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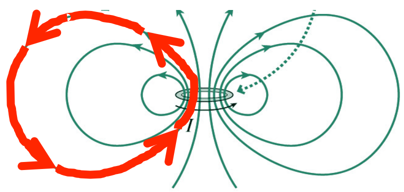

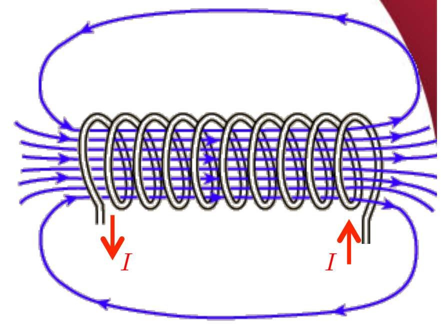

University Physics 227N/232N

Review: Gauss's Law and Ampere's Law
Solenoids and Magnetic Inductors

Lab rescheduled for Wednesday, March 26 in Scale-Up Classroom
Quiz Friday (but only a quiz due to PhD defense)

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Monday, March 24 2014

Happy Birthday to Jessica Chastain, Peyton Manning, Jim Parsons, Harry Houdini, Joseph Liouville, and **Peter Debye** (1936 Nobel)

His first major scientific contribution was the application of the concept of dipole moment to the charge distribution in asymmetric molecules in 1912, developing equations relating dipole moments to temperature and dielectric constant.

Testing for Rest of Semester

- Past Exams
  - Full solutions promptly posted for review (done)

- Quizzes
  - Similar to (but not exactly the same as) homework
  - Full solutions promptly posted for review

- Future Exams (including comprehensive final)
  - I'll provide copy of cheat sheet(s) at least one week in advance
  - Still no computer/cell phone/interwebz/Chegg/call-a-friend
  - **Will only be homework/quiz/exam problems you have seen!**
    - So no separate practice exam (you'll have seen them all anyway)
  - Extra incentive to do/review/work through/understand homework
  - Reduces (some) of the panic of the (omg) comprehensive exam
    - But still tests your comprehensive knowledge of what we've done

Review: Magnetism

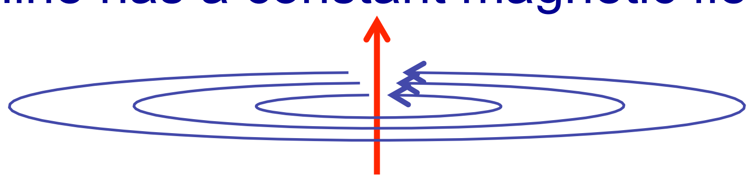
- Magnetism exerts a force on moving electric charges

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \text{magnitude} \quad F = qvB\sin\theta$$

  - Direction follows right hand rule, perpendicular to both $\vec{v}$ and $\vec{B}$
  - Be careful about the sign of the charge q
- Magnetic fields also originate from moving electric charges
  - Electric currents create magnetic fields!
  - There are no individual magnetic "charges"
  - Magnetic field lines are always closed loops
  - Biot-Savart law: how a current creates a magnetic field:

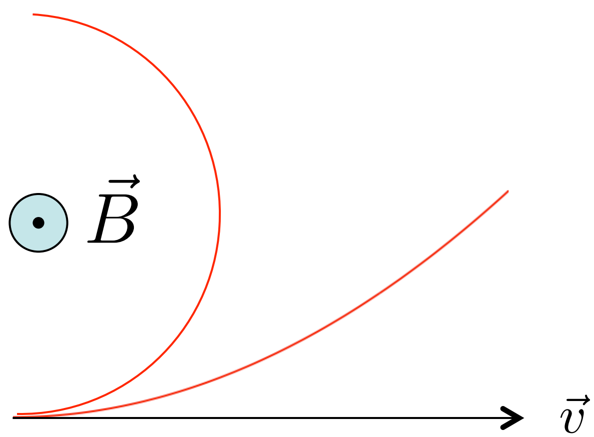
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{L} \times \hat{r}}{r^2} \qquad \mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T-m/A}$$

- Magnetic field field from an infinitely long line of current I
  - Field lines are right-hand circles around the line of current
  - Each field line has a constant magnetic field of $B = \dfrac{\mu_0 I}{2\pi r}$



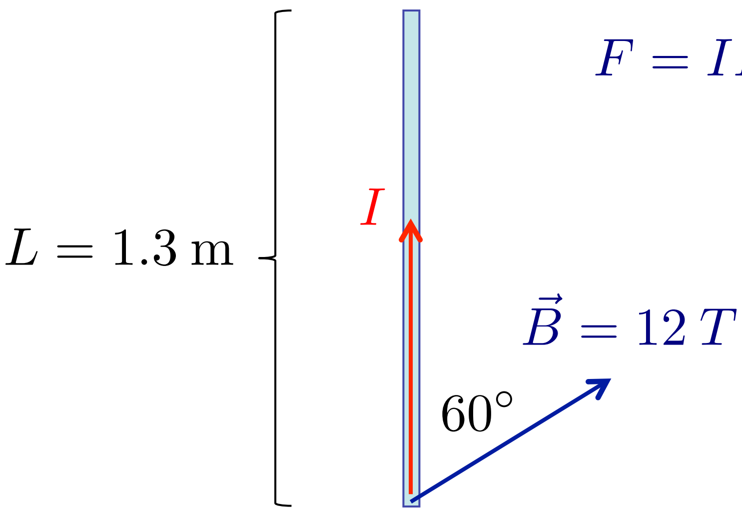
Ponderable (Homework prelecture 26.06)

- A charged particle enters into a uniform magnetic field such that its velocity vector is perpendicular to the magnetic field vector. Ignoring the particle's weight, what type of path will the particle follow?
  - The charged particle will follow a straight-line path.
  - The charged particle will follow a parabolic path.
  - The charged particle will follow a circular path.
  - The charged particle will follow a spiral path.



Homework Question

- You're on a team performing a high-magnetic-field experiment. A conducting bar carrying 2.9 kA will pass through a 1.3m long region containing a 12 T magnetic field, making a 60 degree angle with the field. A colleague proposes resting the bar on wooden blocks. You argue that it will have to be clamped in place, and to back up your argument you claim that the magnetic force will exceed 10,000 pounds. Are you right?



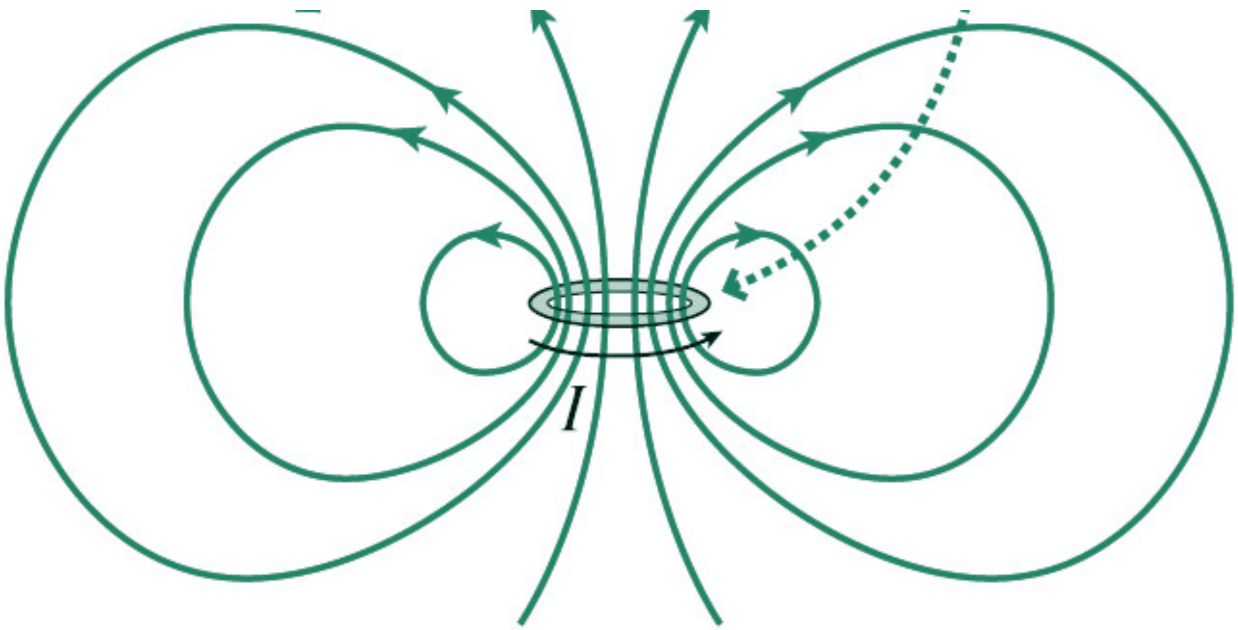
$$F = ILB\sin\theta = (2.9\times 10^3\ \mathrm{A})(1.3\ \mathrm{m})(12\ \mathrm{T})\sin(60^\circ)$$

$$F = 39179\ \mathrm{N} = 8808\ \mathrm{pounds}$$

You are **not** right, but it's pretty close!

Review: Current Loops Again

- Last week we talked about the magnetic fields created by current loops and made a few observations
  - Magnetic field lines are always closed loops
    - they don't start or end anywhere: **there are no magnetic "charges"**
  - Recall: Gauss's Law for electric fields
    - Related electric flux through a closed surface to the total amount of charge inside
    - We can write a similar equation for magnetism but simpler: the total charge enclosed is *always* zero!



Gauss's Law for Magnetism

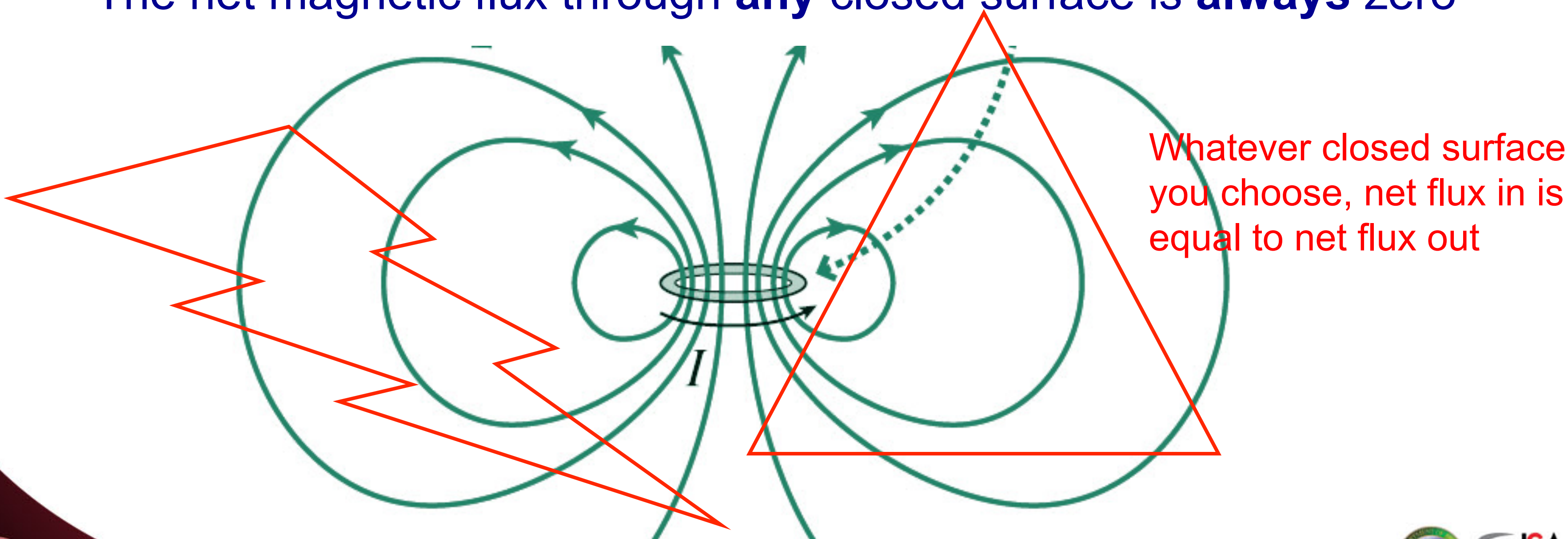
- For electric fields we had Gauss's Law:

$$\Phi_{\text{electric}} = \oint \vec{E} \cdot d\vec{A} = 4\pi k q_{\text{enclosed}}$$

- For magnetic fields this becomes (very generally)

$$\Phi_{\text{magnetic}} = \oint \vec{B} \cdot d\vec{A} = 0$$

  - The net magnetic flux through **any** closed surface is **always** zero

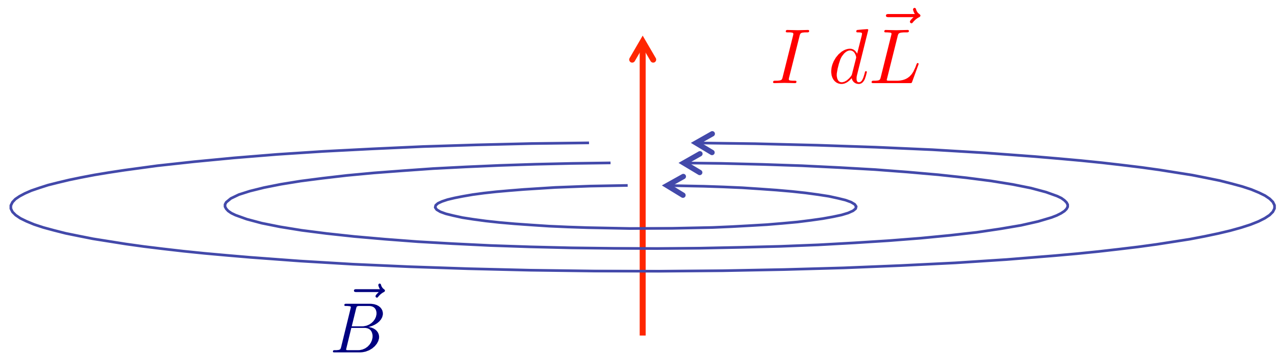


Gauss's Law: Applications?

- Magnetic fields we calculated with Biot-Savart are kinda complicated

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{L} \times \hat{r}}{r^2} \qquad \mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T-m/A}$$

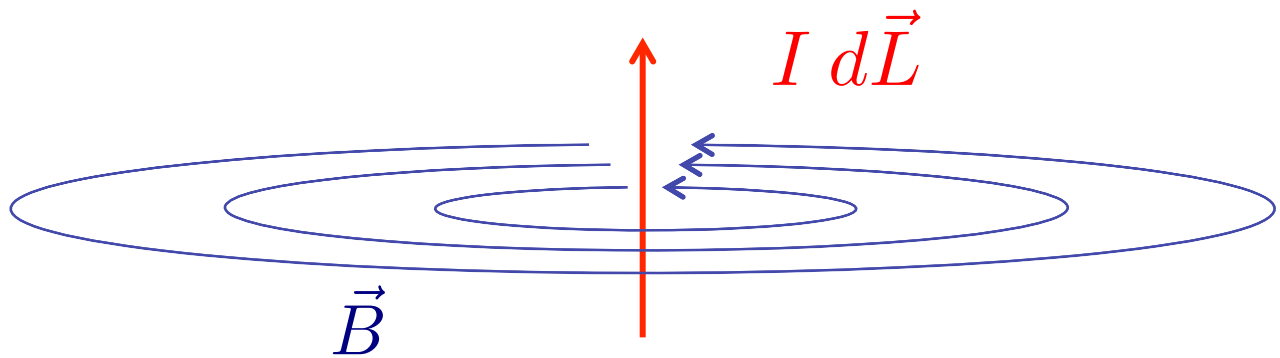
  - The current loop magnetic field is only easily calculated on the axis of the magnetic field
  - The magnetic field from an infinite straight line of current is (right-hand) circles going around the current



  - Here calculating magnetic flux is either boring (cylindrical Gaussian surface) or hard (other surfaces)
  - Gauss's Law for magnetism is more useful as a **concept**

Infinite Line Current and Ampere's Law

- So Gauss's Law isn't very useful in helping us calculate magnetic fields from symmetry
  - But let's go back to the infinite line of current and its magnetic field calculated from Biot-Savart and notice something



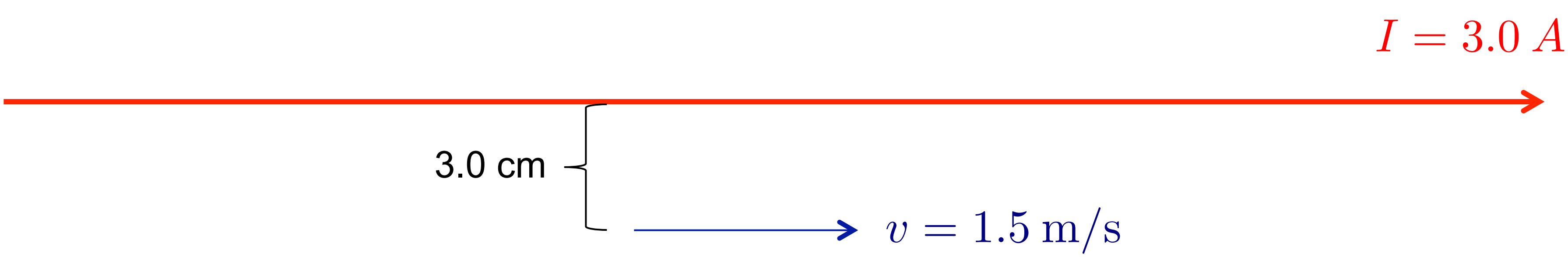
$$B = \frac{\mu_0 I}{2\pi r} \quad (r \text{ is distance from line, direction is right hand around } I)$$

$$(2\pi r)B = \mu_0 I$$

  - The magnetic field lines are just circles -- and 2πr is just the circumference of a circle of radius r.
  - Maybe summing up (integrating) B over the circumference of the circle is related to the total current "enclosed" by that circle

Problem

- A long wire carries a current of 3.0 A. A proton is moving at a velocity of 1.5 m/s in the same direction as the current and parallel to it, a distance of 3.0 cm away. What is the force on the proton (magnitude and direction)?

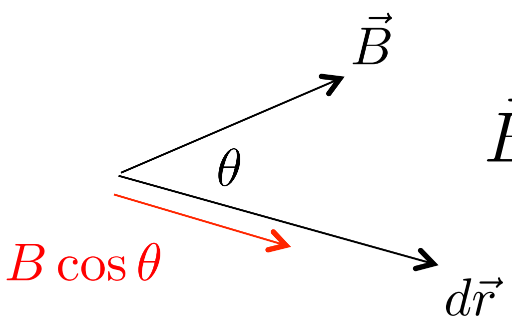


Ampere's Law

- Indeed, this was discovered to be true and is known as **Ampere's Law**:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}}$$

  - I_{enclosed} is the current going **through** the closed 'Amperian" loop
  - This really only applies for steady (constant) currents
    - Changing currents create a complicated mix of B and E fields
  - Remember the **vector dot product** is a scaler:
    - really just taking a "component in the direction of" the other vector



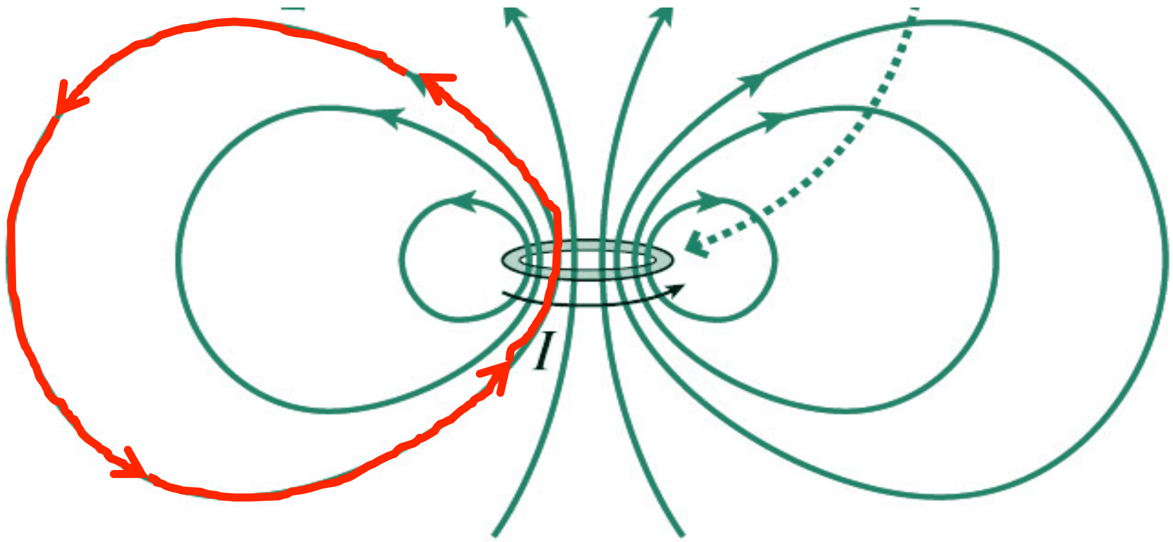
$$\vec{B} \cdot d\vec{r} = B\,dr\;\cos\theta$$

$$\vec{B} \cdot d\vec{r} = B\,dr \;\text{ when } \vec{B} \text{ and } d\vec{r} \text{ are parallel}$$

$$\vec{B} \cdot d\vec{r} = 0 \;\text{ when } \vec{B} \text{ and } d\vec{r} \text{ are perpendicular}$$

Applying Ampere's Law in General Cases

- Ampere's law is always true but it's usually hard to apply
- Example: Magnetic field from a current loop (but generally true)
  - Draw our Amperian loop along a closed magnetic field line
  - Then $\vec{B}$ and $d\vec{r}$ are parallel everywhere on the closed path
  - $\vec{B}$ also has constant magnitude everywhere on the closed path

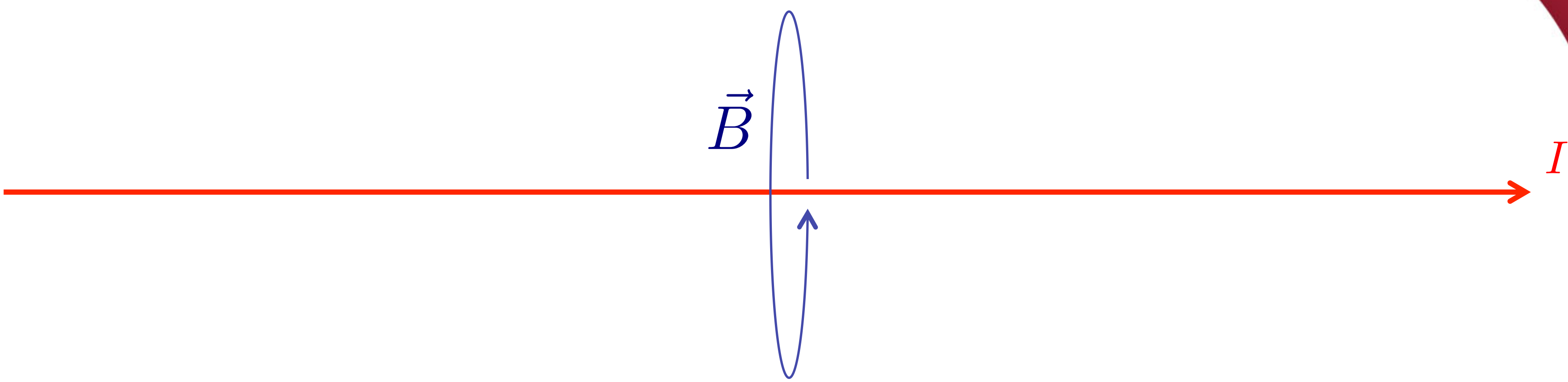


- So if the field line has length $L_{\text{field line}}$, Ampere's Law gives

$$\oint \vec{B} \cdot d\vec{r} = B L_{\text{field line}} = \mu_0 I \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 I}{L_{\text{field line}}}}$$

- But the length of that field line is usually quite hard to calculate!

Applying Ampere's Law: Infinite Line Current



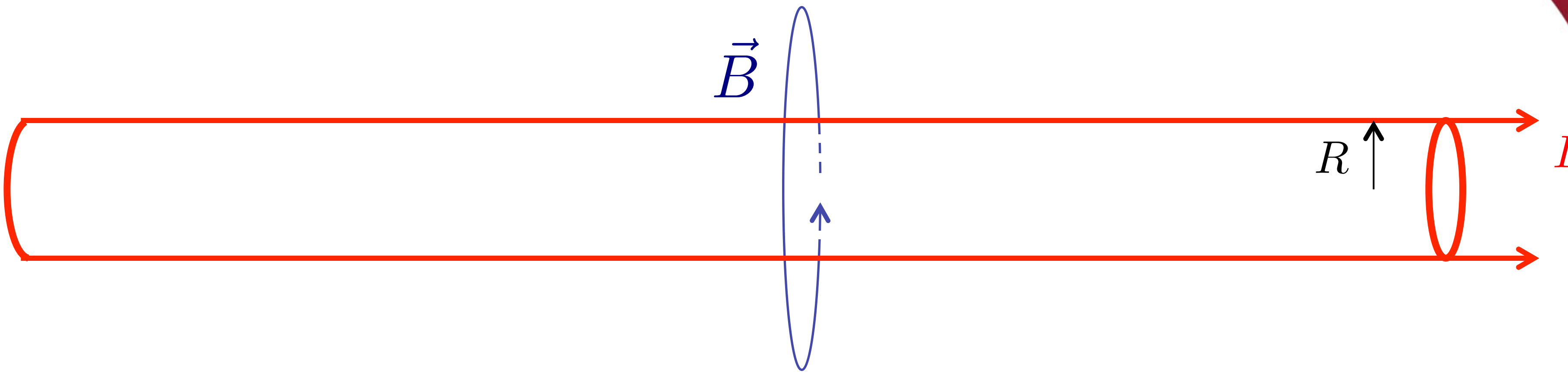
- There is one case where it's easy to calculate the length of the field lines: the line of infinite current
  - Field lines go in (right-hand) circles around the current
  - Rotational symmetry: B is same everywhere on the circle
  - Then Ampere's Law gives

$$\oint \vec{B} \cdot d\vec{r} = B L_{\text{field line}} = \mu_0 I_{\text{enclosed}}$$

$$L_{\text{field line}} = 2\pi r$$

$$B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$$

Applying Ampere's Law: Infinite Cylinder Current

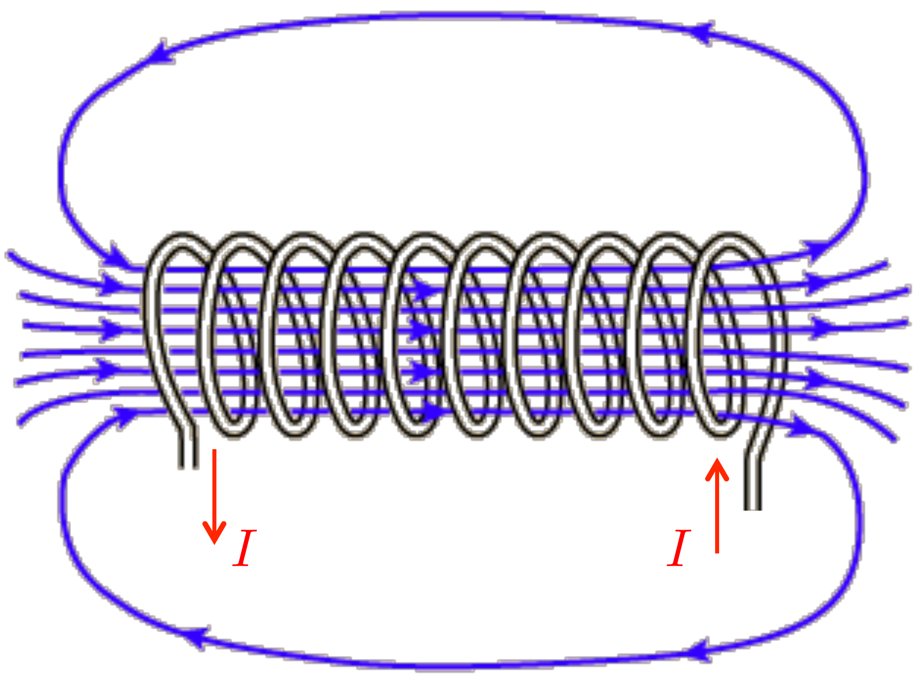


Ampere's Law: $\oint \vec{B} \cdot d\vec{r} = BL_{\text{field line}} = \mu_0 I_{\text{enclosed}}$

- Consider an infinite conducting cylinder of current, with the current evenly distributed over the cross section of radius *R*.
  - The magnetic field lines still are circles (still circularly symmetric)
  - Using Ampere's Law, for a distance *r* from the cylinder axis:
    - What is the magnetic field magnitude for $r > R$?
    - What is the magnetic field magnitude for $r < R$?

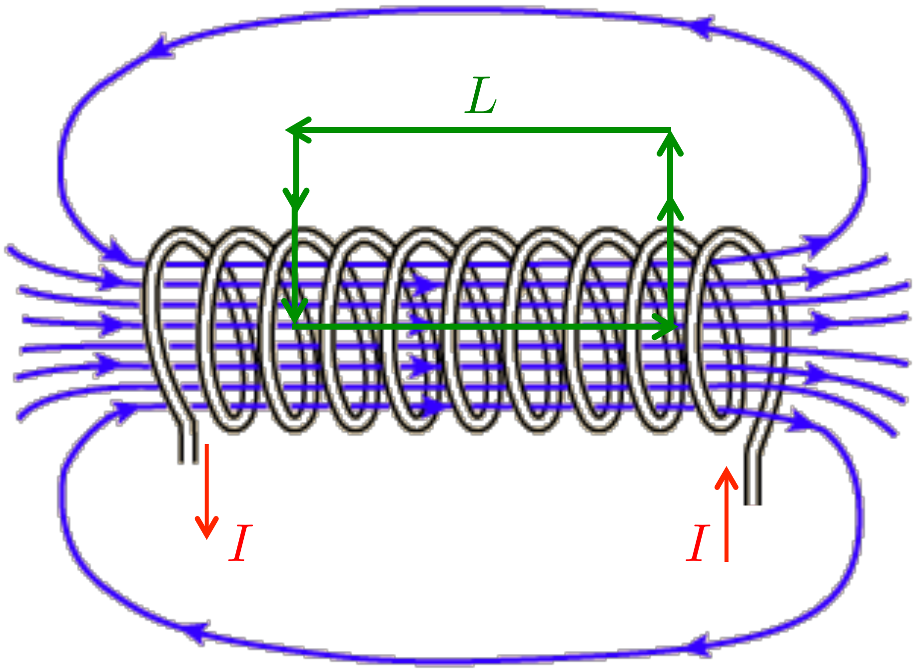
Ampere's Law: Solenoid

- Instead of a current loop, we can make many circular loops of wire with the same radius and evenly spaced
  - This is called a **solenoid**
  - A very long solenoid has a nearly **constant magnetic field** in the center of the loops
  - Outside of the loops the path is long and the field is quite small



B field direction follows right hand rule around current loops

Ampere's Law: Solenoid



Coiling of loops:
n "turns" per unit length

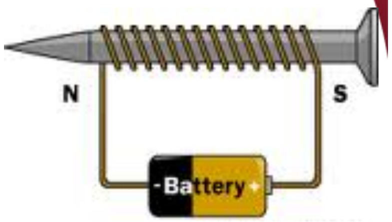
$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}} = \mu_0 (nLI)$$

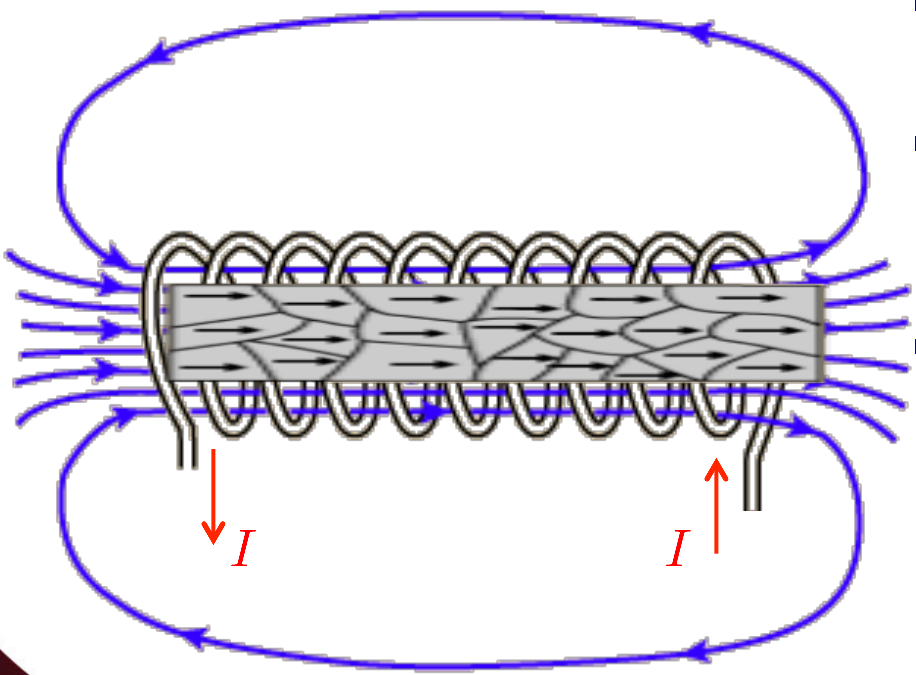
- Draw a rectangular Amperian loop with three parts
  - Outside horizontal path: $B \approx 0$ so $\int \vec{B} \cdot d\vec{r} = 0$
  - Up/down sides: $\vec{B} \perp d\vec{r}$ so $\vec{B} \cdot d\vec{r} = 0$ and $\int \vec{B} \cdot d\vec{r} = 0$
  - Inside: $\vec{B} \parallel d\vec{r}, B \approx \text{constant}$ so $\int \vec{B} \cdot d\vec{r} = BL$

$$B = \mu_0 n I$$

Stronger Magnets

- You may have made a simple electromagnet with a coil of wire like this before
  - Usually the easiest thing is to wrap the coil of wire around something iron and pointed like a nail
  - The iron's atomic magnetic dipoles line up with the magnetic field created by the solenoid, intensifying it





- This is an example of increased **magnetic permeability**
- The sharp tip of the nail also "focuses" the magnetic fields at one end of the solenoid
- This boosting and shaping of the magnetic field is limited by how much the material can be magnetized
  - Leads to **magnetic saturation**

Inductive Reasoning

- We've seen that…
  - Moving charges (currents) experience forces from magnetic fields
  - Moving charges (currents) create magnetic fields
  - Electric and magnetic forces seem deeply intertwined

- It gets even deeper than that
  - A moving electric charge is really creating a changing electric field
  - So a changing electric field is thus really creating a magnetic field

  - Could a changing magnetic field also produce an electric field?

Inductive Reasoning

- We've seen that…
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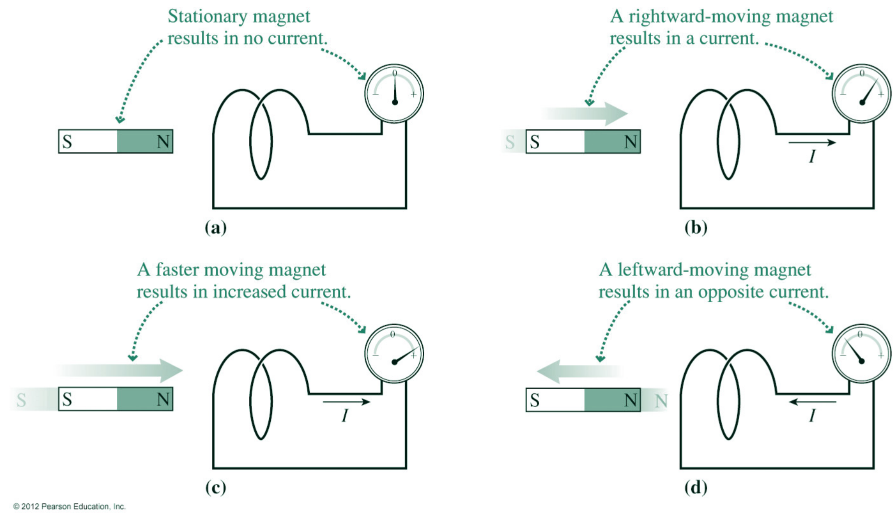
  - Could a changing magnetic field also produce an electric field?

  - **Yes!** Changing magnetic fields create electric fields too!
    - They therefore create EMFs and currents by pushing charges around
    - This symmetry led to Maxwell's equations and one of the first great physics "unifications": **electricity and magnetism became electromagnetism**

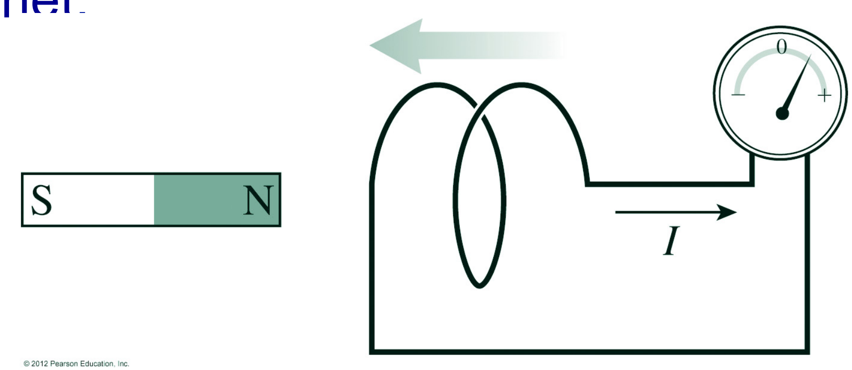
Magnets and Solenoid Coils

- **Electromagnetic induction** involves electrical effects due to *changing magnetic fields.*
- Simple experiments that result in induced current:

1. Move a magnet near a circuit:



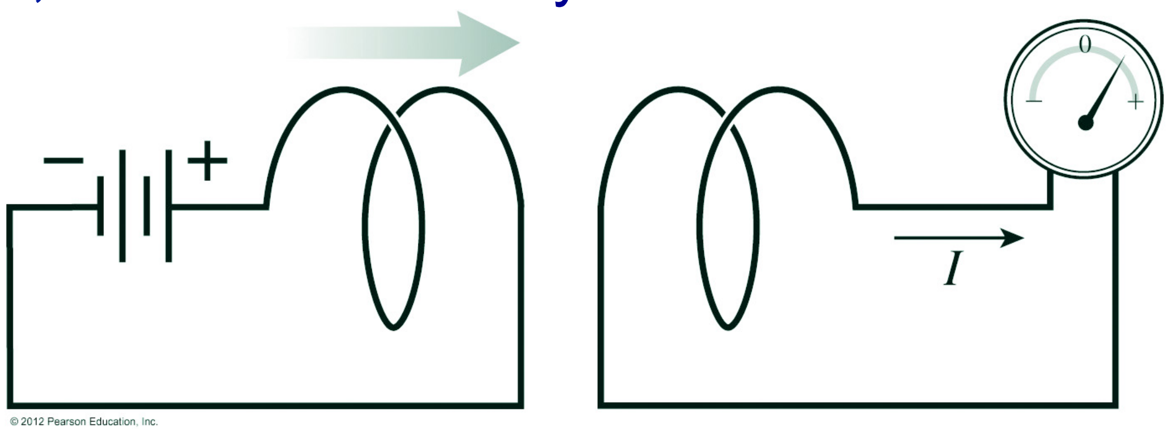
2. Move the circuit near a magnet:



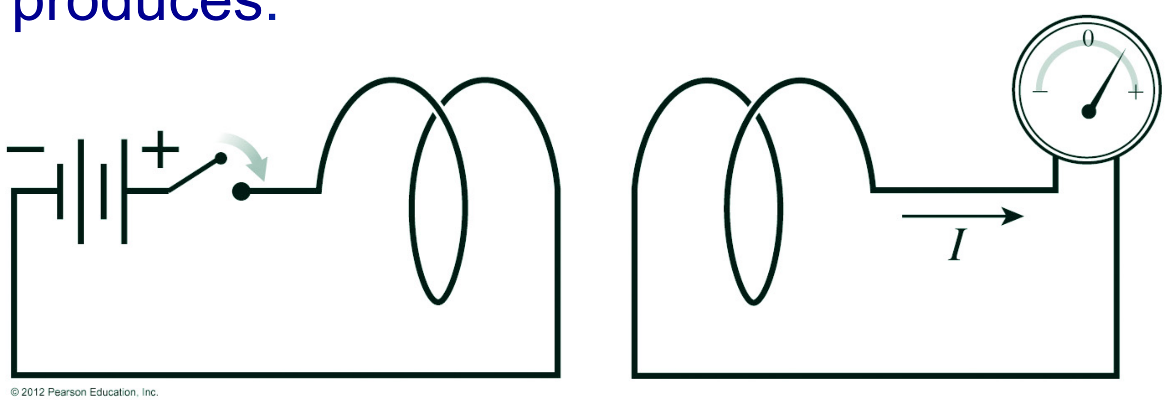
Magnetic Coils Also Interact

- More experiments that result in induced current:

3. Energize one coil to make it an electromagnet; move it near a circuit, or hold it stationary and move a circuit near it:



4. Change the current in one circuit, and thus the magnetic field it produces:



Faraday's Law

- **Faraday's law** describes induction by relating the EMF induced in a circuit to the rate of change of magnetic flux through the circuit:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

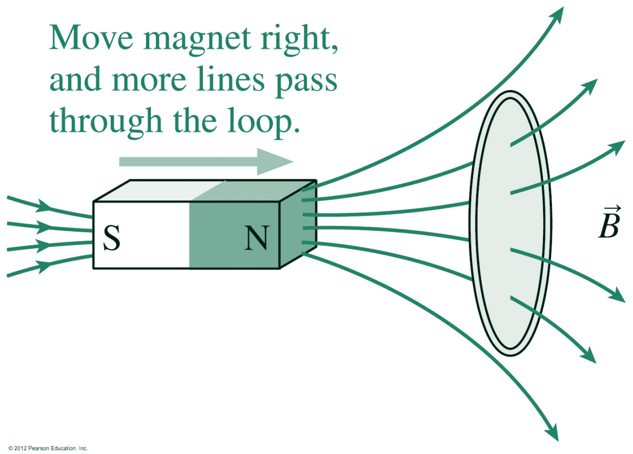
where the **magnetic flux** is given by

$$\Phi_\mathrm{B} = \int \vec{B} \cdot d\vec{A}$$

  - With a flat area and uniform field, this becomes

$$\Phi_B = B\ A\ \cos\theta$$

  - The flux can change by changing the field *B*, the area *A*, or the orientation θ.

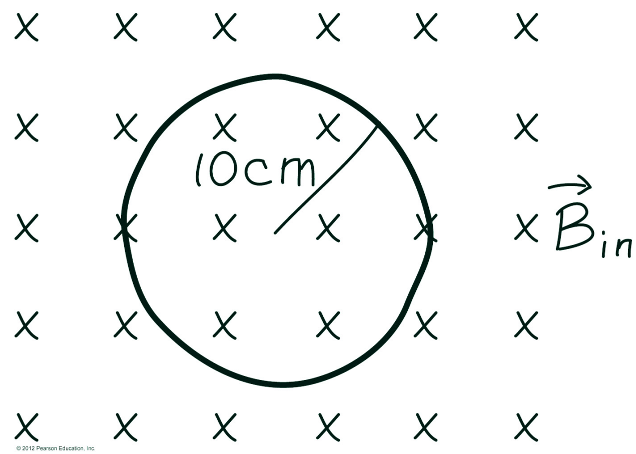


Moving a magnet near a wire loop increases the flux through the loop. The result is an induced EMF given by Faraday's law. The induced EMF drives an induced current in the loop.

Two Examples

- Changing field:
  - The loop has radius *r*, resistance *R*, and is in a magnetic field changing at the rate *dB*/*dt*. The induced emf is

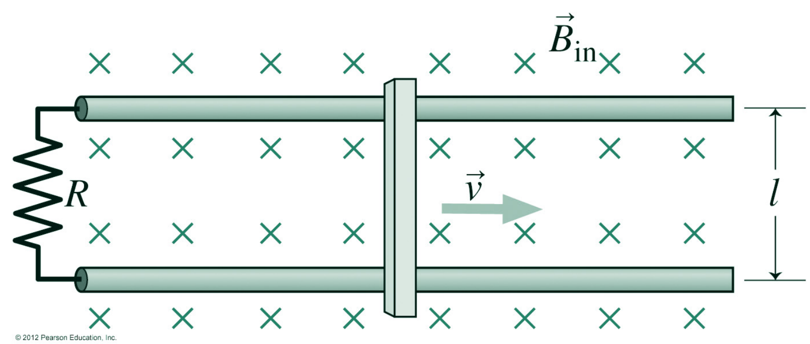
$$\mathcal{E} = -\frac{d\Phi_{\rm B}}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}$$



and the induced current is $I = \frac{|\mathcal{E}|}{R} = \frac{\pi r^2}{R}\frac{dB}{dt}$

- Changing area:
  - The bar slides on the conducting rails, increasing the circuit area at a rate

$$\frac{dA}{dt} = \frac{d(lx)}{dt} = l\frac{dx}{dt} = lv$$



  - The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -Blv$$

  - The induced current is $I = |\varepsilon|/R = Blv/R$.

Direction of Induced Current: Lenz's Law

- The direction of the induced EMF and current is described by the minus sign in Faraday's law, but it's easier to get the direction from conservation of energy.
- **Lenz's law**: The direction of the induced current must be such as to oppose that change that gives rise to it.
  - Otherwise we could produce energy without doing any work!
- **Example**: Here the north pole of the magnet approaches the loop. So the induced current makes the loop a bar magnet with north to the left, opposing the approaching magnet.