms

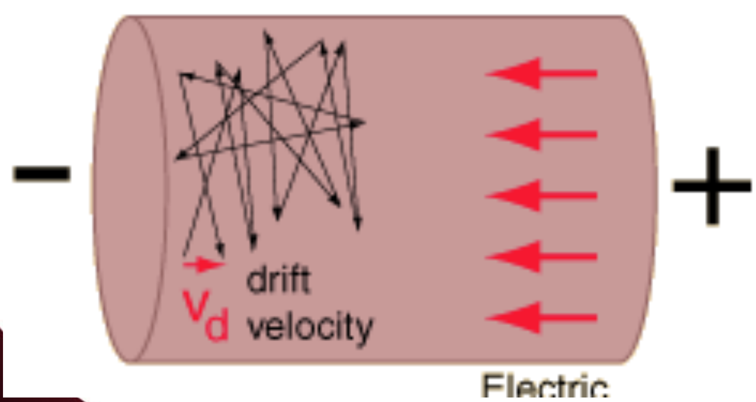
- Conduction occurs differently in different types of materials:
  - In **metallic conductors**, current is carried by free electrons.
  - In **ionic solutions**, current is carried by positive and negative ions.
  - **Plasmas** are ionized gases, with current carried by electrons and ions.
  - **Semiconductors** involve current carried by both electrons and "holes"—absences of electrons in a crystal structure.
    - Semiconductors are at the heart of modern electronics.
    - Their electrical properties can be altered by the controlled addition of small amounts of impurities.

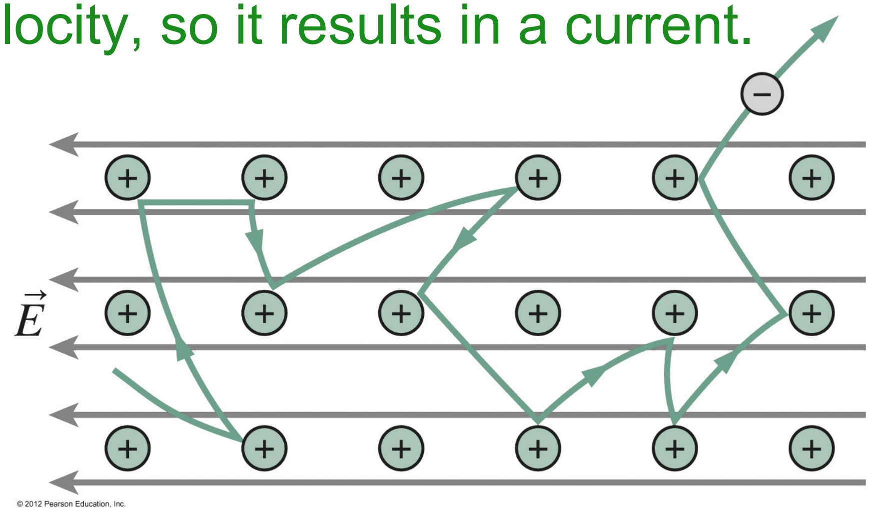


  - **Superconductors** offer zero resistance to the flow of current, and thus can transmit electric power without loss of energy.
    - Known superconducting materials all require temperatures far below typical ambient temperatures.

Conduction in Metals

- A **metal** contains a "sea" or "gas" of free electrons:
  - They're confined to the metal (conductor) but not bound to individual atoms.
  - The electrons move about in random directions with high thermal velocities.
    - On *average*, there's no current associated with thermal motion.
  - Applying an electric field adds a small drift velocity on the electrons' motion.
    - All electrons share the drift velocity, so it results in a current.





Ohm's Law: Microscopic

- Electrons often collide with ions (nuclei) in the metal's crystal structure
  - They usually lose energy this way
  - This limits how easily the electrons "flow" through the material
- This produces resistance to current flow
  - Quantified as **conductivity** σ of the metal
  - Current per unit area, or current density $\vec{J}$ is then

$$\vec{J} = \sigma \vec{E}$$

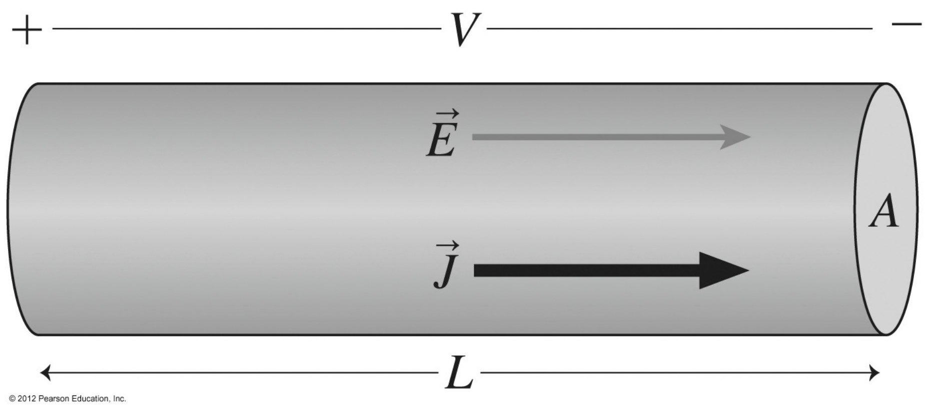
  - **Resistivity**: $\rho \equiv \frac{1}{\sigma}$

Table 24.1 Resistivities

Material	**Resistivity** ($\Omega \cdot m$)
Metallic conductors (20°C)	
Aluminum	2.65×10^{-8}
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Iron	9.71×10^{-8}
Mercury	9.84×10^{-7}
Silver	1.59×10^{-8}
Ionic solutions (in water, 18°C)	
1-molar $CuSO_4$	3.9×10^{-4}
1-molar HCl	1.7×10^{-2}
1-molar NaCl	1.4×10^{-4}
H_2O	2.6×10^{5}
Blood, human	0.70
Seawater (typical)	0.22
Insulators	
Ceramics	10^{11}–10^{14}
Glass	10^{10}–10^{14}
Polystyrene	10^{15}–10^{17}
Rubber	10^{13}–10^{16}
Wood (dry)	10^{8}–10^{14}

Ohm's "Law" for Resistive Devices

- How do we relate this to electric current and voltage?



V : potential difference
$\vec{E}$: Electric field
$\vec{J}$: Current density
I : Electric current

- We can calculate a total resistance to current flowing based on the resistivity ρ and physical properties of the resistor

$$R = \frac{\rho L}{A}$$

- A general rule (called a law though it's really not truly a law):

$$V = IR$$

Ohm's "Law"

Ohm's "Law": Microscopic and Macroscopic

Table 24.2 Microscopic and Macroscopic Quantities and Ohm's Law

Microscopic	Macroscopic	Relation
Electric field, $\vec{E}$	Voltage, V	$\vec{E}$ is defined at each point in a material; V is the integral of $\vec{E}$ over a path. In a uniform field, $V = EL$.
Current density, $\vec{J}$	Current, I	$\vec{J}$ is defined at each point in a material; I is the integral of $\vec{J}$ over an area. With uniform current density, $I = JA$.
Resistivity, ρ	Resistance, R	ρ is a property of a given material; R is a property of a particular piece of that material. In a piece with uniform cross section, $R = \rho L/A$.
Ohm's law $\vec{J} = \frac{\vec{E}}{\rho}$	Ohm's law $I = \frac{V}{R}$	Microscopic version relates current density to electric field at a point in a material. Macroscopic version relates current through to voltage across a given piece of material.

Power Dissipated In A Resistor

- We had a formula for the energy stored in a capacitor

$$\boxed{U_{\text{stored in capacitor}} = \frac{1}{2}CV^2} = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \qquad C \equiv \frac{Q}{V}$$

- This is the energy stored in a capacitor at a particular charge
- Now we're considering circuits where charges is moving

$$\boxed{I = \frac{dQ}{dt}}$$

- The power *P* (energy per unit time!) dissipated by a resistive device with resistance *R* is

$$\boxed{P = IV} = I^2R = \frac{V^2}{R} \qquad V = IR$$