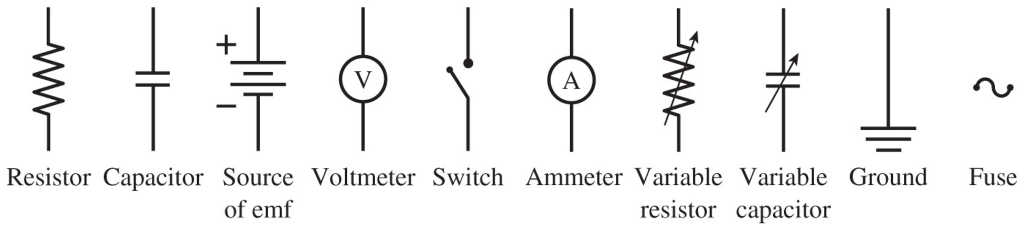
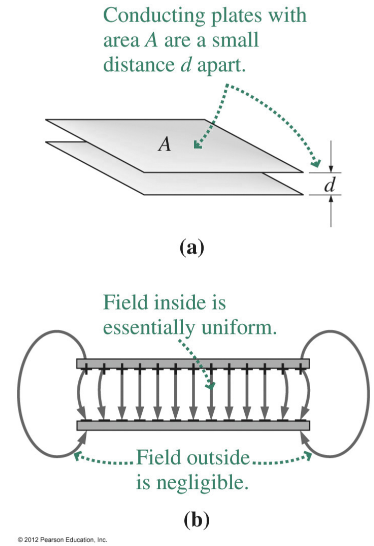




University Physics 227N/232N

Current and Ohm's Law, Resistors, Circuits, and Kirchoff

Lab this Friday, Feb 28
So NO QUIZ this Friday!

Dr. Todd Satogata (ODU/Jefferson Lab) and Fred Miller
satogata@jlab.org
http://www.toddsatogata.net/2014-ODU

Monday, February 24 2014

Happy Birthday to Floyd Mayweather Jr, Bronson Arroyo, Mitch Hedberg, Steve Jobs, and Brian Schmidt (yes, 2011 Nobel Prize)

Review: Capacitors

- A **capacitor** is a pair of conductors, insulated from each other, and used to **store charge and energy**.
  - The two conductors are given equal but opposite charges $\pm Q$
  - Definition of capacitance: $C \equiv Q/V$ $\quad Q = CV$
  - Capacitance is a **physical property** of the capacitor.

- A **parallel plate capacitor** has two parallel conductors of equal area *A* separated by distance *d*, possibly a **dielectric**

$$C_{\text{parallel plate}} = \kappa \frac{A}{4\pi k d} = \kappa C_0 \qquad C_0 = \frac{A}{4\pi k d} \qquad \kappa >= 1$$

  - The dielectric constant κ for a vacuum is 1

- Energy stored in a capacitor

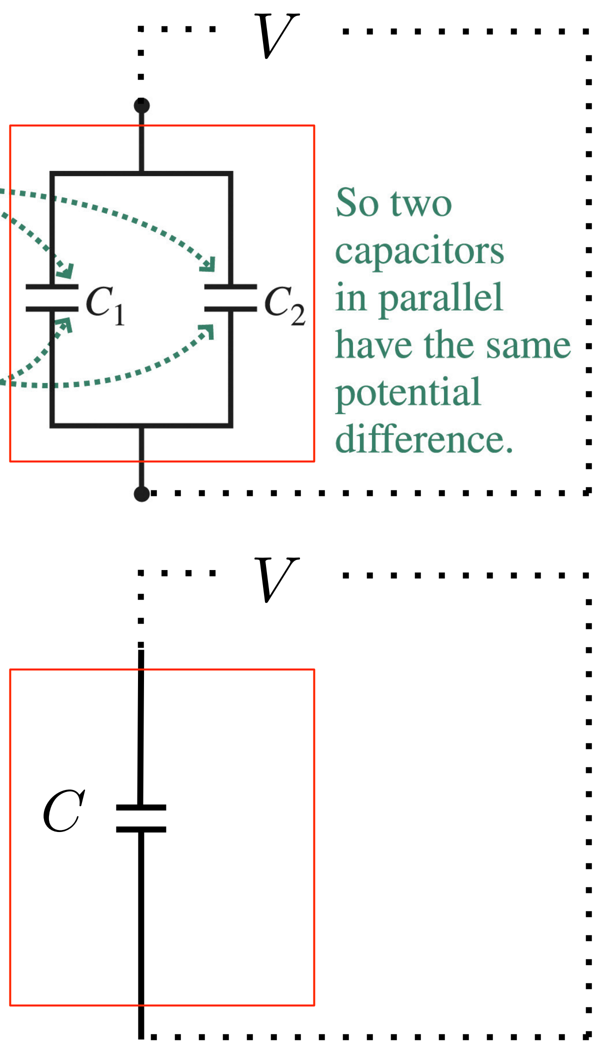
$$U_{\text{stored in capacitor}} = \frac{1}{2} C V^2 \qquad = \frac{1}{2} Q V = \frac{1}{2} \frac{Q^2}{C}$$

Review: Connecting Capacitors in Parallel

We usually just write the voltage difference as *V* even though it's a difference!
Here the capacitors have the **same potential difference *V***.

The two top plates are at the same potential . . .

. . . as are the two bottom plates.



So two capacitors in parallel have the same potential difference.

Equivalent capacitance

$$C_1 = \frac{Q_1}{V}$$

$$C_2 = \frac{Q_2}{V}$$

$$Q = Q_1 + Q_2$$

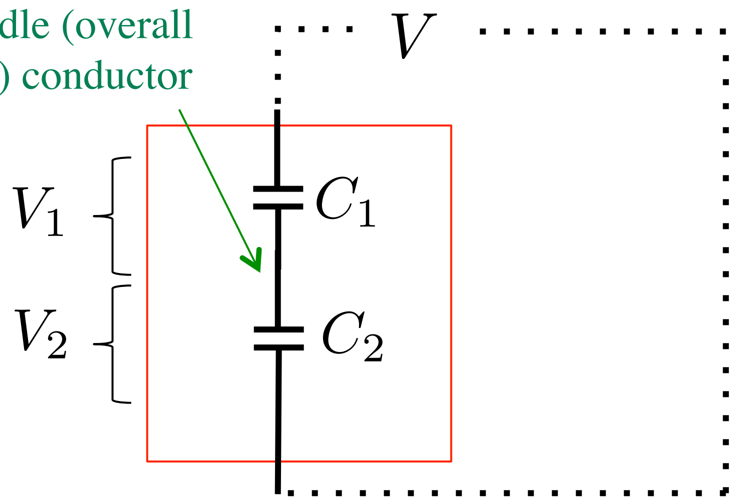
$$C = \frac{Q}{V}$$

$$C = C_1 + C_2$$

Review: Connecting Capacitors in Series

Here the capacitors have the **same charge *Q***.

Same charges ±Q on middle (overall neutral) conductor



$$C_1 = \frac{Q}{V_1}$$

$$C_2 = \frac{Q}{V_2}$$

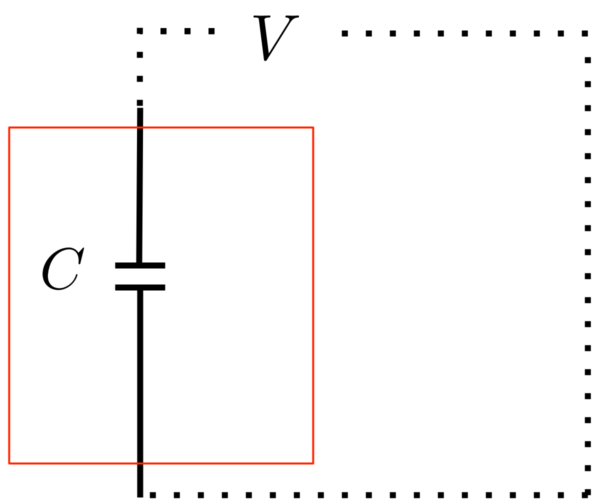
$$V = V_1 + V_2$$

$$V_1 = \frac{Q}{C_1}$$

$$V_2 = \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Equivalent capacitance

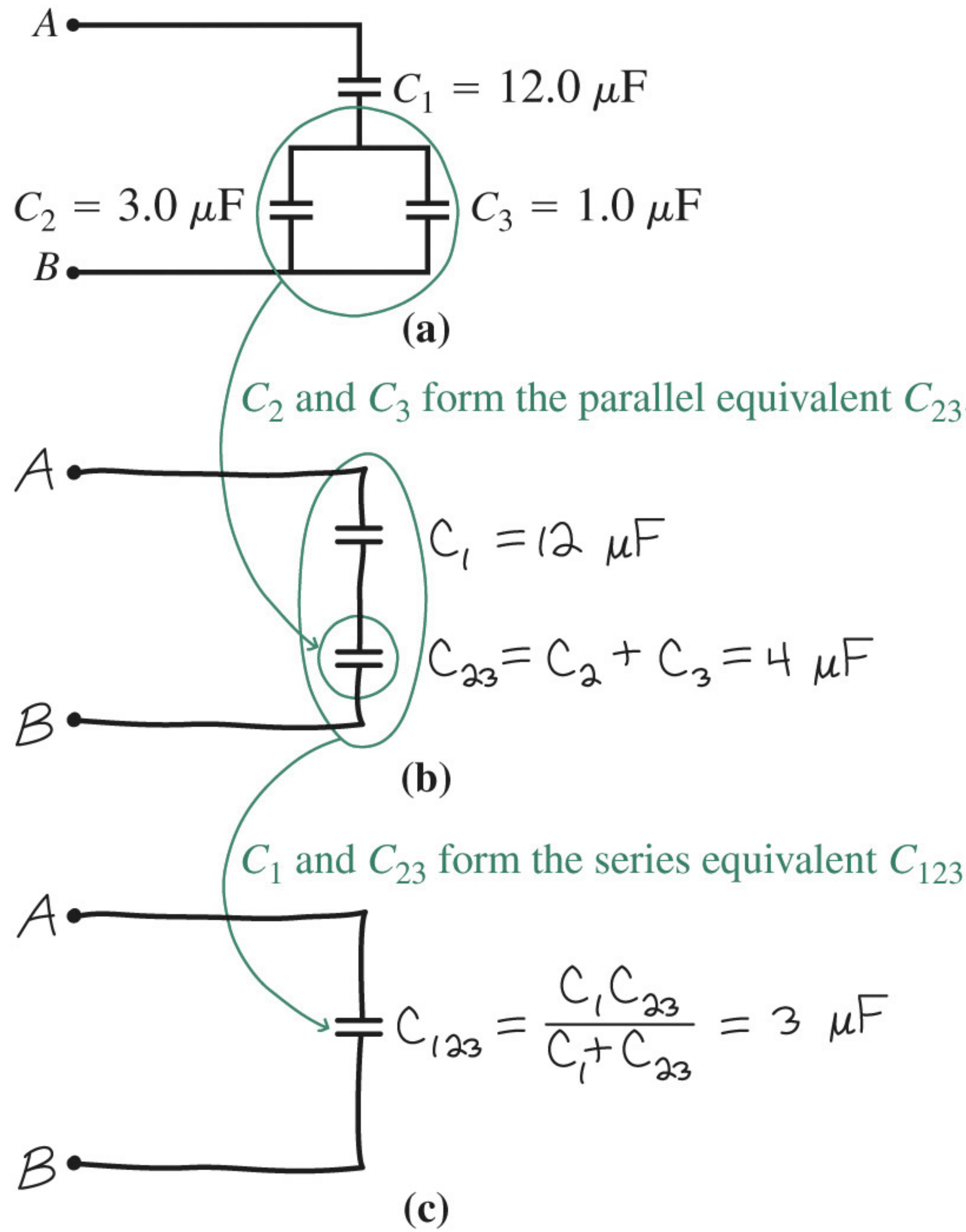


$$C = \frac{Q}{V}$$

$$V = \frac{Q}{C}$$

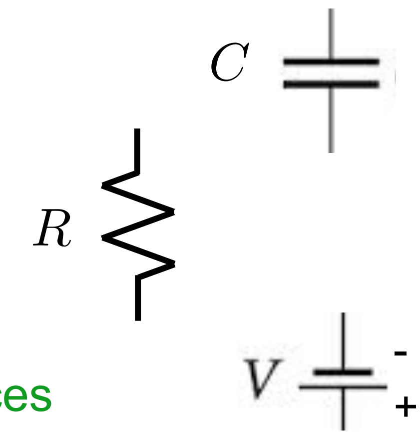
Circuits with Parallel and Series Capacitors

- To analyze a circuit with several capacitors, look for series and parallel combinations.
  - Calculate the equivalent capacitances, and redraw the circuit in simpler form.
  - This technique will work later for more general electric circuits.
  - You got to practice this in your homework and on last week's quiz.



Chapter 24: Current, Resistors, and Ohm's Law

- "Classical" analog electronic circuits are made from four types of elements
  - Capacitors
    - Electrical energy storage: "springs"
  - Resistors
    - Electrical energy dissipation: "friction"
  - Voltage sources (EMF)
    - Electrical energy (potential difference) sources
  - Conductive wires
    - Treated as perfectly conductive
    - (But we know they really have some small resistance too)



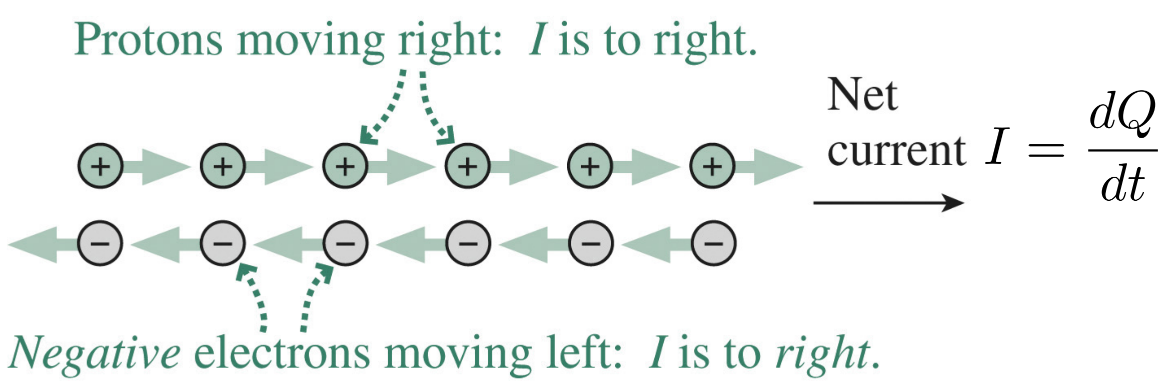
- We characterize electrical circuits with
  - **Voltage** (potential) differences *V* between various points
  - **Current** (electron or charge) flow *I* between various points
  - **DC**: Constant current (including zero) **AC**: time-varying current
  - **Charges** on capacitors

What is Electric Current?

- Electric current relates to the **net flow** of electric charge
  - Now we can talk about how charges move in our conductors that aren't quite ideal (they take time to move)
  - How many charges pass a certain point in a certain amount of time?
  - At a given time

$$I \equiv \frac{dQ}{dt} \qquad \text{Ampere} \equiv \frac{\text{Coulomb}}{\text{sec}}$$

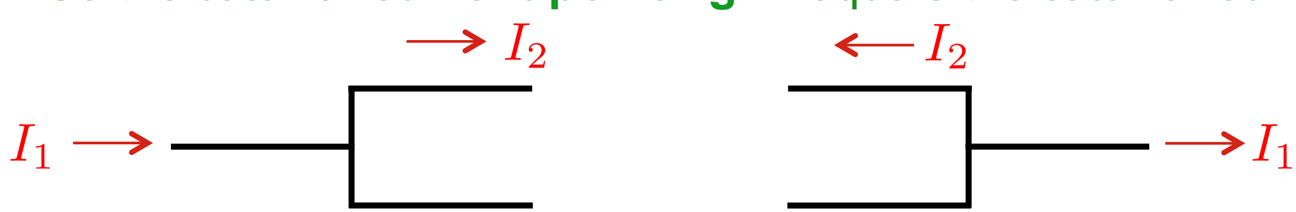
  - *dQ* is really counting charge: + are positive, - are negative
  - Current has a **direction** and doesn't require a conductor to exist
    - Although conductors are materials that allow electrons to move a **lot** more easily than in most other materials



Electric Current in Wires

- We'll be considering electric current *I* in conductive wires for now
  - Remember, current is a flow of charges
  - **Net** charge doesn't build up on wires or other electric circuit elements
    - Even on capacitors! They have +/-Q on plates and stays net neutral
    - But the charges of the plates +/-Q do change as current moves charges onto them
  - So current *I* is the same along a single piece of wire

  - What about wire junctions?
    - There charges go one way or another, or come from one way or another
    - So the **total of current pointing in** equals the **total of current point out**

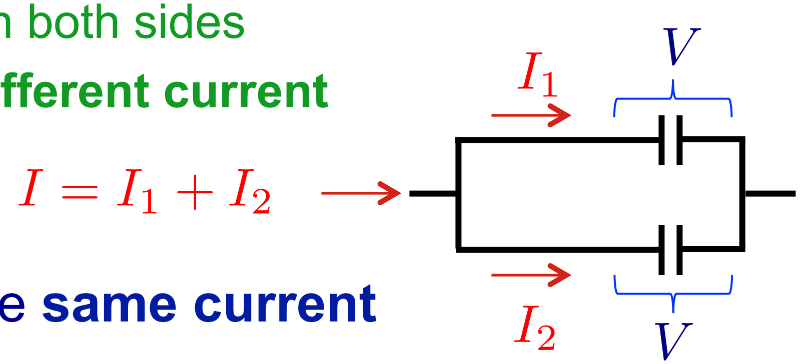


$I_1 = I_2 + I_3$ $\qquad$ $I_1 + I_2 + I_3 = 0$

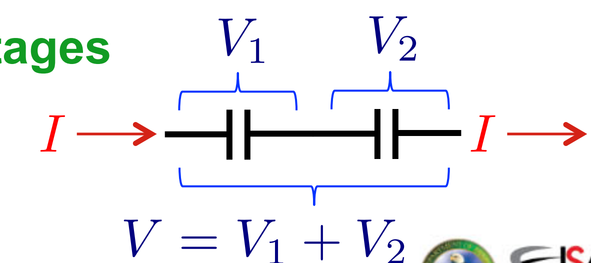
    - Negative current is really just current that's really opposite to your arrow

Multi-Step Problems

- The homework had you do a few problems that are multi-step
  - You have to use multiple applications of the same equations and principles to untangle voltages, capacitances, charges of all elements within a circuit
  - We'll do even more of this as we add in more circuit elements
- Important principles that generally hold true
  - Elements that are in **parallel** have the **same voltage**
    - They share the same conductors on both sides
    - But they can (and often do) have **different current**

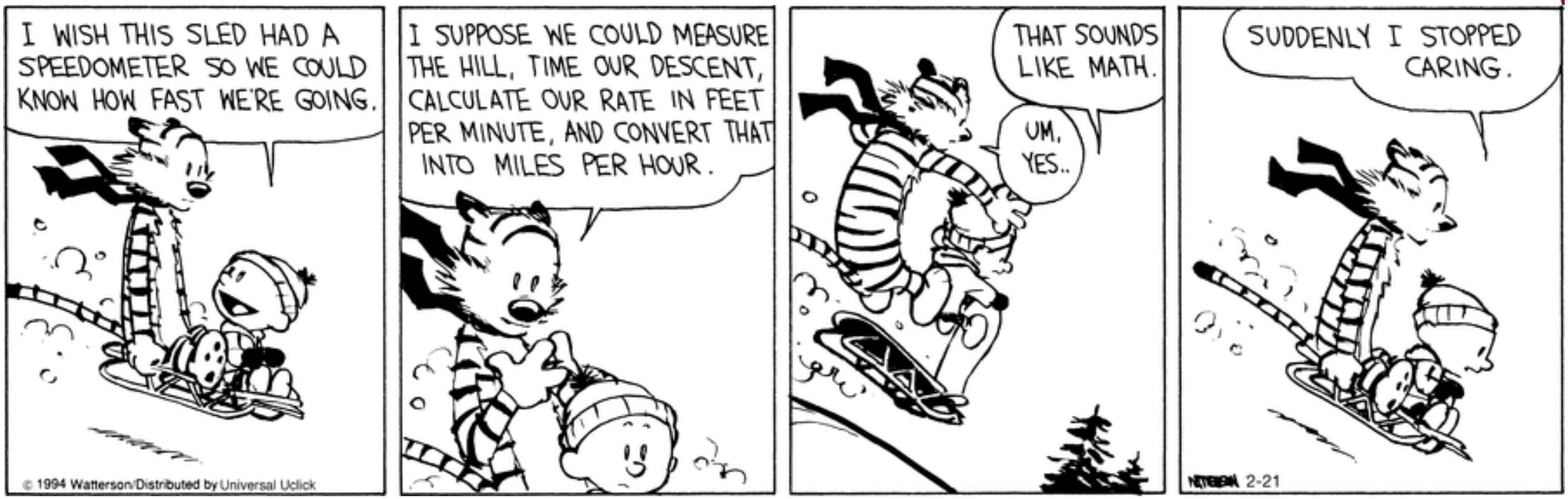


  - Elements that are in **series** have the **same current**
    - Or, for capacitors, the **same charge**
    - But they can (and often do) have **different voltages**



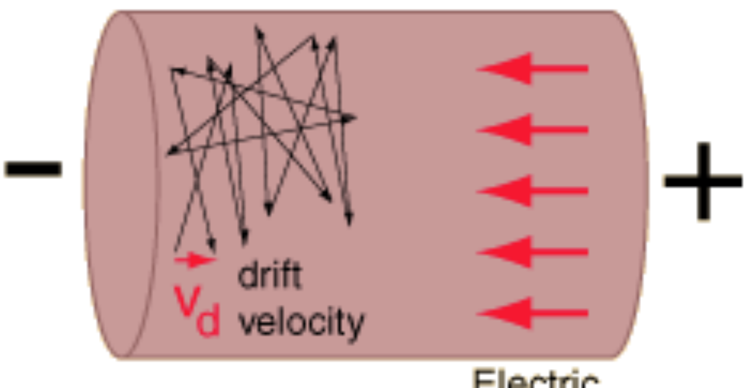
    - Applying these can already get you far!

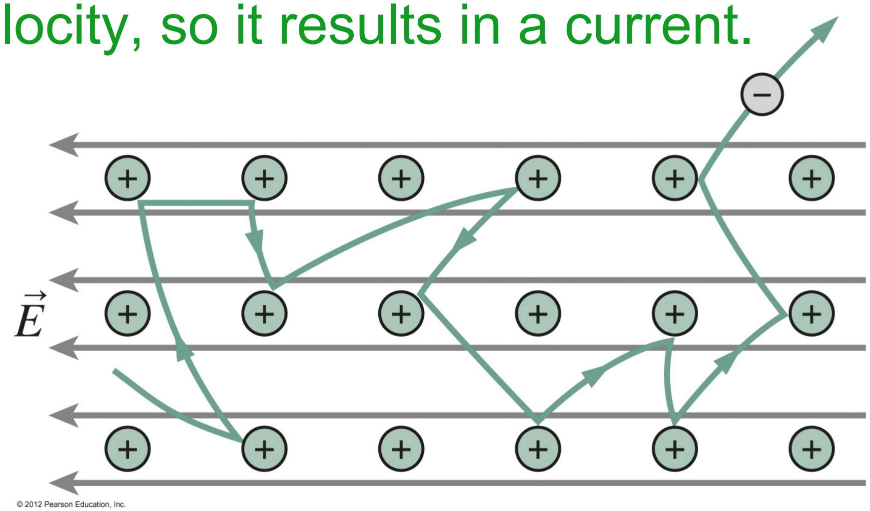
Although it can be a fair amount of work…



Conduction of Current in Metal Wires

- A **metal** contains a "sea" or "gas" of free electrons:
  - They're confined to the metal (conductor) but not bound to individual atoms.
  - The electrons move about in random directions with high thermal velocities.
    - On *average*, there's no current associated with thermal motion.
  - Applying an electric field adds a small **drift velocity** on the electrons' motion.
    - All electrons share the drift velocity, so it results in a current.





Ohm's Law (Microscopic)

- Electrons often collide with ions (nuclei) in the metal's crystal structure
  - They usually lose energy this way
  - This limits how easily the electrons "flow" through the material
- This produces resistance to current flow
  - Quantified as **conductivity** σ of the metal
  - It's defined by how the **current density** $\vec{J}$ is related to the electric field $\vec{E}$

$$\sigma \equiv \frac{J}{E}$$

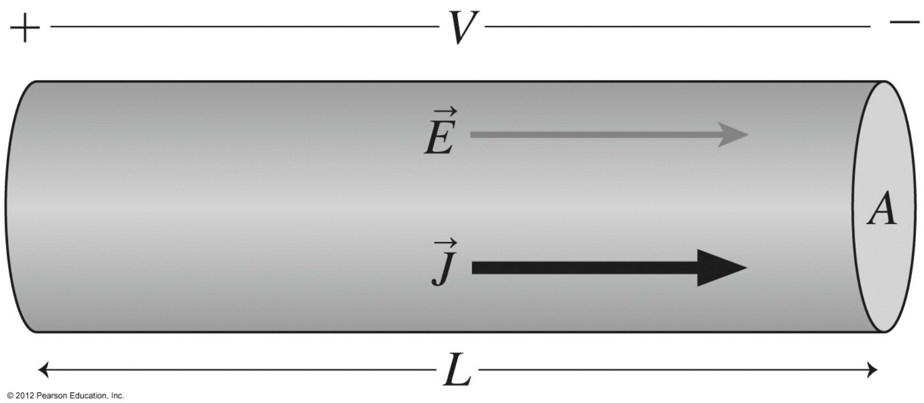
- **Resistivity**: $$\rho \equiv \frac{1}{\sigma}$$

Table 24.1 Resistivities

Material	**Resistivity** $(\Omega \cdot m)$
Metallic conductors (20°C)	
Aluminum	2.65×10^{-8}
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Iron	9.71×10^{-8}
Mercury	9.84×10^{-7}
Silver	1.59×10^{-8}
Ionic solutions (in water, 18°C)	
1-molar $CuSO_4$	3.9×10^{-4}
1-molar HCl	1.7×10^{-2}
1-molar NaCl	1.4×10^{-4}
H_2O	2.6×10^{5}
Blood, human	0.70
Seawater (typical)	0.22
Insulators	
Ceramics	10^{11}–10^{14}
Glass	10^{10}–10^{14}
Polystyrene	10^{15}–10^{17}
Rubber	10^{13}–10^{16}
Wood (dry)	10^{8}–10^{14}

Ohm's "Law" for Macroscopic Resistive Devices

- How do we relate this to electric current and voltage?



V : potential difference
$\vec{E}$: Electric field
$\vec{J}$: Current density
I : Electric current

- We can calculate a total resistance to current flowing based on the resistivity ρ and physical properties of the object

$$R = \frac{\rho L}{A}$$

10 cm long, 1mm^2 Cu wire: $R = 1.7\ \mathrm{m}\Omega$

- A general rule (called a law though it's really not truly a law):

$$\boxed{V = IR}$$

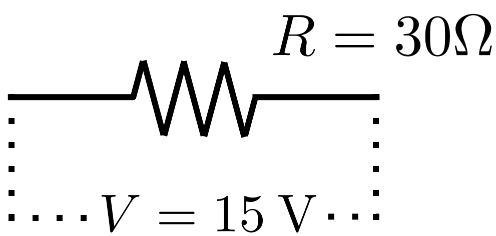
Ohm's "Law"

$$R = \frac{I}{V} \qquad 1\ \mathrm{Ohm} = 1\ \Omega \equiv \frac{\mathrm{Amp}}{\mathrm{Volt}}$$

e.g. this "law" tends to fall apart for very low and very high temperatures

Resistors – and a Quick Calculation

- Macroscopic objects with electric resistance are another basic building block of electric circuits
  - These are **resistors**
  - We assume they have a lot more resistance than the (generally conductive) wires that connect them
    - A modest conductive wire as a resistance of mΩ
    - Electrical resistors have resistances of Ω to kΩ to MΩ



$$V = IR$$

$$I = dQ/dt$$

- A 30 Ω resistor has a voltage of 15 V between its sides
  - What is the current in the resistor?
  - How much charge flows through the resistor in 1 second?

Power Dissipated in a Resistor

- We had a formula for the energy stored in a capacitor

$$U_{\text{stored in capacitor}} = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} \qquad C \equiv \frac{Q}{V}$$

- This is the energy stored in a capacitor at a particular charge
- Now we're considering circuits where charges is moving

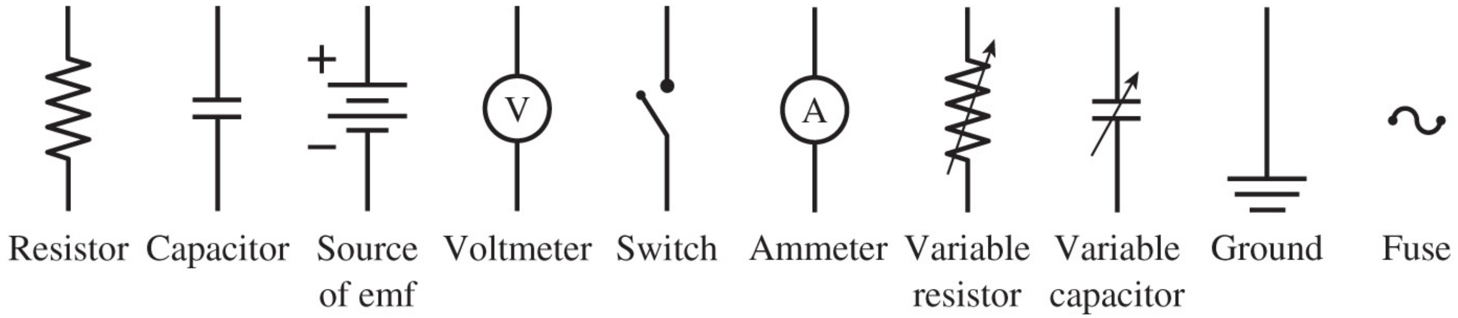
$$I = \frac{dQ}{dt}$$

- The power *P* (energy per unit time!) dissipated by a resistive device with resistance *R* is

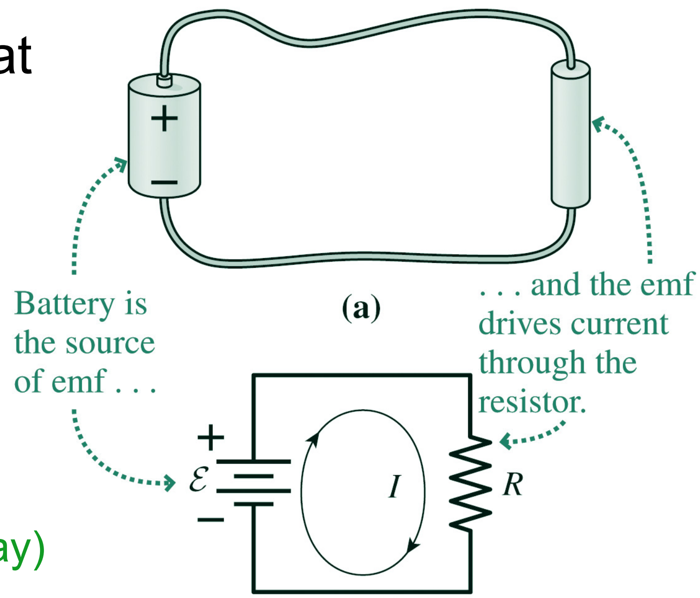
$$P = IV = I^2 R = \frac{V^2}{R} \qquad V = IR$$

Circuits, Symbols, and EMF

- Electric circuits are portrayed with diagrams using standard symbols, showing interconnections among their components.
  - We've seen wires, capacitors, and resistors so far



- EMF, or **electromotive force**, is a device that creates a potential difference between its sides.
  - For example, a battery or power supply
  - It supplies **electrical energy**
  - It has a + side and a – side
    - Current flows from + to – on an EMF or battery
    - (Remember that electrons flow the opposite way)



Ground and Grounding

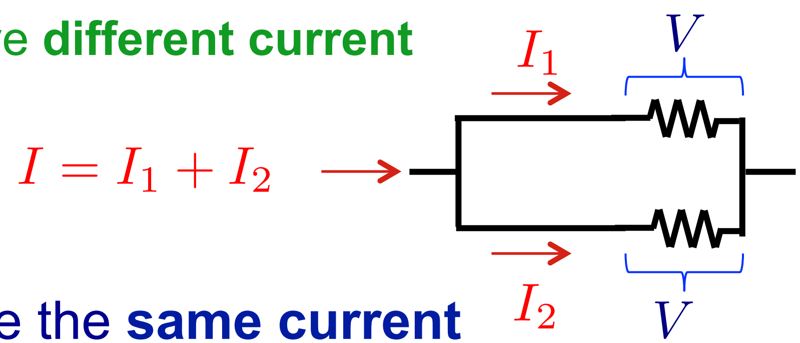
- One of the electrical symbols above was for **ground**
  - All wires connected to ground have an electric potential of zero
  - Current flows from positive potential to ground
  - Current flows from ground to negative potential
- This is the electrical equivalent of us setting electric potential to zero at infinity (or very far away)
  - "Ground wants to make stuff electrically neutral"
- Ground can be a source or sink of an effectively infinite amount of electrical charges or current
  - Think of it as a vast conductor with many charges
    - Perhaps as vast as the earth
      - Hmm, maybe that's why it's called "ground"
        - » ya think?

I wonder how small this font gets…

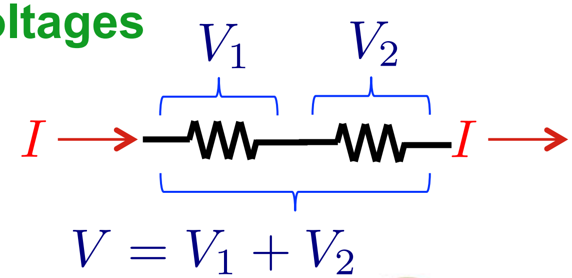
Back to Resistors

- Just like capacitors, we'll have to deal with resistors in series and parallel

- Our important principles still hold true, but now for resistors!
  - Elements that are in **parallel** have the **same voltage**
    - They share the same conductors on both sides
    - But they can (and often do) have **different current**



  - Elements that are in **series** have the **same current**
    - Or, for capacitors, the **same charge**
    - But they can (and often do) have **different voltages**

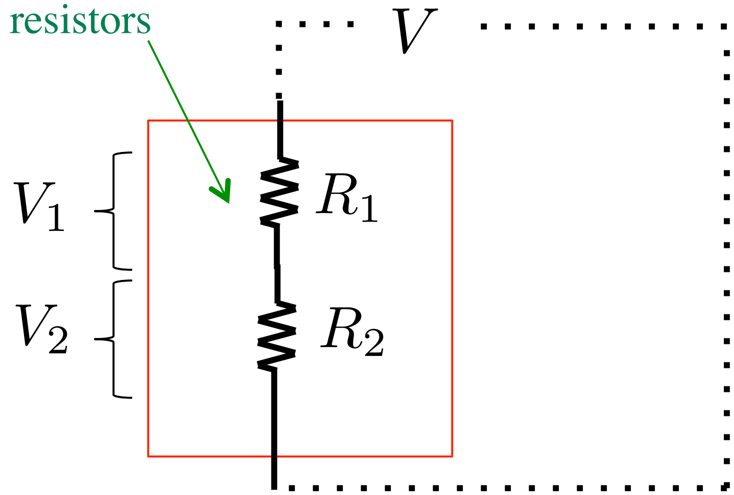


Connecting Resistors in Series

Here the resistors have the **same current *I*** but **different voltages *V_1 and V_2***

Same current I
in both resistors

$V = IR$



$$V_1 = IR_1$$

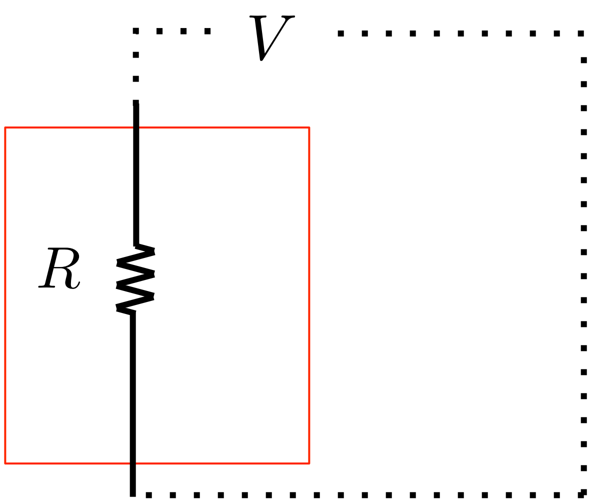
$$V_2 = IR_2$$

$$V = V_1 + V_2$$

$$IR = IR + 1 + IR_2$$

$$R = R_1 + R_2$$

Equivalent
resistance

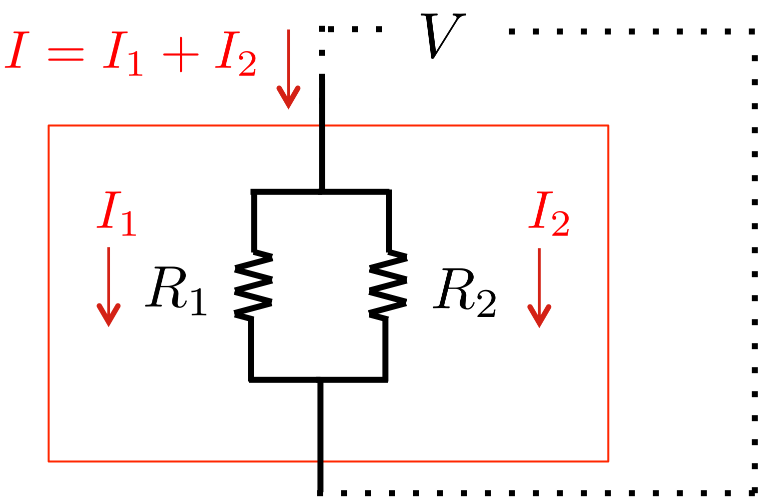


$$V = IR$$

Connecting Resistors in Parallel

Here the resistors have the **same potential difference *V*** but **different currents *I***

$$V = IR$$



$$I_1 = \frac{V}{R_1}$$

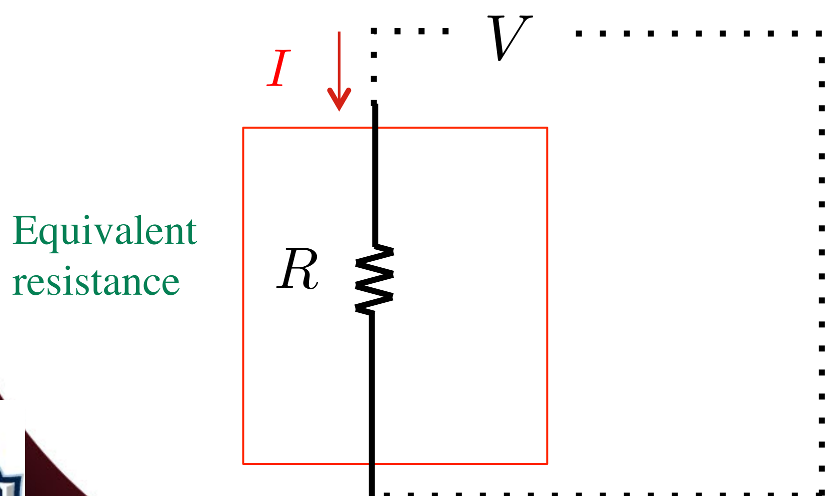
$$I_2 = \frac{V}{R_2}$$

$$I = I_1 + I_2$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = \frac{V}{R}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$



Analyzing Circuits with Series and Parallel Components

TACTICS 25.1 Analyzing Circuits with Series and Parallel Components

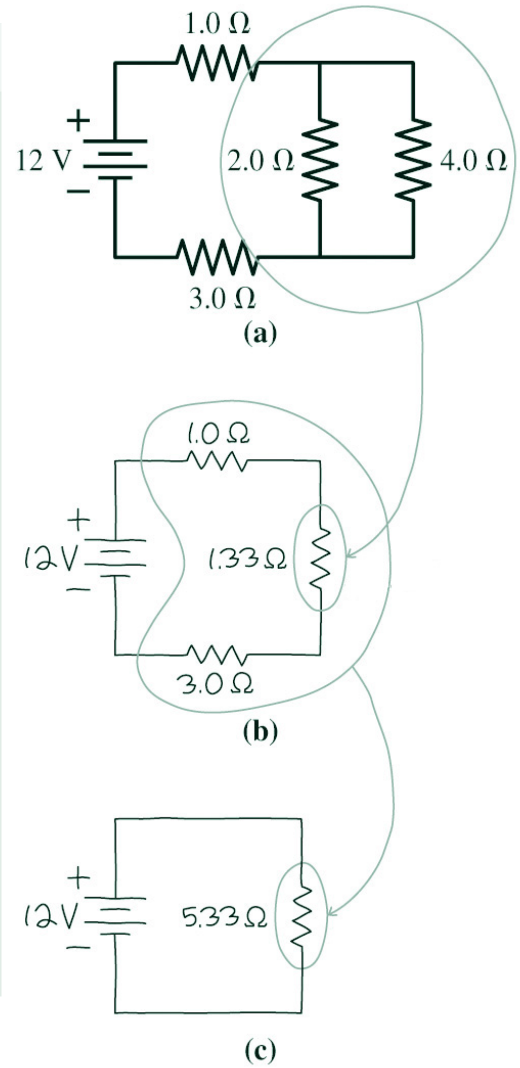
1. Identify series and parallel combinations. Remember that components are in parallel *only* if they're connected directly together at each end. Components are in series *only* if current through one component has no place to go but through the next component. If you can't find at least one series or parallel combination, then you have to use the methods of Section 25.3.
2. Solve for the series and parallel equivalents using Equations 25.1 and 25.3 for resistors:

$$R_{\text{series}} = R_1 + R_2 + R_3 + \cdots \quad (25.1)$$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \quad (25.3a)$$

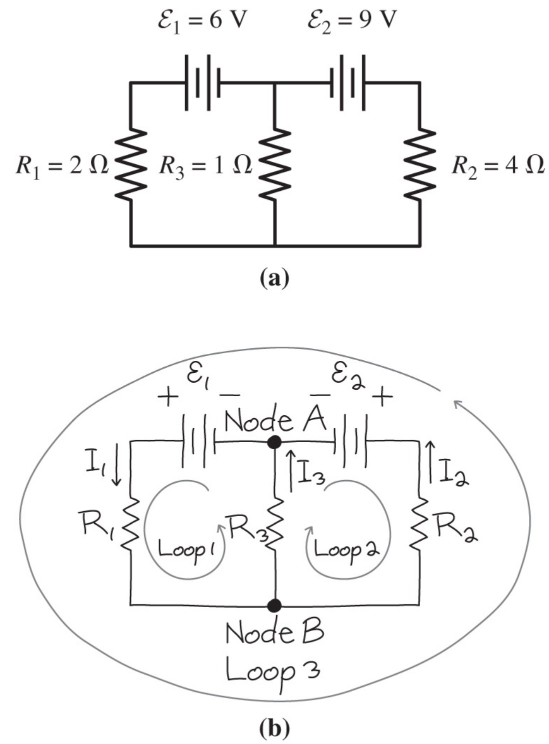
$$R_{\text{parallel}} = \frac{R_1 R_2}{R_1 + R_2} \quad (25.3b)$$

   If you're dealing with capacitors, use Equations 23.6 and 23.5, respectively.
3. Redraw the circuit, replacing series and parallel combinations with their one-component equivalents.
4. Repeat Steps 1–3, each time identifying series and parallel combinations and then reducing each to a single equivalent. Continue until either you've found the quantity you're asked for or the circuit consists of just an emf and one other component. You can then solve for the current in this component.
5. Work backward, replacing series and parallel equivalents with combinations of individual components. At each point apply Ohm's law, $I = V/R$, to find the currents through and/or the voltages across the individual components. As you work backward, remember that series components carry the same current as their series equivalent, and parallel components have the same voltage as their parallel equivalent. Continue until you're able to evaluate the quantity you're asked for.



Multiloop Circuits

- Some circuits aren't amenable to series-parallel analysis.
- Then it's necessary to use **Kirchhoff's loop** and **node** laws:
  - The loop law states that the sum of voltage drops around any circuit loop is zero.
    - The loop law expresses conservation of energy.
  - The node law states that the sum of currents at any circuit node is zero.
    - The node law expresses conservation of charge.

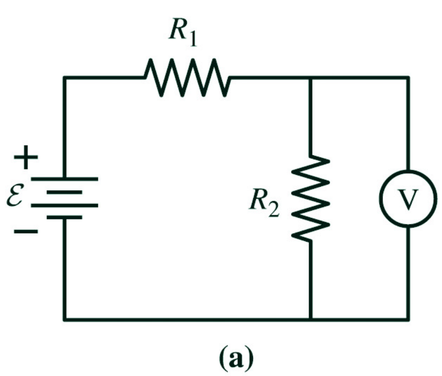


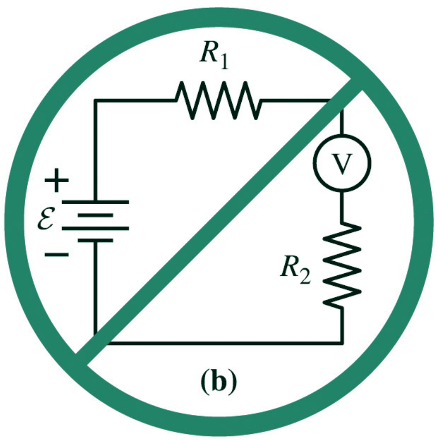
Node and loop equations for this circuit:

$$-I_1 + I_2 + I_3 = 0 \qquad \text{(node A)}$$
$$6 - 2I_1 - I_3 = 0 \qquad \text{(loop 1)}$$
$$9 + 4I_2 - I_3 = 0 \qquad \text{(loop 2)}$$

Electrical Measurements: Voltage

- A **voltmeter** measures the potential difference between its two terminals.
  – Connect a voltmeter in parallel with the component whose voltage you're measuring.
  – An ideal voltmeter has infinite resistance so it doesn't affect the circuit being measured.
  – A real voltmeter should have a resistance much greater than resistances in the circuit being measured.

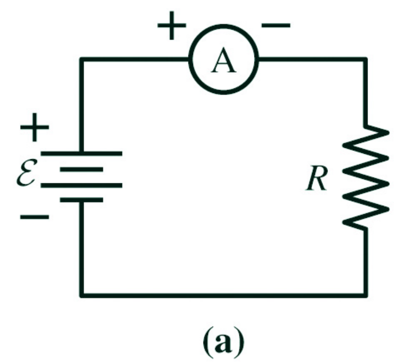


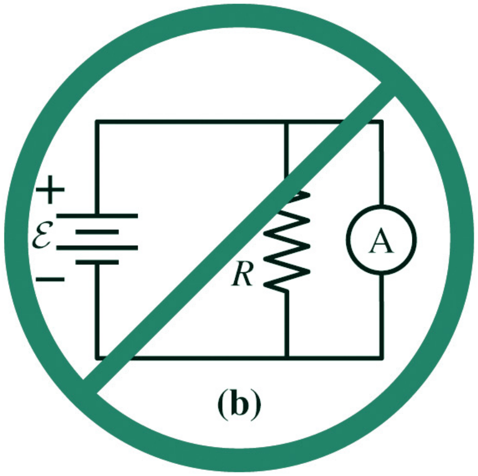


Correct (a) and incorrect (b) ways to measure the voltage across R_2

Electrical Measurements: Current

- An **ammeter** measures the current flowing through itself.
  - Connect an ammeter in series with the component whose current you're measuring.
  - An ideal ammeter has zero resistance so it doesn't affect the circuit being measured.
  - A real ammeter should have a resistance much less than resistances in the circuit being measured.





Correct (a) and incorrect (b) ways to measure the current in the series circuit

Capacitors in Circuits

- Capacitors introduce time-dependent behavior to circuits.
- The voltage across a capacitor is proportional to the charge on the capacitor.
- The charge can't change instantaneously, because that would require an infinite current to move a finite amount of charge onto the capacitor in zero time.
- Therefore, **the voltage across a capacitor cannot change instantaneously.**

The *RC* Circuit

- The capacitor voltage V_C is initially zero.
- Therefore, current flows through the resistor, putting charge on the capacitor.
- As the capacitor charges, V_C increases and the voltage across the resistor decreases.
- Therefore, the current decreases.
- Eventually a steady state is reached, with zero current and capacitor voltage equal to the battery emf.