University Physics, PHYS 227/232 Quiz 4 Solution Fri Mar 28, 2014

Your name and table number: ______________________________

Please show your work, write neatly, write units, and box your answers.

Magnetic force on moving charged particle: $\vec{F} = q\vec{v} \times \vec{B}$

Magnetic force on a current: $\vec{F} = I\vec{L} \times \vec{B}$

Magnetic field magnitude from a line current: $B = \frac{\mu_0 I}{2\pi r}$

Biot-Savart Law: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{L}\times\hat{r}}{r^2}$

$\mu_0 \equiv 4\pi \times 10^{-7}$ T − m/A

Cyclotron radius: $r = mv/qB$

Cyclotron frequency: $f = \frac{qB}{2\pi m}$

Electron charge: $e = -1.602 \times 10^{-19}$ C

Electron mass: $m_e = 9.109 \times 10^{-31}$ kg

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1. (2 points) A positively charged particle enters a region with a uniform magnetic field, such that the particle's velocity is perpendicular to the magnetic field. Ignoring the particle's weight (the force of gravity), what type of path will the particle follow?

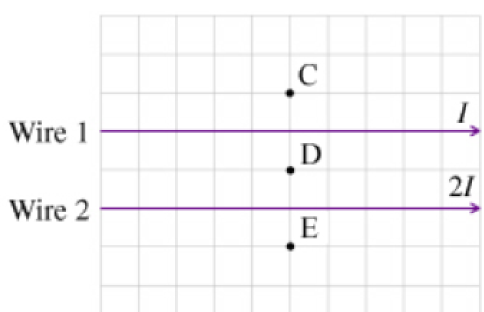
   (a) The charged particle will follow a straight-line path.

   (b) The charged particle will follow a parabolic path.

   (c) The charged particle will follow a circular path.

   (d) The charged particle will follow a spiral path.

   Solution: The force from a magnetic field on a moving charged particle is given by $\vec{F} = q\vec{v} \times \vec{B}$. This force is always perpendicular to both the particle's velocity and the magnetic field. Since there is no component of the particle's velocity along the direction of the magnetic field, the particle is moving in a plane with a constant force that is always perpendicular to its velocity. This direction of a constant force will make the charged particle move on a circular path.



2. Consider the wires in the above figure, with the top wire carrying current I to the right and the bottom wire carrying current $2I$ to the right. In what direction is the *total* magnetic field from these two wires...

(a) (1 point) ... at point C? out of the page

(b) (1 point) ... at point D? out of the page

(c) (1 point) ... at point E? into the page

Solution: Magnetic field lines go in right-handed circles around currents; if you wrap your right hand around the wire with your thumb pointed in the direction of the current, your fingers will curl in the direction of the magnetic field. Here at point C, this rule gives the magnetic field lines from both currents as pointing out of the page, so at point C the total magnetic field points out of the page. At point E, similarly, the total magnetic field points into the page.

The only question then is about point D, where the field from the top current points into the page and the field from the bottom current points out of the page. However, you were given the magnetic field from a line current:

$$B = \frac{\mu_0 I}{2\pi r}$$

Here r is the same for the magnetic field magnitude from each current (they are both equidistant from point D), but the current for the bottom wire is large than the current from the top wire. The magnitude of the magnetic field from the bottom current is therefore larger, and the total magnetic field at D points out of the page.

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3. Microwave oven microwaves are produced by electrons moving around in a magnetron at a frequency of 2.6 GHz. The magnetron can accommodate electron orbits with a maximum *diameter* of 3.0 mm.

   (a) (3 points) What is the magnetic field strength, in mT?

   Solution: The cyclotron frequency can be used to calculate the magnetic field:

$$\begin{aligned} f &= \frac{qB}{2\pi m_e} \quad \Rightarrow \quad B = \frac{2\pi m_e f}{e} \\ B &= \frac{2\pi(9.109 \times 10^{-31}\text{ kg})(2.6 \times 10^{9}\text{ Hz})}{(1.6 \times 10^{-19}\text{ C}} \\ &= 0.093\text{ T} = \boxed{93\text{ mT} = B} \end{aligned}$$

   (b) (2 points) What is the maximum electron speed, in m/s?

   Solution: The cyclotron radius can be used to give us the electron speed:

$$\begin{aligned} r &= \frac{m_e v}{qB} = \frac{m_e v}{eB} \quad \Rightarrow \quad v = \frac{qBr}{m_e} \\ v &= \frac{qBr}{m_e} = \frac{(1.6 \times 10^{-19}\text{ C})(0.093\text{ T})(1.5 \times 10^{-3}\text{ m})}{9.109 \times 10^{-31}\text{ kg}} = \boxed{2.45 \times 10^{7}\text{ m/s} = v} \end{aligned}$$