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Happy Birthday to Rob Karsashian, Mia Hamm, and Billy Corgan!

Happy St Patrick’s Day!!!
Exam #2 Score Distribution

number of students

Total midterm score, PHYS232 Second Midterm 2014

ODU University Physics 227N/232N
The first problem seemed to have the lowest average score

Main concepts:
- **Parallel** components have the **same voltage** across them
  - They have the same conductors on both sides
  - But current “splits” through the paths it can take through parallel components
- **Serial** components have the **same current** going through them
  - Where else could the current go?
  - But voltage across each item in a series may be different
On to Magnetism

- You probably have some experience with magnets
  - They all have north and south poles
    - (But what about fridge magnets? We’ll get to that magic later…)
  - “Like poles repel, unlike poles attract”
  - The force between them gets weaker with larger separation
  - Some metals are magnetic (iron) while some are not (copper)

- What creates magnetic fields?
  - How are they like (and unlike) electric fields?
All magnets have two poles: they are magnetic dipoles!

- Recall: Electric dipoles, separated equal +/- electric charges
- But unlike electric dipoles, we cannot separate the poles of a magnet into individual poles
  - There are no “magnetic charges” or “magnetic monopoles”
  - So we concentrate on the magnetic field rather than charges

- Magnet poles are called “North” (like + charge) and “South” (like – charge)
  - Some of the earliest magnet observations: Earth’s magnetic field
    - (China ~200 BC! Navigation: ~1000-1200 AD)
We draw **magnetic field lines** like we did with electric field lines

- Just like electric dipole field lines
- They start at “north” and point towards “south”: poles are their only endpoints
- Greater density = larger magnetic field

**Opposite poles attract**: magnetic field lines hook up together and the magnetic poles attract each other

**Same poles repel**: magnetic field lines “push” each other apart and The magnetic poles repel each other
Magnetic Forces: A Confusing Convention

- The north pole of a magnet is attracted to the south pole of another magnet…

- But a compass needle’s north pole points north!
  - Towards the earth’s north pole which must be a magnetic north pole, right? But but but…

- There’s only one conclusion: someone named magnetic poles north/south before they understood magnetism.

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The Earth’s (confusing) Magnetic Field

- This took a while to figure out
  - We had to adopt certain sign conventions in studying magnets
  - We had to connect magnets to electric fields etc
  - Conclusion: When field lines go from north to south (like an electric dipole), the compass bar magnet is labeled correctly
    - Earth’s north geographic pole is actually a magnetic south pole, and vice versa

Variable over decades, flips about every 100k-50M years!
Geomagnetism and You

- The Earth’s magnetic field shields us from solar radiation and potentially destructive solar flares.
Back to (mundane) Terrestrial Concerns

- Wait! We haven’t had any formulas yet in this lecture!!
  - How are we going to do our homework!?!?

- No worries! Unless…

- What about the forces between bar magnets?
  - Bar magnets are common to our intuition
  - We should be able to calculate these forces, right?
  - But there are no magnetic charges or magnetic monopoles!
    - So there are no point charges, no obvious simple distances
    - It depends on how big the magnets are, all sorts of gory details
      - It ends up requiring calculus and, well, is hard and not very illustrative
      - It doesn’t even have a simple scaling law 🙄
Bar Magnet Relationship Status: “It’s Complicated”

- It’s even complicated enough to have its own Wikipedia page

![Diagram of magnetic field and force between two bar magnets]

\[
F = \left[ \frac{B_0^2 A^2 (L^2 + R^2)}{\pi \mu_0 L^2} \right] \left[ \frac{1}{x^2} + \frac{1}{(x + 2L)^2} - \frac{2}{(x + L)^2} \right]
\]

where

- \(B_0\) is the magnetic flux density very close to each pole, in T,
- \(A\) is the area of each pole, in m²,
- \(L\) is the length of each magnet, in m,
- \(R\) is the radius of each magnet, in m, and
- \(x\) is the separation between the two magnets, in m

\(B_0 = \frac{\mu_0}{2} M\) relates the flux density at the pole to the magnetization of the magnet.

- So instead let’s concentrate on magnetic fields, their effect on charged particles, and how magnetic fields are actually created
Magnetic Fields

- **Magnetic field is a vector field**
  - Recall: vector fields have magnitude and direction at all points in space
  - Like the electric field that we considered earlier this semester
    - It’s denoted with the symbol $\vec{B}$
    - It’s in units called **Tesla [T]** (or Gauss [G], 1 G = $10^{-4}$ T)
      - Earth’s surface magnetic field is $B = 0.3 - 0.6 \text{ G} = 3 - 6 \times 10^{-5} \text{ T}$
      - Bar magnet: $B \approx 0.01 - 0.02 \text{ T}$
      - Particle accelerator magnets: $B \approx 0.1 - 10 \text{ T}$
  - Let’s consider the **force** from this magnetic field on an electrically charged point particle
    - This ends up having simple enough rules that we can draw some conclusions
    - The ability of magnetic fields to influence electrically charged particles foreshadows **Maxwell’s unification of electric and magnetic forces and fields** (1865)
Magnetic Forces on Charged Particles

- A magnetic field $\vec{B}$ exerts forces on charged particles
  - Magnetic force, however, is not as simple as electric force

- Recall: electric forces were like gravity
  - Acts in straight lines between electric charges (or masses)

\[ \vec{F}_{\text{electric}}(q_1 \text{ on } q_2) = \frac{kq_1q_2}{r_{12}} \hat{r}_{12} = \vec{E}_1 q_2 \]

- Magnetic fields exert force only on moving electric charges
  - This force is proportional to the moving charge’s velocity
  - The direction of the force is **perpendicular** to both the charge’s velocity and the magnetic field vector

\[ \vec{F}_{\text{magnetic}}(\text{on } q) = q\vec{v} \times \vec{B} \]

Vector cross product!
\[ 1 \text{ N} = (1 \text{ C})(1 \text{ m/s})(1 \text{ T}) \]
Magnetic Forces on Charged Particles

\[ \vec{F}_{\text{magnetic}}(\text{on } q) = q\vec{v} \times \vec{B} \]

Vector cross product!
(1 N) = (1 C)(1 m/s)(1 T)

\[ F_{\text{magnetic}}(\text{on } q) = |q|vB \sin \theta \quad \theta \text{ measured between } v \text{ and } B \]

- To figure out the direction of a **cross product vector**, we use the **right hand rule**
  - Fingers in direction of first, curl to direction of second, follow thumb
  - First and second vectors parallel (no curl) = no force!

- Force points **out of screen** for \( q > 0 \)
- Force points **into screen** for \( q > 0 \)
Word Problem Example

A charge of $0.3 \, \mu C$ is moving in the $-\hat{i}$ direction at $10 \, m/s$. A magnetic field of $0.3 \, T$ is pointing in the $+\hat{j}$ direction. What is the magnitude and direction of the force on the charge? How does your answer change if the charge is $-0.3 \, mC$?

$$F_{\text{magnetic (on q)}} = |q|vB \sin \theta \quad \theta \text{ measured between } v \text{ and } B$$

$$F = qvB \sin \theta = (0.3 \times 10^{-6} \, C)(10 \, m/s)(0.3 \, T) = 9 \times 10^{-7} \, N = F$$

Using the right hand rule, the force is in the $-\hat{k}$ direction.

If the charge is reversed, the direction of the force is reversed and therefore in the $+\hat{k}$ direction.
Example: Find the magnetic forces on these charges

\[ \vec{B} = 0.3 \, \text{T (constant)} \]

\[ \vec{F}_{\text{magnetic (on q)}} = q\vec{v} \times \vec{B} \]

- \( q_1 = 0.3 \, \mu\text{C} \) \( \vec{v}_1 = (10 \, \text{m/s}) \hat{i} \)
- \( q_2 = -0.3 \, \mu\text{C} \) \( \vec{v}_2 = (10, 10) \, \text{m/s} \)
- \( q_3 = 0.3 \, \mu\text{C} \) \( \vec{v}_3 = (10 \, \text{m/s}) \hat{k} \)
- \( q_4 = 0.3 \, \mu\text{C} \) \( \vec{v}_4 = (10 \, \text{m/s}) (\hat{i} + \hat{k}) \)

(\text{indicates } B \text{ field lines pointing OUT of page, } +\hat{k} \text{ direction})
Problem (10 minutes): Find Forces (magnitude/direction)

\[ \vec{B} = 0.3 \text{ T (constant)} \]

\[ \vec{F}_{\text{magnetic (on q)}} = q\vec{v} \times \vec{B} \]

\[ q = 0.3 \mu \text{C}, \quad v = 10 \text{ m/s for all} \quad \text{(in plane of page)} \]
Problem: Circular motion!

\[ \vec{B} = 0.3 \text{ T (constant)} \]

\[ \vec{F}_{\text{magnetic}}(\text{on } q) = q\vec{v} \times \vec{B} \]

- Constant acceleration towards the center of a circle
  - Circular motion!

\[ F = qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB} \]

“Cyclotron radius”
Cyclotron Radius and Frequency

The revolution frequency is

\[ f = \frac{v}{2\pi r} = \frac{qB}{2\pi m} = f \]

- This is the **cyclotron frequency** of a particle of charge \( q \) and mass \( m \) moving in a magnetic field of strength \( B \)
- A cool observation: This is **independent of particle velocity**
- Larger velocity particles move around in larger circles
- We can use this to build a particle accelerator

**Cyclotron radius**

\[ F = qvB = \frac{mv^2}{r} \implies r = \frac{mv}{qB} \]

Dots represent magnetic field lines coming out of the page.

The magnitude of the velocity is constant.

Charged particles tend to spiral around the magnetic field lines.
The cyclotron alternates electric field between two “D" shaped pieces of metal
- Like a big capacitor
- Creates alternating electric field between gaps
- Alternate electric field at cyclotron frequency
- Then particles always come around at the right time to catch the next peak electric field
- Particles spiral out, gaining energy with every spiral as they go
  - Be careful about velocity pointing out of the page!
Ponderable

- Solar wind ions (atomic nuclei stripped bare of their electrons) would continuously bombard Earth’s surface if most of them were not deflected by Earth’s magnetic field. Given that Earth is, to an excellent approximation, a magnetic dipole (a bar magnet), the intensity of these ions bombarding its surface is greatest at the
  1. poles.
  2. mid-latitudes.
  3. equator.
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It’s the Aurora Borealis!
Review Ponderable

- Magnetic field lines start and stop at individual north and south poles (magnetic charges) just like electric field lines start and stop at individual electric charges.
  - True
  - False
A magnetic field exerts a force on an electrically charged particle…

- Always
- Never
- If the particle is moving parallel the field lines
- If the particle is moving at an angle to the field lines
- If the particle is at rest
An electron and proton are moving in the same direction, perpendicular to a uniform magnetic field. They are both charged particles moving in a magnetic field so they each experience a magnetic force. What can you say about these magnetic forces?

- They are the same magnitude and direction.
- They are the same magnitude but opposite directions.
- They are the same direction but different magnitude because the masses of the electron and proton are different.
- They are opposite directions and different magnitudes because the masses of the electron and proton are different.
Determine the direction of the magnetic field for each case, assuming the particle is positively charged.

(a) 

(b) 

(c)
Reviewing So Far

- Magnetic fields point from north to south pole
  - There are no magnetic monopoles
  - Bar magnets have north and south poles (magnetic dipoles)
  - Earth’s magnetic north pole is its geographical south pole
- Magnetic field is a vector field, denoted by $\vec{B}$ [T]

\[ \vec{F}_{\text{magnetic}} = q\vec{v} \times \vec{B} \]

- Its direction is given by the right hand rule and sign of the charge, and magnitude by

\[ F_{\text{magnetic}} = |q|vB \sin \theta \]

- Charged particles with velocity perpendicular to $B$ move in circles or arcs of circles
  - The revolution frequency is independent of particle velocity: cyclotron motion
- A component of velocity along $B$ will make this path into a spiral or corkscrew motion
Moving Charges = Currents

- We often have a lot of moving charges together in conductors
  - This is a current, \( I \equiv \frac{dQ}{dt} \)
  - A current-carrying conductor experiences a magnetic force
  - This is similar to the \( \vec{F} = q\vec{v} \times \vec{B} \) equation

\[
\vec{F}_B \text{ field on current } = I\vec{L} \times \vec{B} = ILB \sin \theta
\]
Example

- A square wire loop of side length $L=33\text{cm}$ is placed in an area of magnetic field (shaded) as shown on the right, and can turn around the vertical dotted axis. The loop is flat and the field $B=0.3 \text{ T}$ points to the right. A constant current of $I=1 \text{ A}$ is run through the loop.
  - What is the torque on the loop around the vertical axis?
  - As seen from the power supply end, does it turn clockwise or counterclockwise?
Magnetic field not only produces forces on moving electric charges, magnetic field also **arises from moving electric charge**.

- The **Biot-Savart law** gives the magnetic field arising from an infinitesimal current element:
  \[
  d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{L} \times \hat{r}}{r^2}
  \]

- The field of a finite current follows by integrating:
  \[
  \vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi} \frac{I \, d\vec{L} \times \hat{r}}{r^2}
  \]

where \( \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \) is the **permeability constant**.