University Physics, PHYS 227/232 Quiz 3 Fri Feb 21, 2014

Your name and table number: ______________________________

Please show your work, write neatly, write units, and $\boxed{\text{box}}$ your answers.

Definition of capacitance: $C \equiv \kappa Q/V$ where κ is the dielectric constant
Capacitance of a parallel plate capacitor: $C = A/(4\pi k d)$, $k = 9 \times 10^9\ \mathrm{N\,m^2/C^2}$
Capacitors in parallel: $C_{\text{tot}} = C_1 + C_2 + \ldots$
Capacitors in series: $1/C_{\text{tot}} = 1/C_1 + 1/C_2 + \ldots$

1. You build a capacitor out of two parallel thin rectangular conductors, each of length 1.5 cm and width 2.0 cm, separated by a distance of 1.5 μm.

   (a) (2 points) What is its capacitance?
   Solution: Capacitance of a parallel plate capacitor is $C = A/(4\pi k d)$. Here the area of these rectangular conductors is $A = (0.015\,\mathrm{m})(0.020\,\mathrm{m}) = 3 \times 10^{-4}\,\mathrm{m}^2$. So we have

$$C = \frac{A}{4\pi k d} = \frac{(3 \times 10^{-4}\,\mathrm{m}^2)}{4\pi(9 \times 10^9\,\mathrm{N\,m^2/C^2})(1.5 \times 10^{-6}\,\mathrm{m})} = 1.768\,\mathrm{nF} \approx \boxed{1.8\,\mathrm{nF} = C}$$

   (b) (2 points) What is the potential difference between the conductors if there is a charge of 2.0×10^{-7} C on the capacitor?
   Solution: $C \equiv Q/V$, so

$$V = \frac{Q}{C} = \frac{(2.0 \times 10^{-7}\,\mathrm{C})}{(1.768 \times 10^{-9}\,\mathrm{F})} = 113\,\mathrm{V} \approx \boxed{110\,\mathrm{V} = V}$$

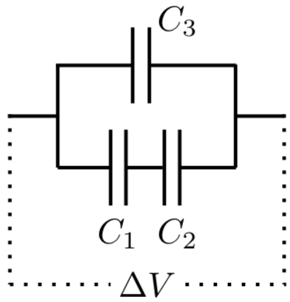
   (c) (1 point) If you now separate the (charged) plates of the capacitor to a new separation of 4.5 μm, what is the new potential difference between the plates?
   Solution: The new capacitance of the capacitor is

$$C_{\text{new}} = \frac{A}{4\pi k d_{\text{new}}} = \frac{A}{4\pi k d}\left(\frac{d}{d_{\text{new}}}\right) = C\left(\frac{1}{3}\right)$$

   It has the same charge, so the new potential difference (voltage) is

$$V_{\text{new}} = \frac{Q}{C_{\text{new}}} = \frac{(2.0 \times 10^{-7}\,\mathrm{C})}{(1.768 \times 10^{-9}\,\mathrm{F})(1/3)} = 339\,\mathrm{V} \approx \boxed{340\,\mathrm{V} = V_{\text{new}}}$$

   The capacitance goes down by a factor of three, so the voltage to hold the same charge must go *up* by a factor of three.



2. Consider the above arrangement of capacitors, where $C_1 = 2.0\,\mu$F and $C_2 = 2.0\,\mu$F are in series, and $C_3 = 1.0\,\mu$F is parallel to the series combination.

   (a) (3 point) What is the overall equivalent capacitance of this combination of capacitors?
   Solution: Capacitors C_1 and C_2 are in series, so their equivalent capacitance C_{12} is

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.0\,\mu\text{F}} + \frac{1}{2.0\,\mu\text{F}} = \frac{1}{1.0\,\mu\text{F}} \qquad \Rightarrow \qquad C_{12} = 1.0\,\mu\text{F}$$

   This equivalent capacitance C_{12} is in parallel with C_3, so the total equivalent capacitance C_{123} is

$$C_{123} = C_{12} + C_3 = (1.0\,\mu\text{F}) + (1.0\,\mu\text{F}) = \boxed{2.0\text{ F} = C_{123}}$$

   (b) (2 points) If there is a voltage difference $\Delta V = 20$ V across this capacitor combination, what is the charge on each capacitor?
   Solution: C_3 has wires that connect to the terminals of ΔV, so the potential difference across C_3 is also ΔV. Then we can use the definition of capacitance to find its charge:

$$Q_3 = C_3 \Delta V = (1.0\,\mu\text{F})(20\text{ V}) = \boxed{20\,\mu\text{C} = Q_3}$$

   We can also use this logic to find the charge on the equivalent capacitance of the series capacitors, C_{12}:

$$Q_{12} = C_{12}\Delta V = (1.0\,\mu\text{F})(20\text{ V}) = 20\,\mu\text{C}$$

   How does Q_{12} relate to the charges on the individual capacitors Q_1 and Q_2? The capacitors in series must have the *same* charge. All charge that goes onto the left plate of C_1 is balanced by an equal and opposite charge on the rightmost plate of C_1. But the central conductor is neutral, so there must also be an equal charge over on the other side of the conductor, on the left plate of C_2, which is then balanced by an equal and opposite charge on the right plate of C_2.
   The conclusion to all that logic is that the charges Q_1 and Q_2 are the both equal to the Q_12 you calculated for the equivalent capacitance of the two capacitors in series.

$$\boxed{Q_1 = Q_2 = Q_{12} = 20\,\mu\text{C}}$$

   Yes, the charges on all three capacitors are the same!