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University Physics 227N/232N

Ch: 26-27: Magnetism and Magnetic Induction

Lab this Friday, Mar 21: Ohms Law and DC RC Circuits

So NO QUIZ this Friday! But you have homework!

Dr. Todd Satogata (ODU/Jefferson Lab)

satogata@jlab.org

http://www.toddsatogata.net/2014-ODU

Wednesday, March 19 2014

Happy Birthday to Clayton Kershaw, Harvey Weinstein, Glenn Close,
David Livingstone, and Frederic Joliot-Curie (1935 Nobel)

Reviewing So Far

- Magnetic fields point from north to south pole
  - There are no magnetic monopoles
  - Bar magnets have north and south poles (magnetic dipoles)
  - Earth's magnetic north pole is its geographical south pole
- Magnetic field is a vector field, denoted by $\vec{B}$ $[\mathrm{T}]$

$$\vec{F}_{\mathrm{magnetic}} = q\vec{v} \times \vec{B}$$

  - Its direction is given by the **right hand rule** and sign of the charge, and magnitude by

$$F_{\mathrm{magnetic}} = |q| v B \sin\theta$$

  - Charged particles with velocity perpendicular to *B* move in circles or arcs of circles
    - The revolution frequency is independent of particle velocity: **cyclotron motion**
  - A component of velocity along *B* will make this path into a spiral or corkscrew motion

Review Ponderable

- Magnetic fields can be used to make charged particles go faster or slower.
  - True
  - False

Review Ponderable

- A magnetic field exerts a force on an electrically charged particle…
  - Always
  - Never
  - If the particle is moving parallel the field lines
  - If the particle is moving at an angle to the field lines
  - If the particle is at rest

Ponderable

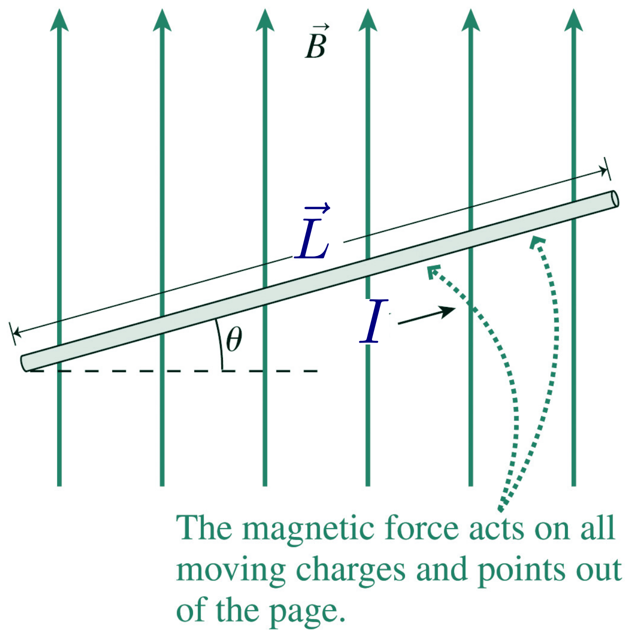
- A uniform magnetic field points out of this page. An electron that's moving in the plane of the page will circle _______________ as viewed from above the page.

A. clockwise
B. counterclockwise

Review: Moving Charges = Currents

- We often have a lot of moving charges together in conductors
  - This is a current, $I \equiv \frac{dq}{dt}$ $\qquad q\vec{v} = q\frac{d\vec{L}}{dt} \quad \Rightarrow \quad q\vec{v} = \frac{dq}{dt}\vec{L} = I\vec{L}$
  - A current-carrying conductor experiences a magnetic force
  - This is similar to the $\vec{F} = q\vec{v} \times \vec{B}$ equation (just move the dt over)

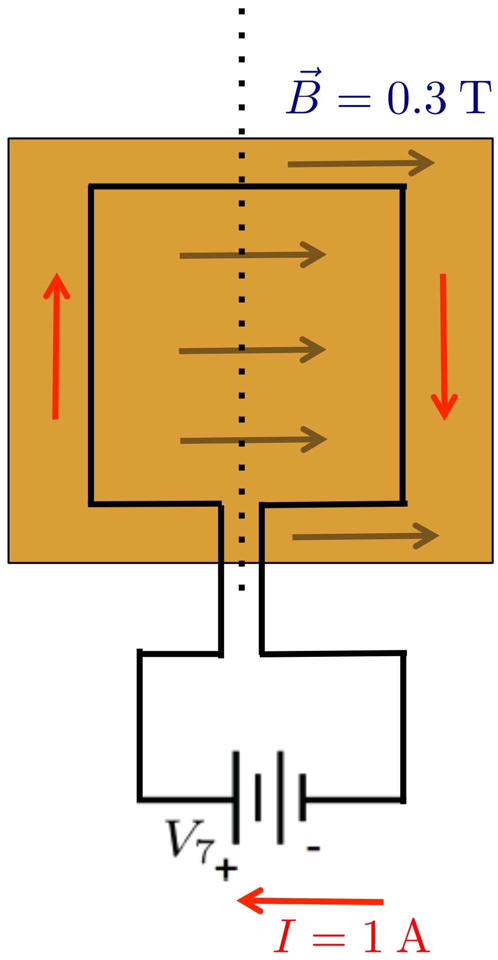
$$\boxed{\vec{F}_{\text{B field on current}} = I\vec{L} \times \vec{B}} = ILB\sin\theta$$



$\vec{L}$ points in the direction of the current *I*

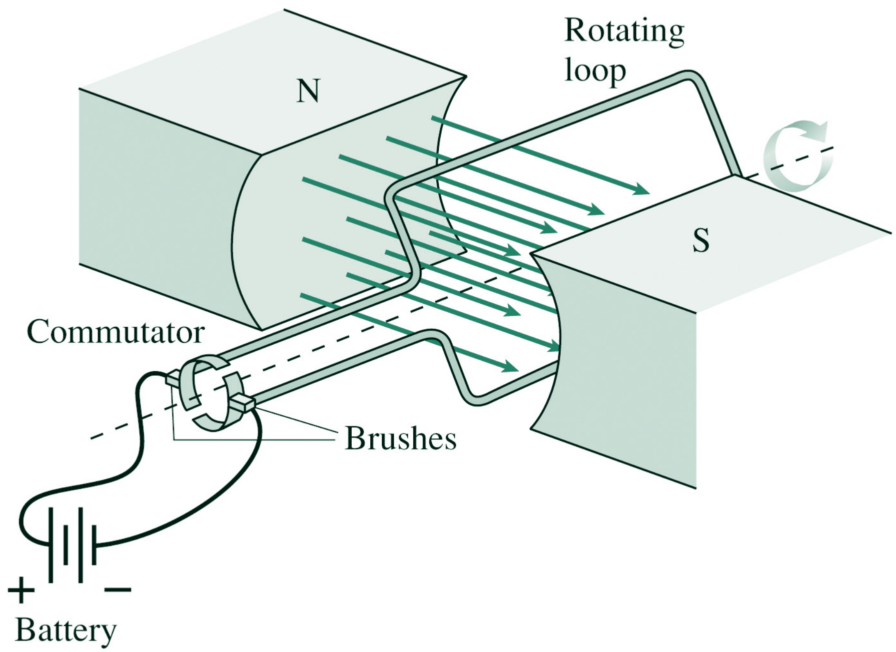
Example

- A square wire loop of side length *L*=33 cm is placed in an area of magnetic field (shaded) as shown on the right, and can turn around the vertical dotted axis. The loop is flat and the field *B*=0.3 T points to the right. A constant current of *I*=1 A is run through the loop.
  - What is the **torque** on the loop around the vertical axis?
  - As seen from the power supply end, does it turn clockwise or counterclockwise?



Concept: DC Electric Motors

- The electric motor is a vital technological application of the torque on a current loop.
- A current loop spins between permanent magnet poles.
- In a DC motor, the **commutator** keeps reversing the current direction to keep the loop spinning in the same direction.



How To Create A Magnetic Field

- Recall: (Stationary) electric charges produce electric fields

$$\vec{E} = \sum \frac{kq_i}{r_i^2}\hat{r}_i$$

- Magnets don't have point charges. Indeed, **all magnetic fields come from currents** (moving electric charges)
  - This is another deep connection between electricity/magnetism
- The equation for this is known as the **Biot-Savart Law**

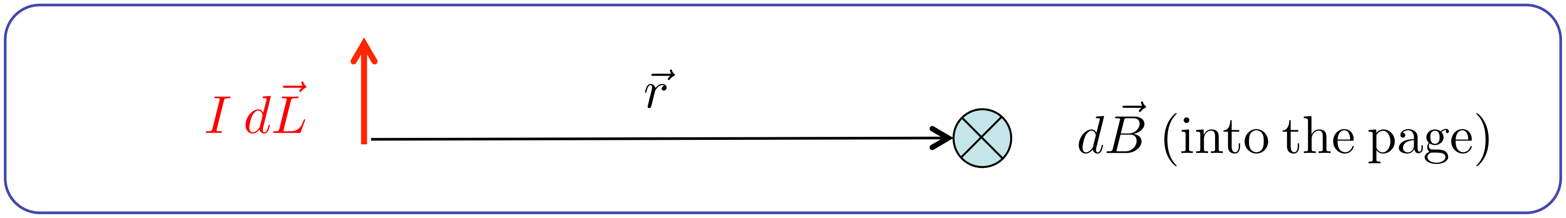
$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2}} \qquad \mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T-m/A}$$

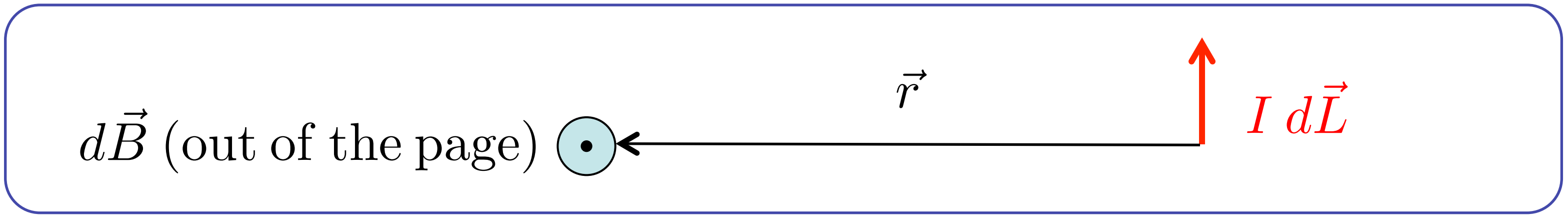
  - Here $\vec{r}$ is measured to the location of the magnetic field $\vec{B}$ that is created by the current I in a small length $d\vec{L}$

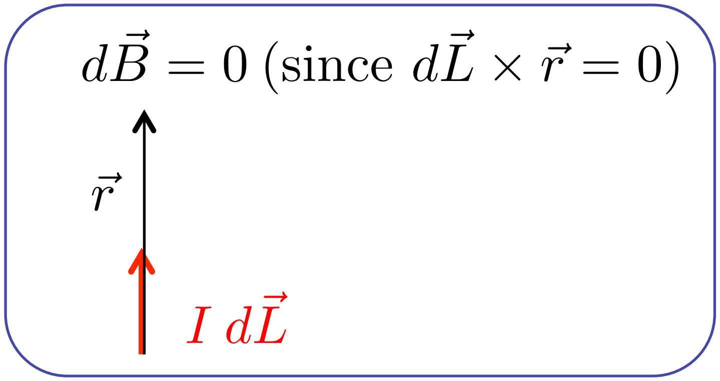
More Biot-Savart Field Direction Examples

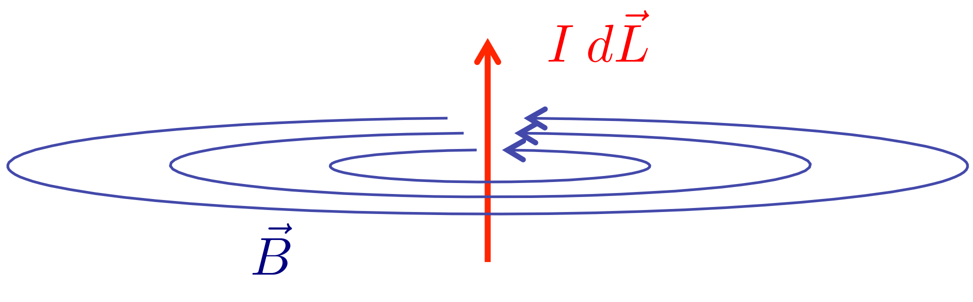
$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L} \times \hat{r}}{r^2}}$$

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T-m/A}$$



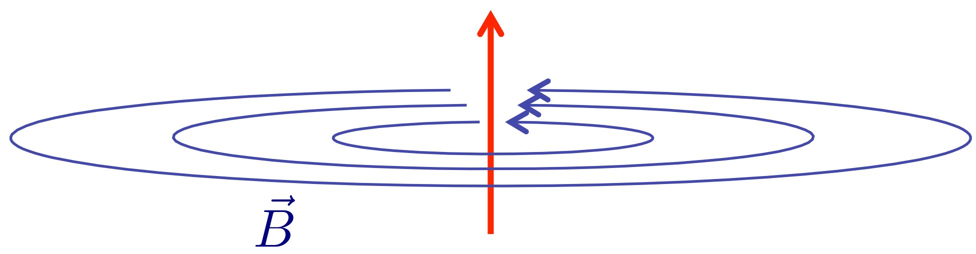




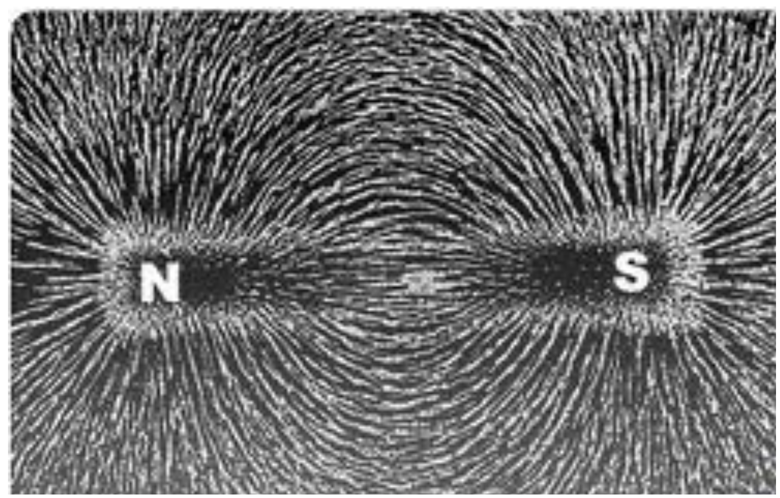


It sure looks like the magnetic field goes in right-handed circles (follows your right fingers around your right thumb) "around" the current element *I dL*

Concept: Magnetic Field Lines Are Closed Loops

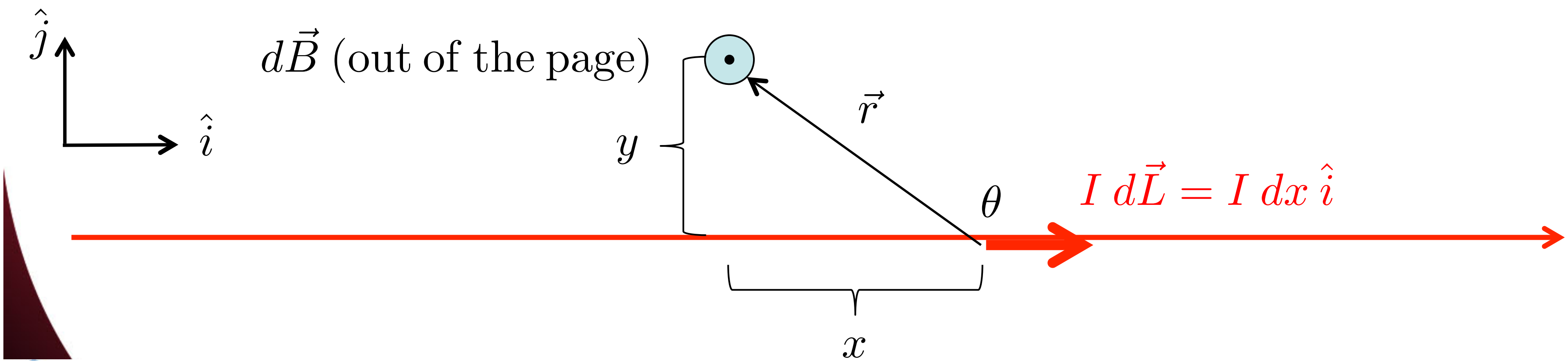


- Recall: there are **no magnetic monopoles** (charges)
  - Electric field lines started and stopped at electric charges
  - So magnetic field lines don't have anywhere to start or stop
  - **All magnetic field lines are closed loops (some "close" at infinity)**
- This even includes what look like starts and stops at north and south poles of bar magnets
  - The field lines here are actually connected inside the magnet. The magnetic fields are created by lots of little lined up loops of current at the atomic level.



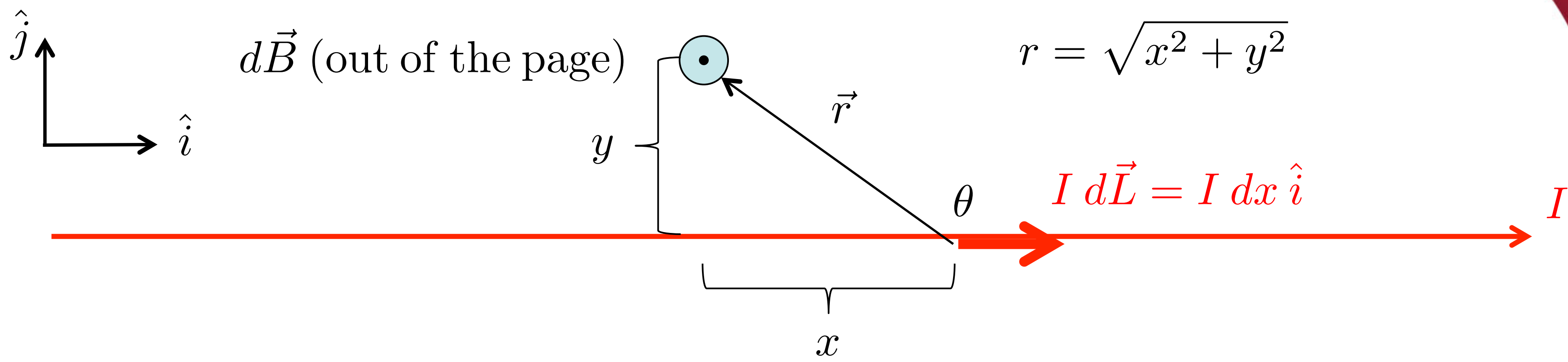
Back to Biot-Savart: Magnetic Field fom Current Line

- What's the magnetic field from an infinite line of current?
  - We've got the magnetic field from a small section of that line – Biot-Savart gives us that.
  - Add up all of those contributions to find the total magnetic field at a certain distance away from the line of current.
  - Recall: We did this for a line of (unmoving) electric charge before and got an electric field that pointed radially out or in from the line of charge, with a 1/r field strength dependence.



$$\vec{r} = x\hat{i} + y\hat{j} \qquad r = \sqrt{x^2 + y^2} \qquad \hat{r} = \frac{\vec{r}}{r} = \left(\frac{x}{r}\right)\hat{i} + \left(\frac{y}{r}\right)\hat{j}$$

Back to Biot-Savart: Magnetic Field from Current Line



$$r = \sqrt{x^2 + y^2}$$

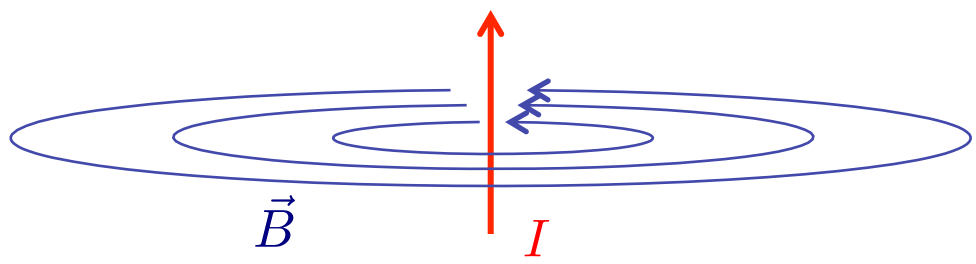
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{L} \times \hat{r}}{r^2} \qquad \hat{r} = \frac{\vec{r}}{r} = \left(\frac{x}{r}\right)\hat{i} + \left(\frac{y}{r}\right)\hat{j}$$

$$d\vec{L} \times \hat{r} = dx\left(\frac{x}{r}\cancelto{0}{(\hat{i} \times \hat{i})} + \frac{y}{r}(\hat{i} \times \hat{j})\right) = \frac{y\,dx}{r}\,\hat{k}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{y\,dx}{r^3} = \frac{\mu_0 I y}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\frac{\mu_0 I}{2\pi y} = B}$$

I will not ask you to figure out this integral. ☺

Problem: Infinite Lines of Current

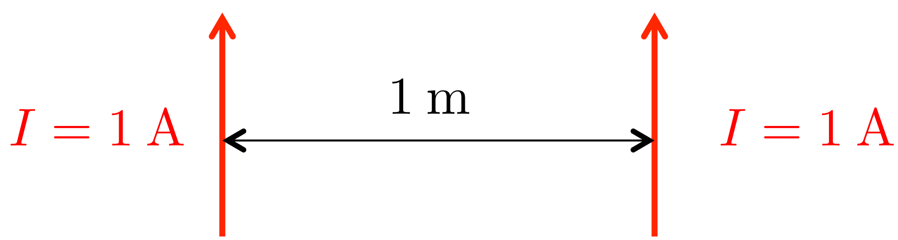


$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \quad (r \text{ is distance from line, direction is right hand around } I)$$

… and remember
$$\boxed{\vec{F}_{\text{B field on current}} = I\vec{L} \times \vec{B}}$$

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T-m/A}$$

Now consider two parallel infinite lines of *I*=1A current separated by 1m.

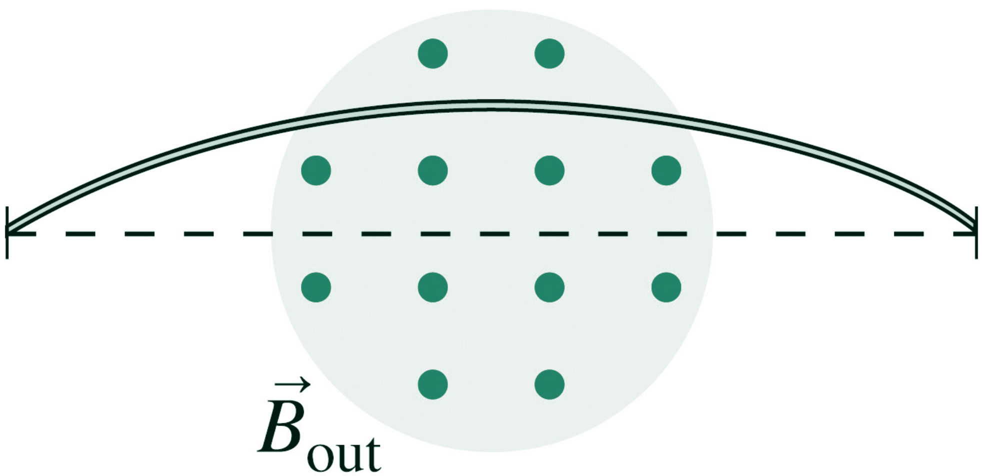


- Do they attract or repel each other? (remember right hand rules)
- With what force per meter do they attract or repel each other?
- What happens if you reverse the direction of one of the currents?

Ponderable

- The figure shows a flexible conducting wire passing through a magnetic field that points out of the page. The wire is deflected upward, as shown. In which direction is current flowing in the wire?

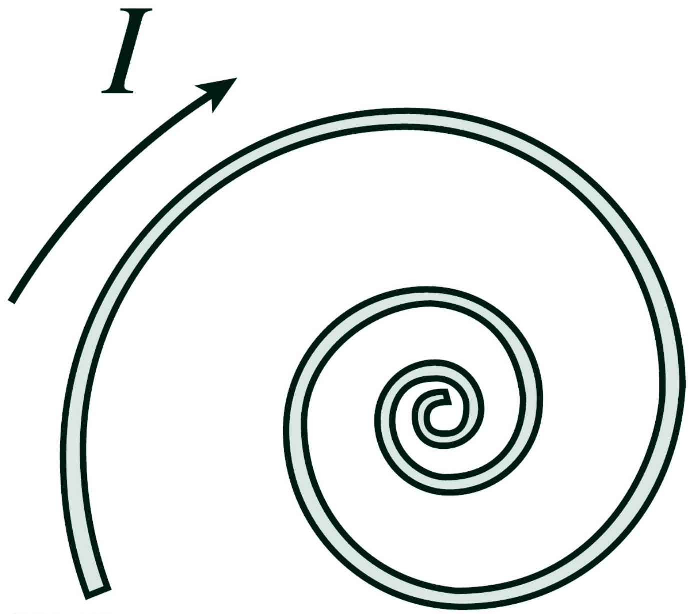
A. To the left
B. To the right



Ponderable

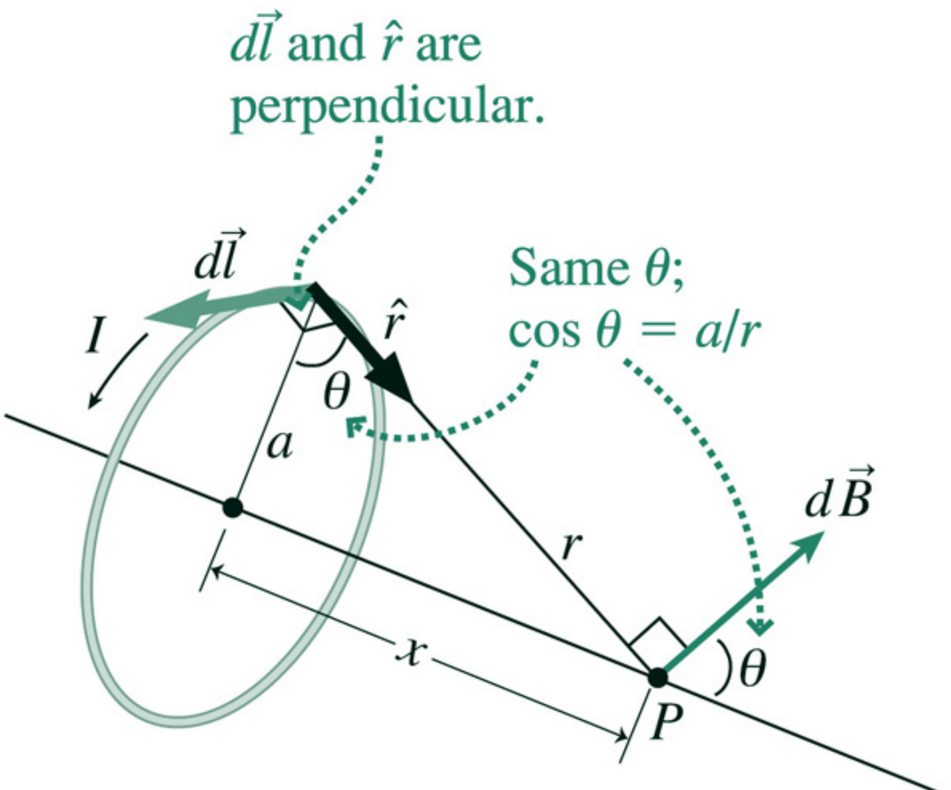
- A flexible wire is wound into a flat spiral as shown in the figure. If a current flows in the direction shown, will the coil tighten or loosen?

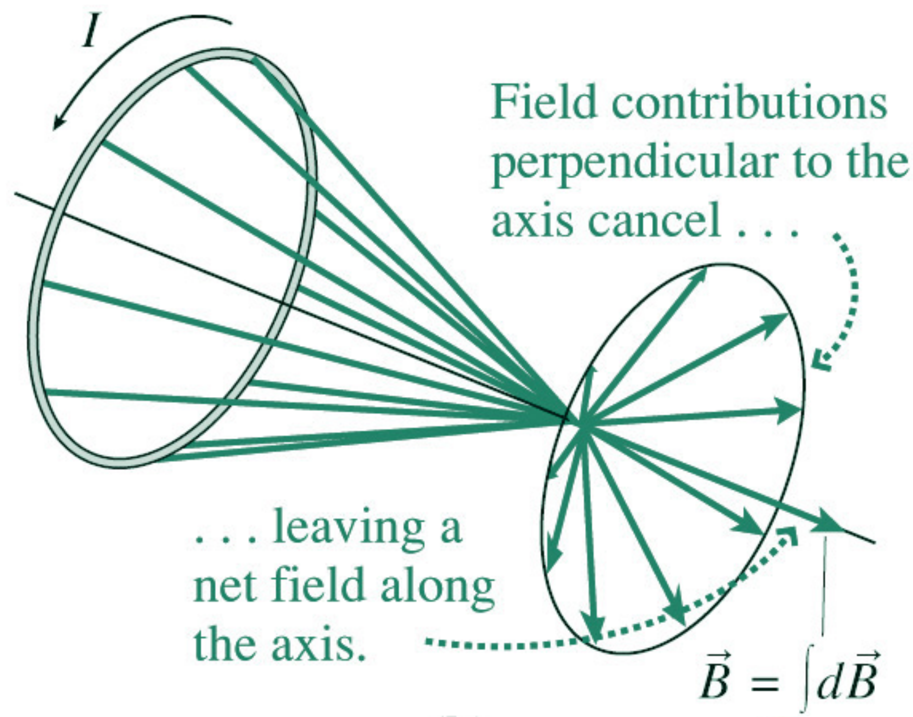
A. The coil will tighten.
B. The coil will loosen.



Lines and Loops

- We can also work through the math for the magnetic field from a current going in a circle (a "current loop")
  - This requires fancier art than I can easily draw… ☺
  - Symmetry: The B field along the axis must point along the axis!





And Then A Miracle Occurs (or, “Todd skips the math”)

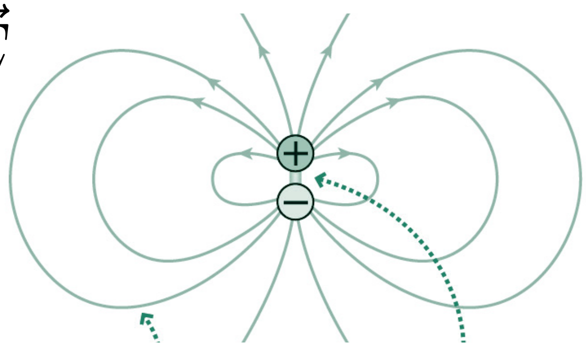
- We could work through all the math (yawn)
  - Or those who are interested (or masochists) can work through it and ask me questions via email (yay!)

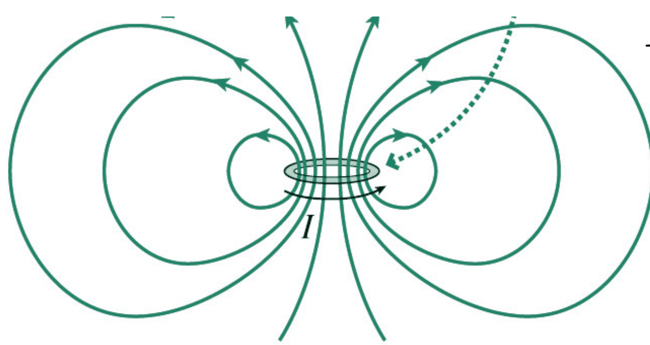
Along axis of current loop with radius a : $\boxed{B = \dfrac{\mu_0 I a^2}{2x^3}}$

(*x* is distance from center of loop)

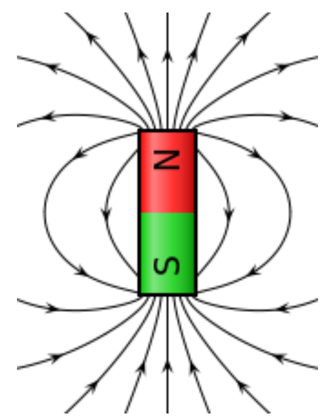
- The $1/x^3$ dependence looks (kinda) like an **electric dipole**
- We can make the loop very small – this becomes the simplest type of magnet (like our bar magnets), a **magnetic dipole**
- Really like a little (infinitely short) bar magnet

$\vec{E}$





$\vec{B}$



Concept: Dipole Moment

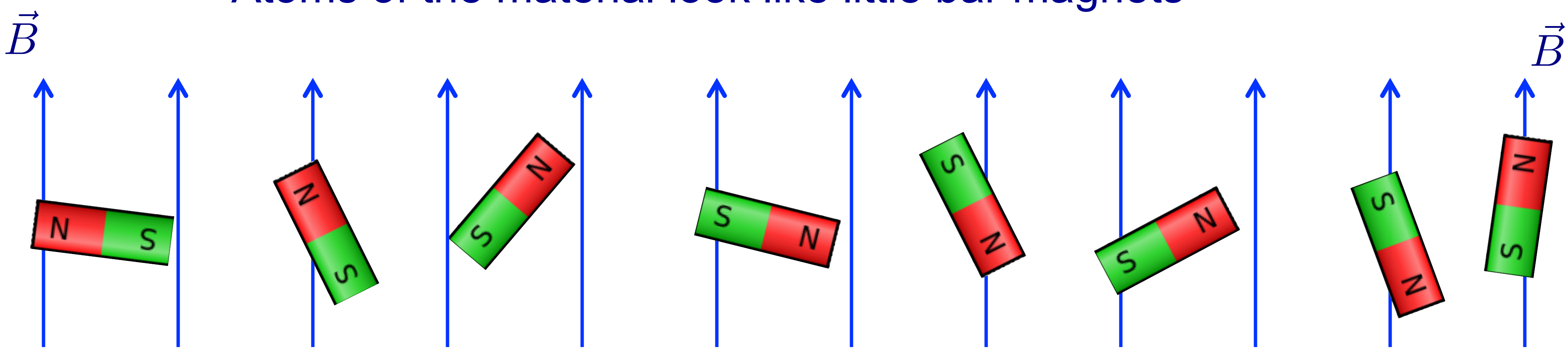
- Physicists said "hey, this looks kinda fundamental"
  - Can't really make any isolated closed current to create a magnetic field simpler than a small loop
  - Let's put all the stuff about the loop in one quantity and call it something new: **magnetic dipole moment**
  - Let's also give it the symbol μ just to confuse future students.
    - This is **not** related to μ_0 - doesn't even have the same units!

$$\boxed{\vec{\mu} = I\vec{A}} \quad (\vec{A}\text{ is area of loop})$$

  - Then for a very small loop, along the axis: $B = \dfrac{\mu_0\,\mu}{2\pi x^3}$
- "Kinda fundamental" at the level of atoms and beyond
  - Atoms have moving electrons: current loops!! Magnetic dipoles!
  - Some atoms look like little bar magnets!
  - Even weirder: charged subatomic particles have magnetic moments, even the electron!

Ponderable

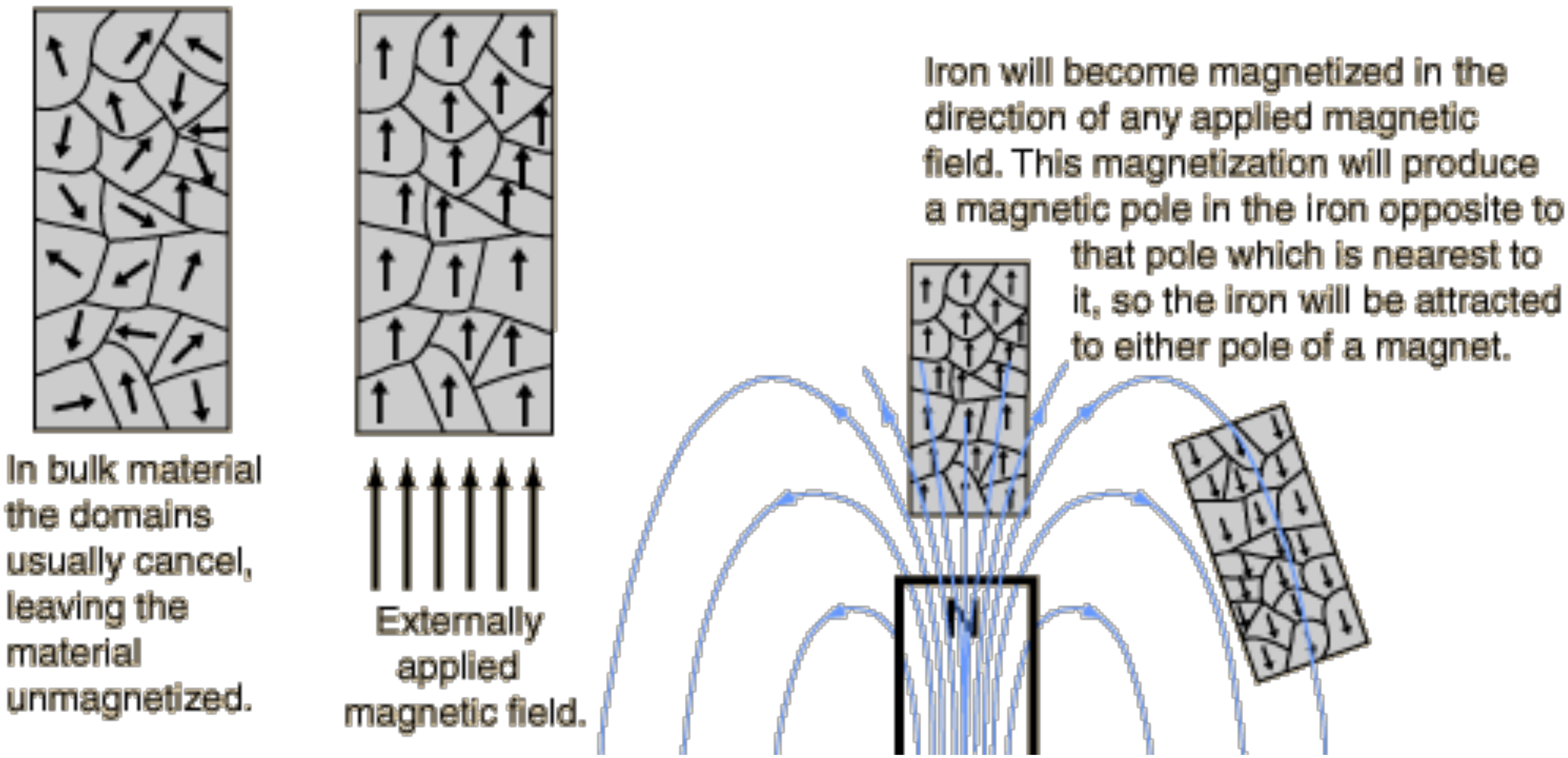
- A constant magnetic field is applied through a material that's not the vacuum
  - Atoms of the material look like little bar magnets



  - What kind of force do they experience in the magnetic field?
    - Are the north poles attracted in one direction?
    - Are the south poles attracted in one direction?
    - Are those two directions the same or different?

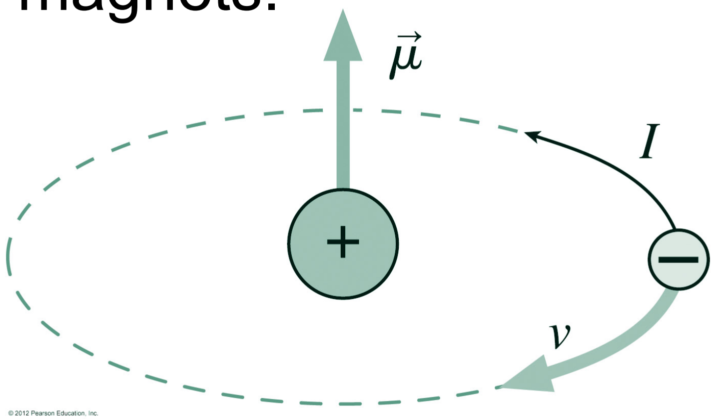
Magnetization and Magnetic Domains

- Usually groups of atoms ("domains") align to each other's magnetic fields anyway in a ferromagnetic material

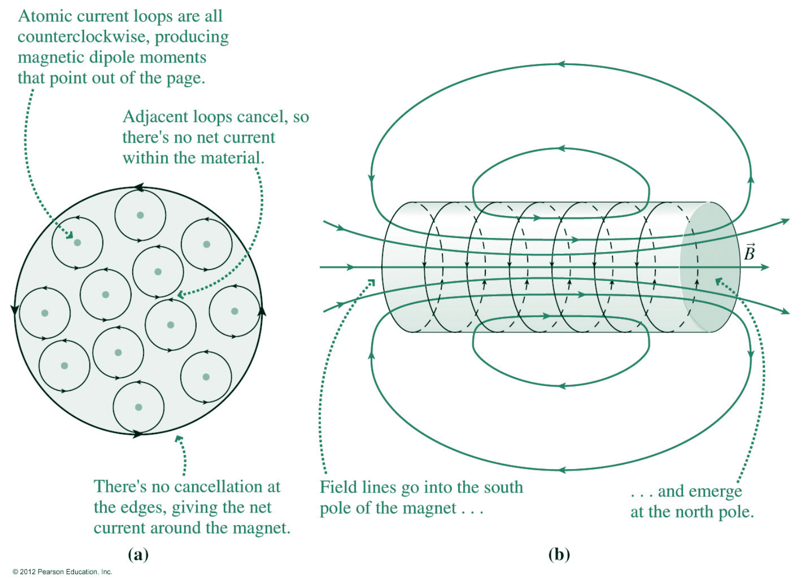


Concept: Magnetism in Matter

- Magnetism in matter arises from atomic current loops associated with orbiting and spinning electrons.
- In **ferromagnetic** materials like iron, strong interactions among individual magnetic dipoles result in large-scale magnetic properties, including strong attraction to magnets.
- **Paramagnetic** materials exhibit much weaker magnetism.
- **Diamagnetic** materials respond oppositely, and are repelled by magnets.



Classical picture of magnetic dipole moment arising from orbiting electron



Concept: Ampère's Law

- Gauss's law for electricity provides a global description of the electric field in relation to charge that is equivalent to Coulomb's law.
- Analogously, Ampère's law provides a global description of the magnetic field in relation to moving charge that is equivalent to the Biot-Savart law.
  - But where Gauss's law involves a surface integral over a closed surface, Ampère's law involves a line integral around a closed loop.
  - For steady currents, Ampère's law says

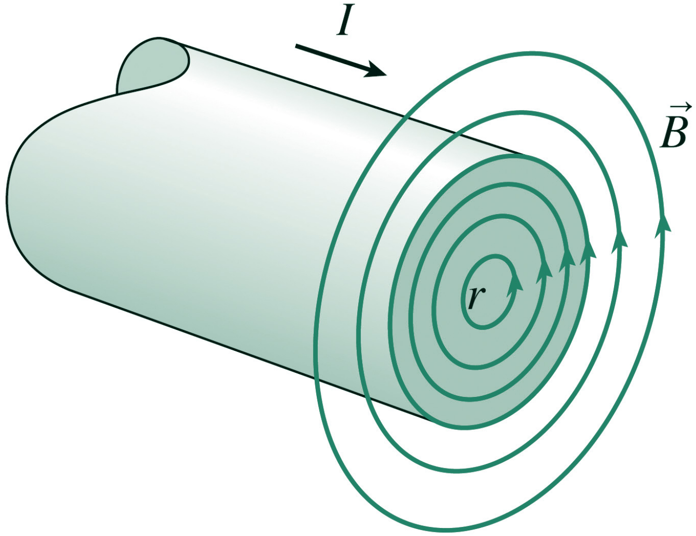
$$\boxed{\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\mathrm{enclosed}}}$$

  - where the integral is taken around *any* closed loop, and $I_{\mathrm{encircled}}$ is the current encircled *by that loop.*

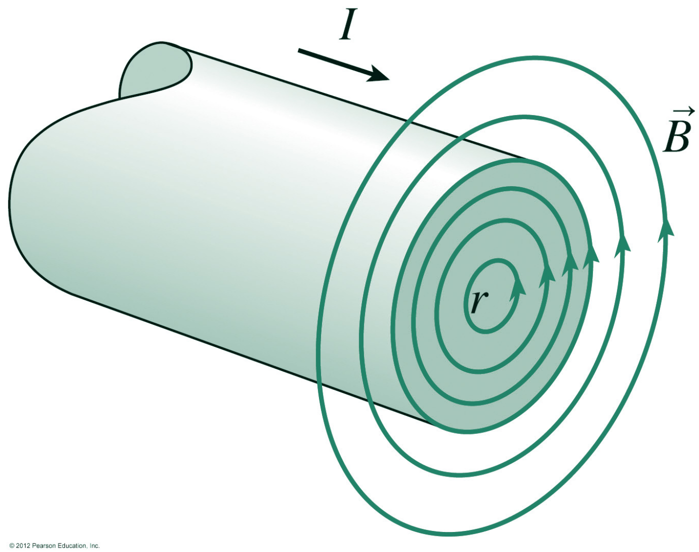
Using Ampère's Law

- Ampère's law is always true, but it can be used to calculate magnetic fields only in cases with sufficient symmetry.
  - Then it's possible to choose an "amperian" loop around which $\oint \vec{B} \cdot d\vec{r}$ can be evaluated in terms of the unknown *B.*
  - An example: Ampère's law quickly gives the 1/*r* field of a line current—or outside *any* current distribution with line symmetry.

Cross section of a long cylindrical wire. Any field line can serve as an amperian loop, for evaluating the field both outside and inside the wire.

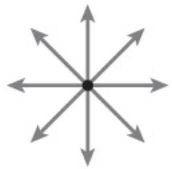
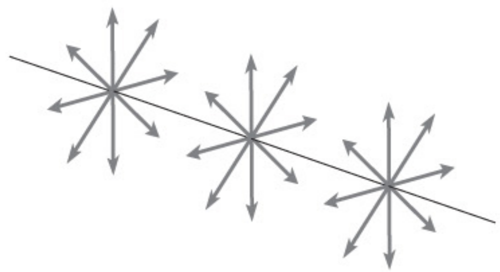
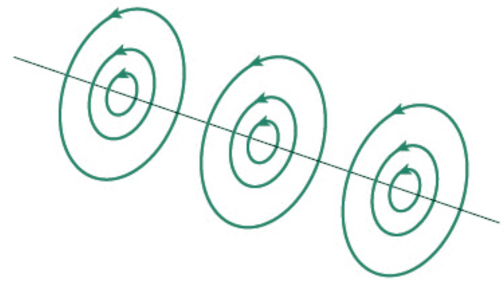
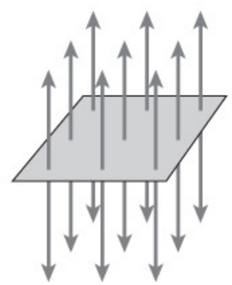
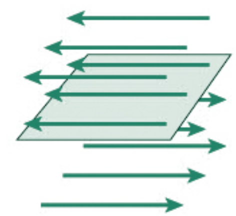


Problem



- A current of I=1A is evenly distributed through a long cylindrical perfectly conducting wire of radius r=3 mm.
  - Find the magnetic field 10 cm from the center of the wire.
  - Find the magnetic field 1 mm from the center of the wire.

Fields of Charge/Current Distributions

Field Dependence on Distance[a]	Charge Distribution	Electric Field	Current Distribution	Magnetic Field
$\frac{1}{r^3}$	Electric dipole		Magnetic dipole	
$\frac{1}{r^2}$	Point charge or spherically symmetric		Impossible for steady current	
$\frac{1}{r}$	Charge distribution with line symmetry		Current distribution with line symmetry	
Uniform field; no variation	Infinite flat sheet of charge		Current sheet	

Ponderable

- Which of the following statements is TRUE?

  A. A metal wire is bent into a square and carries a uniform current throughout it. The net magnetic field at the center of this square is zero.

  B. The net magnetic field inside a conductor must be zero.

  C. Two long, current-carrying wires run parallel to each other. If these wires tend to push away from each other, the currents in them must be going in opposite directions.