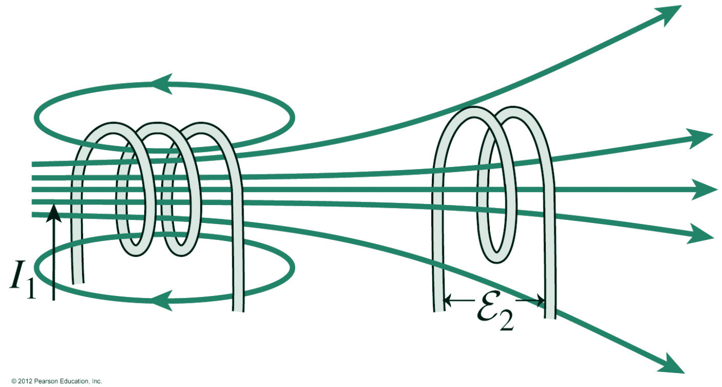

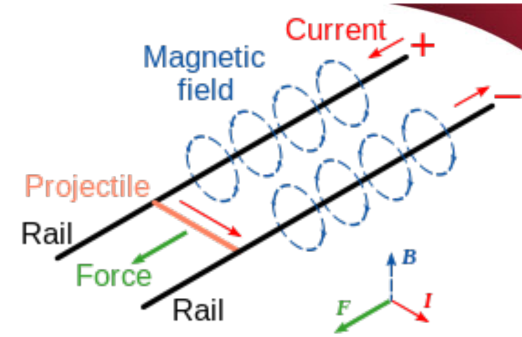

University Physics 227N/232N

Ch 27: Inductors, towards Ch 28: AC Circuits

Quiz and Homework This Week

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http://www.toddsatogata.net/2014-ODU

Monday, March 31 2014

Happy Birthday to Jack Antonoff, Kate Micucci, Ewan McGregor,
Christopher Walken, Carlo Rubbia (1984 Nobel), and Al Gore (2007 Nobel)
(and Sin-Itiro Tomonaga and Rene Descartes and Johann Sebastian Bach too!)

Testing for Rest of Semester

- Past Exams
  - Full solutions promptly posted for review (done)

- Quizzes
  - Similar to (but not exactly the same as) homework
  - Full solutions promptly posted for review

- Future Exams (including comprehensive final)
  - I'll provide copy of cheat sheet(s) at least one week in advance
  - Still no computer/cell phone/interwebz/Chegg/call-a-friend
  - **Will only be homework/quiz/exam problems you have seen!**
    - So no separate practice exam (you'll have seen them all anyway)
  - Extra incentive to do/review/work through/understand homework
  - Reduces (some) of the panic of the (omg) comprehensive exam
    - But still tests your comprehensive knowledge of what we've done

Review: Magnetism

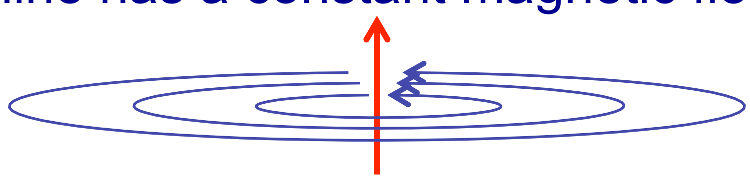
- Magnetism exerts a force on moving electric charges

$$\vec{F} = q\vec{v} \times \vec{B} \qquad \text{magnitude} \quad F = qvB\sin\theta$$

  - Direction follows right hand rule, perpendicular to both $\vec{v}$ and $\vec{B}$
  - Be careful about the sign of the charge q
- Magnetic fields also originate from moving electric charges
  - Electric currents create magnetic fields!
  - There are no individual magnetic “charges”
  - Magnetic field lines are always closed loops
  - Biot-Savart law: how a current creates a magnetic field:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{L} \times \hat{r}}{r^2} \qquad \mu_0 \equiv 4\pi \times 10^{-7}\,\mathrm{T-m/A}$$

- Magnetic field field from an infinitely long line of current I
  - Field lines are right-hand circles around the line of current
  - Each field line has a constant magnetic field of $B = \frac{\mu_0 I}{2\pi r}$



Review: Magnetism II

- Gauss's Law and flux for magnetic fields
  - Flux defined same way as defined for electric fields $\Phi_{\mathrm{B}} \equiv \int \vec{B} \cdot d\vec{A}$
  - Integral equals zero for a *closed surface* $\oint \vec{B} \cdot d\vec{A} = 0$
- Ampere's Law
  - Use symmetry to calculate magnetic field from current going through surface defined by a closed path

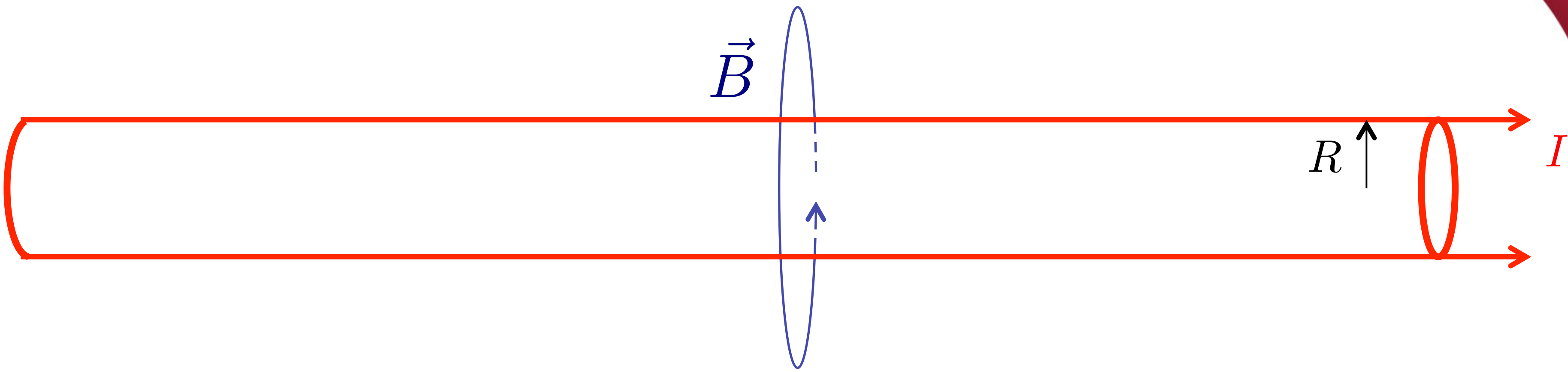
$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\mathrm{enclosed}}$$

- Faraday's Law
  - Changing magnetic flux produces electric fields/EMF/current

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Lenz's Law
  - The EMF/current "induced" by Faraday's law creates a changing flux that *opposes* the original changing flux

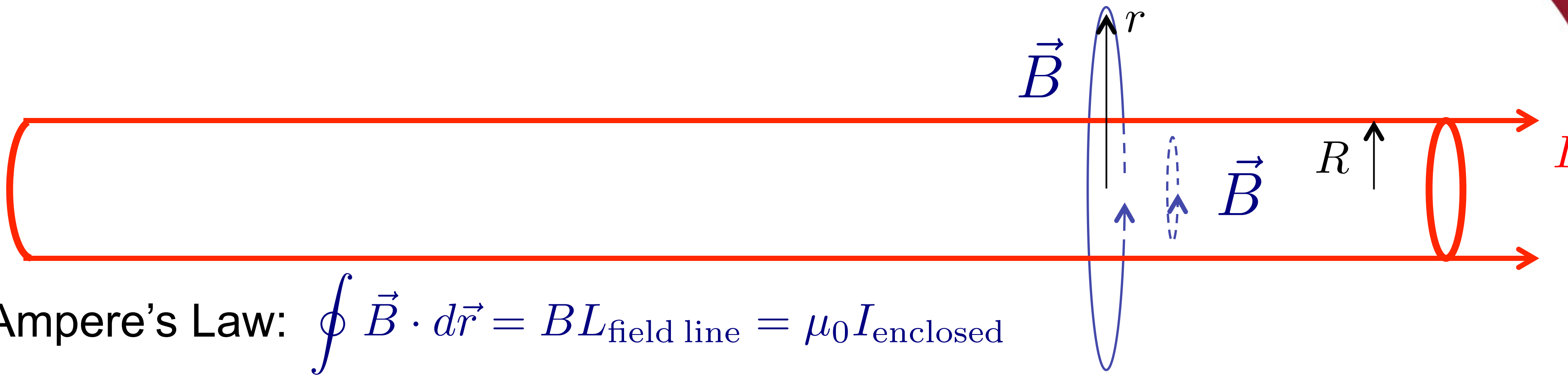
Applying Ampere's Law: Infinite Cylinder Current



Ampere's Law: $\oint \vec{B} \cdot d\vec{r} = BL_{\text{field line}} = \mu_0 I_{\text{enclosed}}$

- Consider an infinite conducting cylinder of current, with the current evenly distributed over the cross section of radius *R*.
  - The magnetic field lines still are circles (still circularly symmetric)
  - Using Ampere's Law, for a distance *r* from the cylinder axis:
    - What is the magnetic field magnitude for $r > R$?
    - What is the magnetic field magnitude for $r < R$?

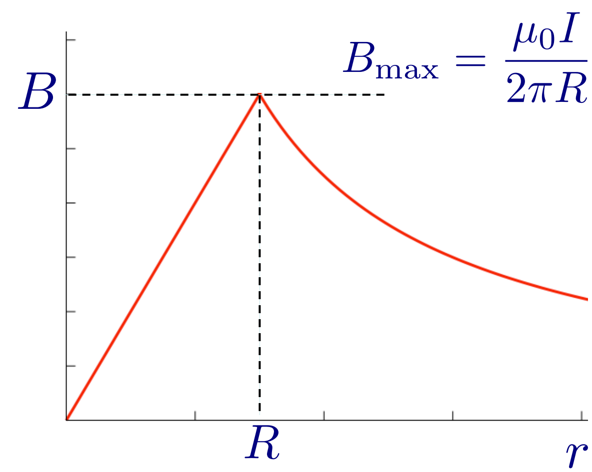
Applying Ampere's Law: Infinite Cylinder Current



Ampere's Law: $\oint \vec{B} \cdot d\vec{r} = BL_{\text{field line}} = \mu_0 I_{\text{enclosed}}$

- What is the magnetic field magnitude for $r > R$?
  - I_{enclosed} is the total current I

$$r > R: \quad 2\pi r B_{r>R} = \mu_0 I \quad \Rightarrow \quad \boxed{B_{r>R} = \frac{\mu_0 I}{2\pi r}}$$

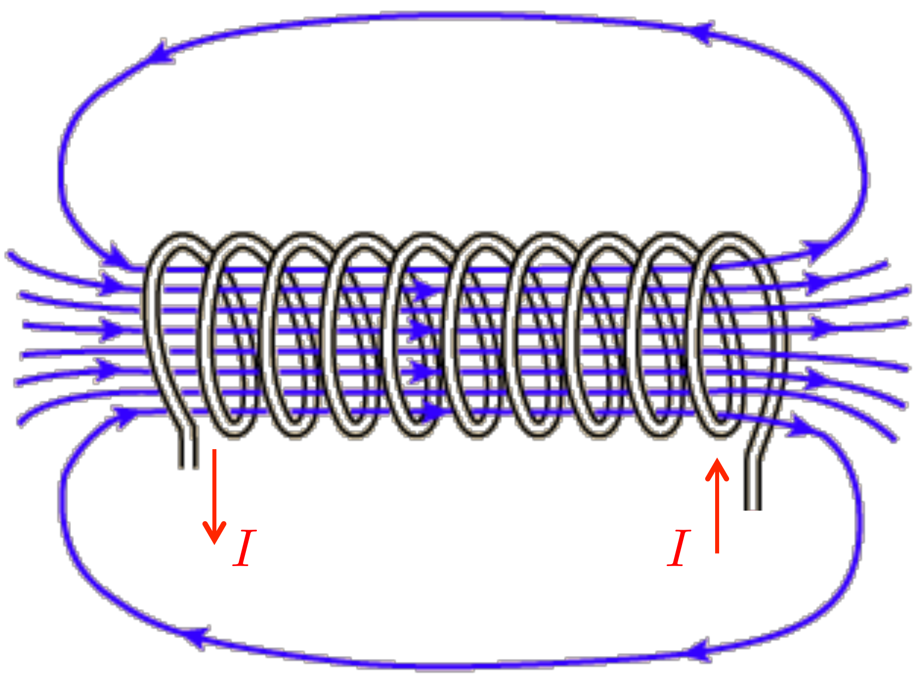


- What is the magnetic field magnitude for $r < R$?
  - I_{enclosed} is the only a fraction of the total current
  - Fractional current is given by the ratio of cross section areas

$$r < R: \quad 2\pi r B_{r<R} = \mu_0 \left(\frac{\pi r^2}{\pi R^2}\right) I \quad \Rightarrow \quad \boxed{B_{r<R} = \frac{\mu_0 I r}{2\pi R^2}}$$

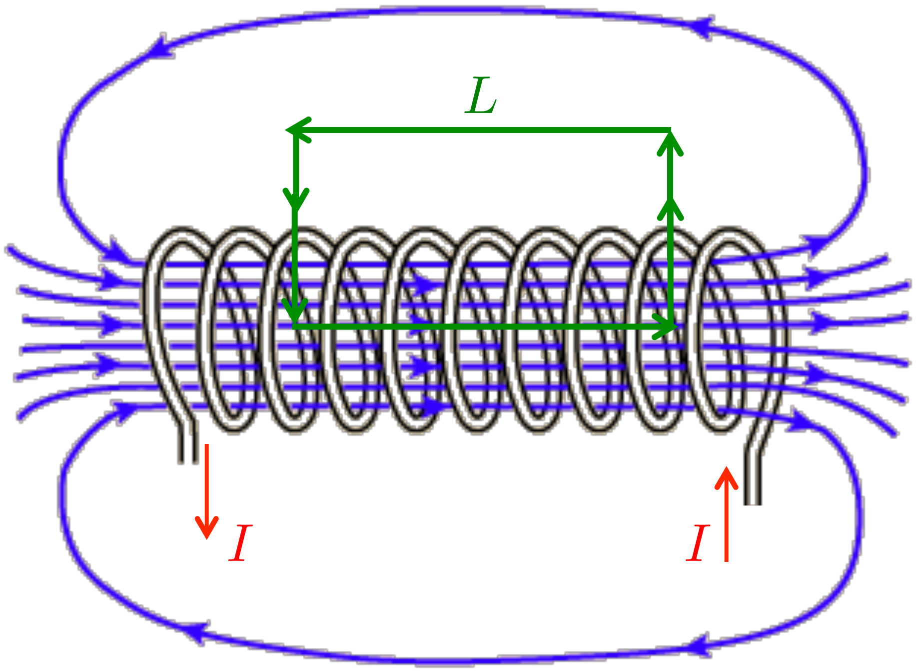
Ampere's Law: Solenoid

- Instead of a current loop, we can make many circular loops of wire with the same radius and evenly spaced
  - This is called a **solenoid**
  - A very long solenoid has a nearly **constant magnetic field** in the center of the loops
  - Outside of the loops the path is long and the field is quite small



B field direction follows right hand rule around current loops

Ampere's Law: Solenoid



Coiling of loops:
n "turns" per unit length

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}} = \mu_0 (nLI)$$

- Draw a rectangular Amperian loop with three parts
  - Outside horizontal path: $B \approx 0 \text{ so } \int \vec{B} \cdot d\vec{r} = 0$
  - Up/down sides: $\vec{B} \perp d\vec{r} \text{ so } \vec{B} \cdot d\vec{r} = 0 \text{ and } \int \vec{B} \cdot d\vec{r} = 0$
  - Inside: $\vec{B} \parallel d\vec{r}, B \approx \text{constant so } \int \vec{B} \cdot d\vec{r} = BL$

$$B = \mu_0 n I$$

Inductive Reasoning

- We've seen that…
  - Moving charges (currents) experience forces from magnetic fields
  - Moving charges (currents) create magnetic fields
  - Electric and magnetic forces seem deeply intertwined

- It gets even deeper than that
  - A moving electric charge is really creating a changing electric field
  - So a changing electric field is thus really creating a magnetic field

  - Could a changing magnetic field also produce an electric field?

Inductive Reasoning

- We've seen that…
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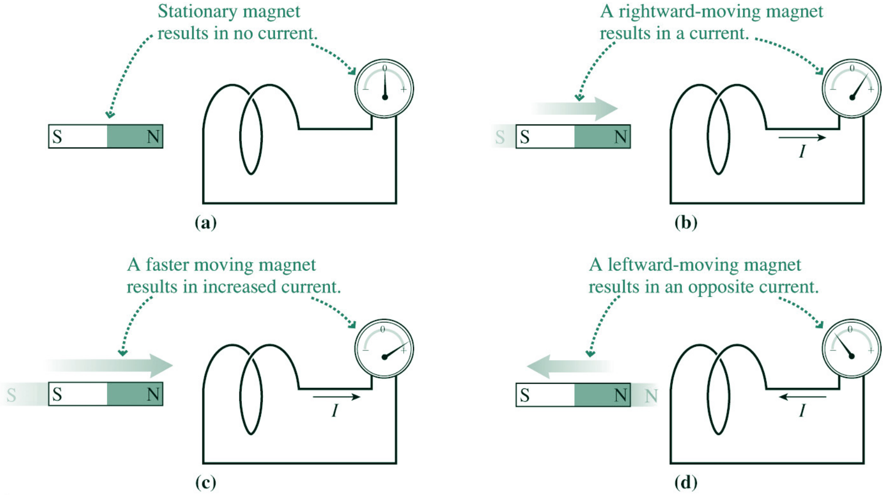
  - Could a changing magnetic field also produce an electric field?

  - **Yes!** Changing magnetic fields create electric fields too!
    - They therefore create EMFs and currents by pushing charges around
    - This symmetry led to Maxwell's equations and one of the first great physics "unifications": **electricity and magnetism became electromagnetism**

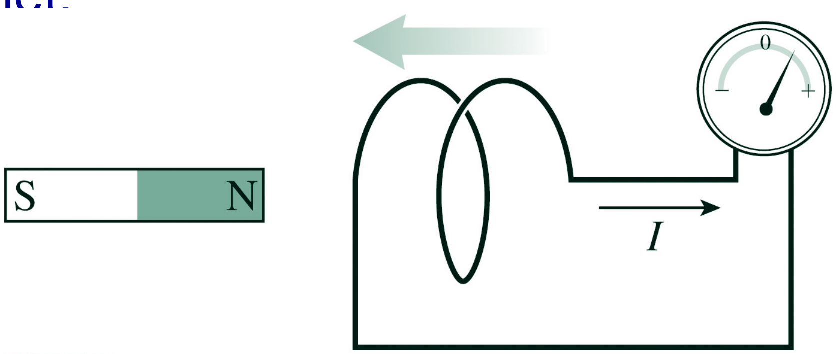
Magnets and Solenoid Coils

- **Electromagnetic induction** involves electrical effects due to *changing magnetic fields.*
- Simple experiments that result in induced current:

1. Move a magnet near a circuit:



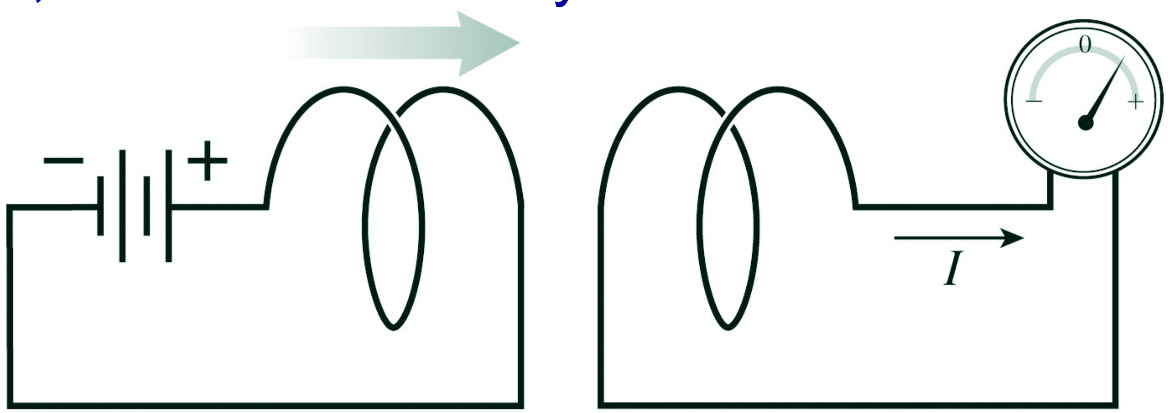
2. Move the circuit near a magnet:



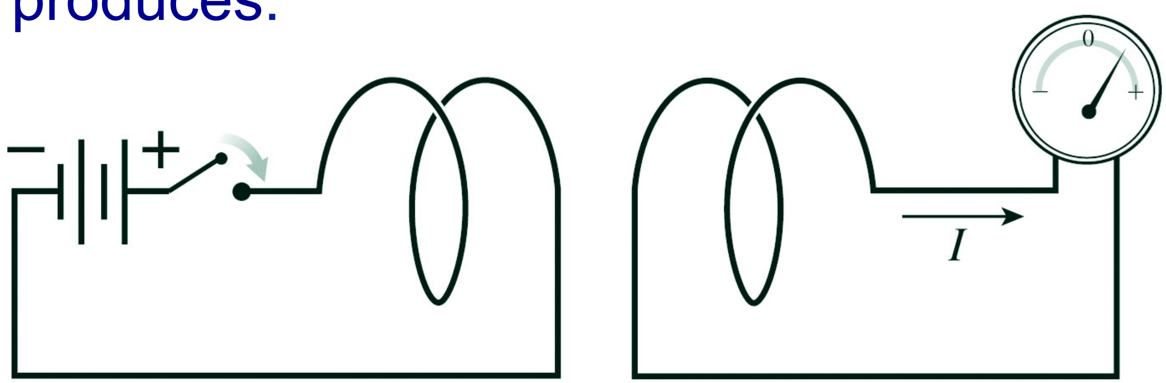
Magnetic Coils Also Interact

- More experiments that result in induced current:

3. Energize one coil to make it an electromagnet; move it near a circuit, or hold it stationary and move a circuit near it:



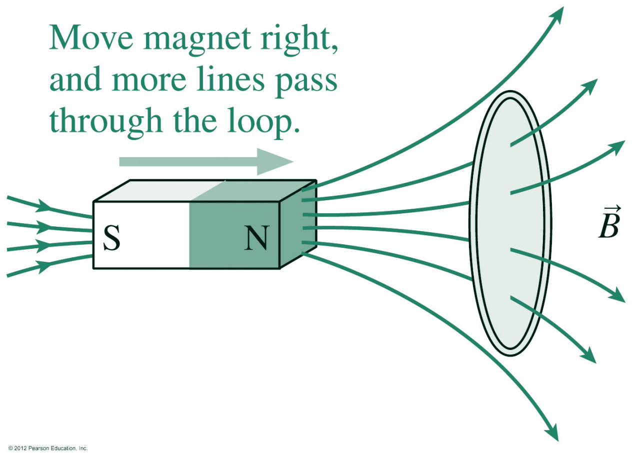
4. Change the current in one circuit, and thus the magnetic field it produces:



Faraday's Law

- **Faraday's law** describes induction by relating the EMF induced in a circuit to the rate of change of magnetic flux through the circuit:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$



where the **magnetic flux** is given by

$$\Phi_\mathrm{B} = \int \vec{B} \cdot d\vec{A}$$

- With a flat area and uniform field, this becomes

$$\Phi_B = B\ A\ \cos\theta$$

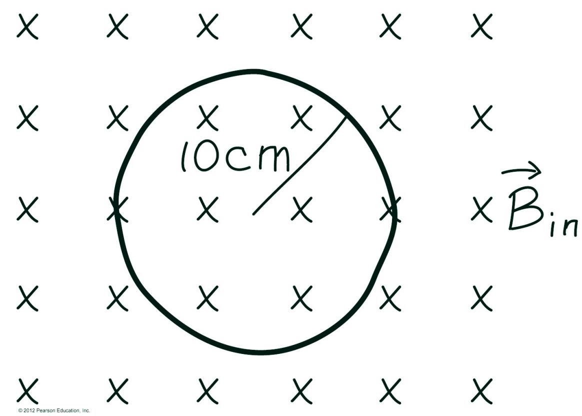
- The flux can change by changing the field *B*, the area *A*, or the orientation θ.

Moving a magnet near a wire loop increases the flux through the loop. The result is an induced EMF given by Faraday's law. The induced EMF drives an induced current in the loop.

Two Examples

- Changing field:
  - The loop has radius *r*, resistance *R*, and is in a magnetic field changing at the rate *dB*/*dt*. The induced emf is

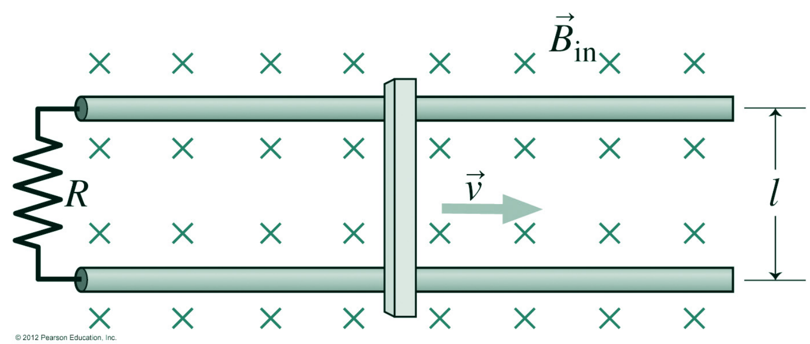
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -\pi r^2\frac{dB}{dt}$$



and the induced current is $I = \frac{|\mathcal{E}|}{R} = \frac{\pi r^2}{R}\frac{dB}{dt}$

- Changing area:
  - The bar slides on the conducting rails, increasing the circuit area at a rate

$$\frac{dA}{dt} = \frac{d(lx)}{dt} = l\frac{dx}{dt} = lv$$



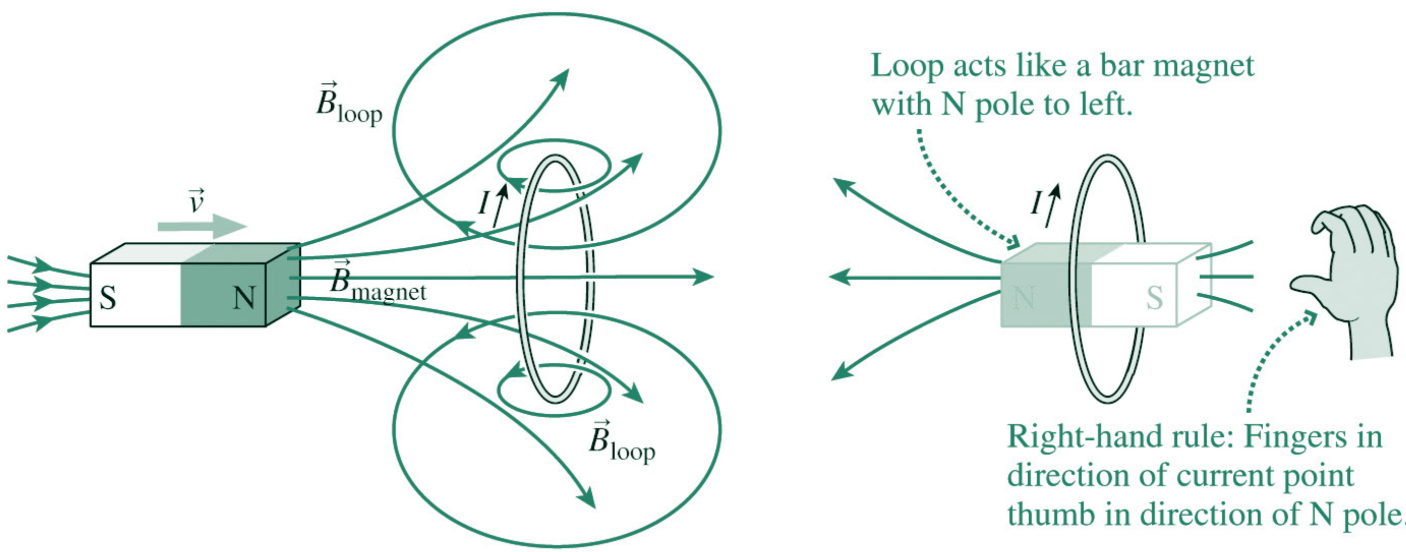
  - The induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -Blv$$

  - The induced current is $I = |\varepsilon|/R = Blv/R$.

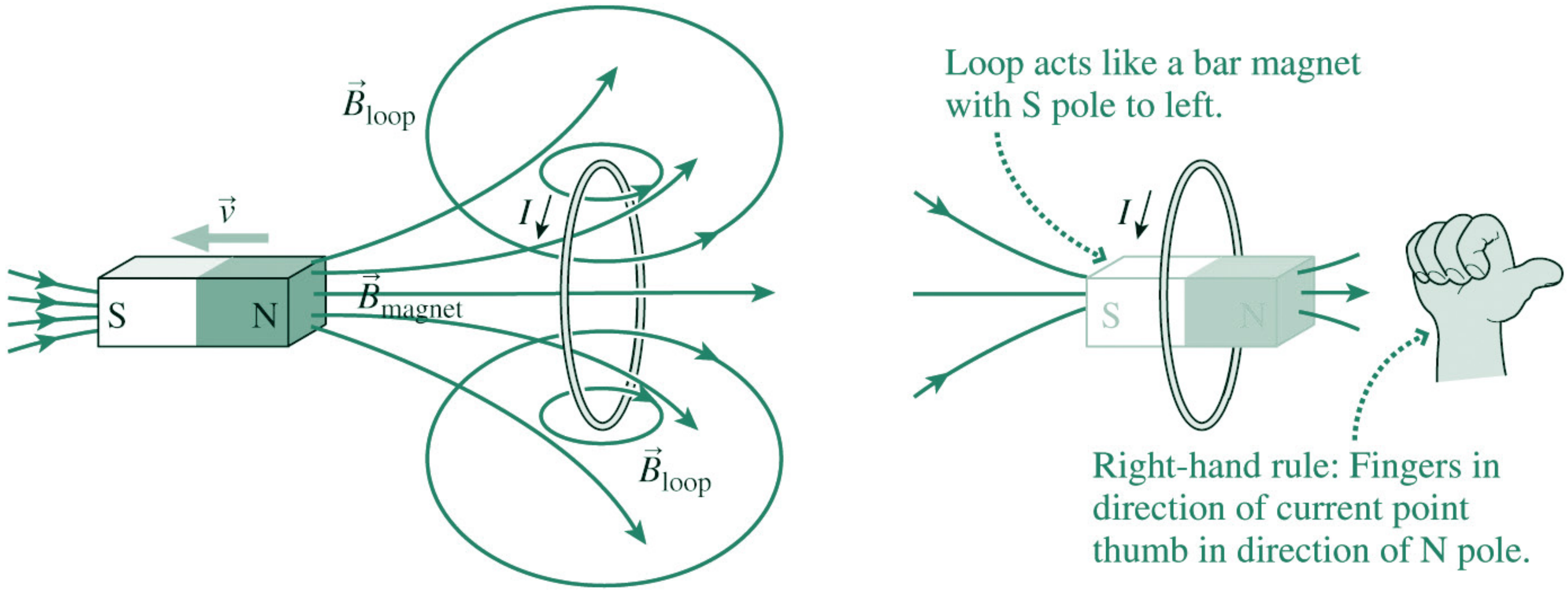
Direction of Induced Current: Lenz's Law

- The direction of the induced EMF and current is described by the minus sign in Faraday's law, but it's easier to get the direction from conservation of energy.
- **Lenz's law**: The direction of the induced current must be such as to oppose that change that gives rise to it.
  - Otherwise we could produce energy without doing any work!
- **Example**: Here the north pole of the magnet approaches the loop. So the induced current makes the loop a bar magnet with north to the left, opposing the approaching magnet.



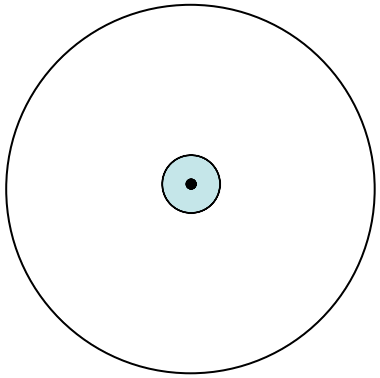
It's the *Change* in Flux that Matters

- Lenz's law says that induced effects oppose the changes that give rise to them.
  - Now the induced current flows the opposite way, making the loop's south pole to the left and opposing the withdrawal of the magnet.



  - You'll end up practicing this type of logic quite a bit in this week's homework.

Induction and Lenz's Law Ponderables



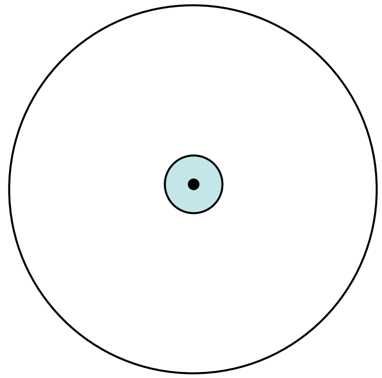
B points out of paper and is increasing

- Which way does current flow in the loop of wire?
  - Clockwise
  - Counterclockwise
  - Neither

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_\mathrm{B} \equiv \int \vec{B} \cdot d\vec{A}$$

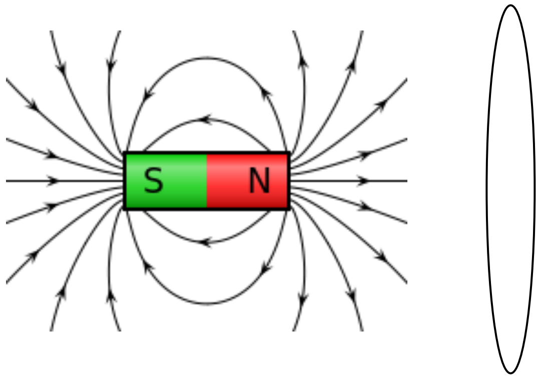
Induction and Lenz's Law Ponderables



B points out of paper and is decreasing

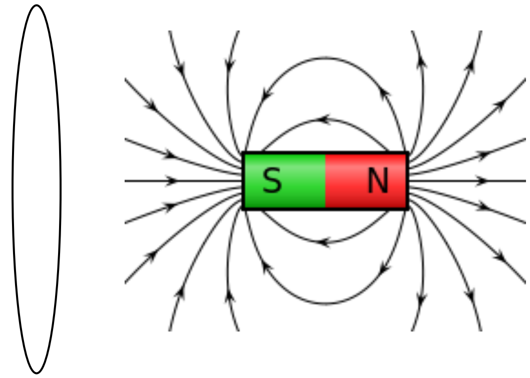
- Which way does current flow in the loop of wire?
  - Clockwise
  - Counterclockwise
  - Neither

Induction and Lenz's Law Ponderables



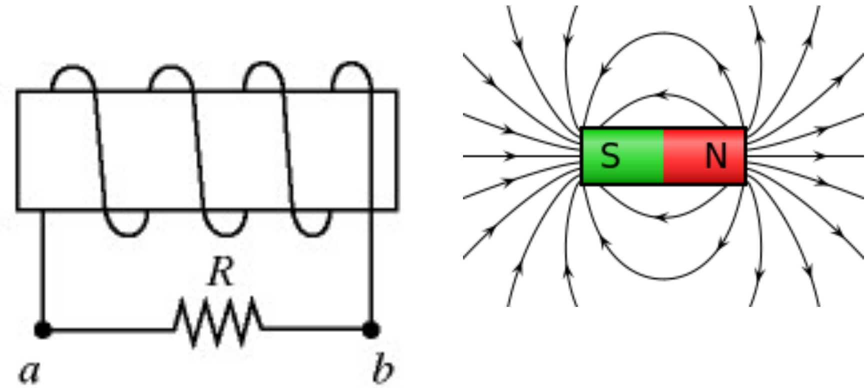
- A magnet is being pushed towards the right through a wire loop.
- Which way does current flow at the top of the loop of wire?
  - Out of the screen
  - Into the screen
  - Neither

Induction and Lenz's Law Ponderables



- A magnet is being pushed towards the right away from a wire loop.
- Which way does current flow at the top of the loop of wire?
  - Out of the screen
  - Into the screen
  - Neither
- So which way is the current flowing when the magnet is moving to the right and is halfway through the wire?
  - Out of the screen, into the screen, or neither?

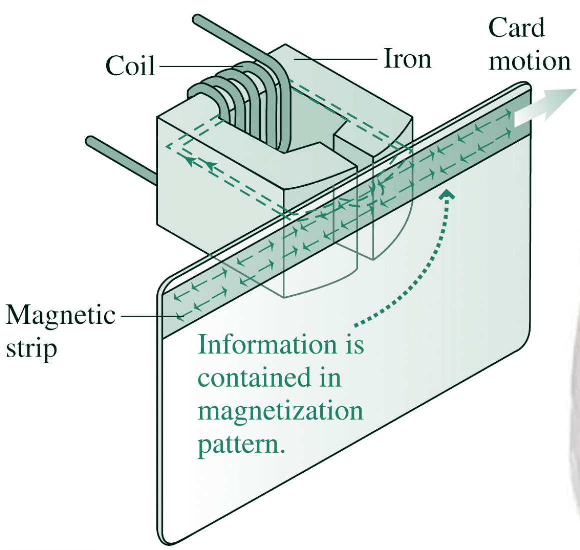
Induction and Lenz's Law Ponderables

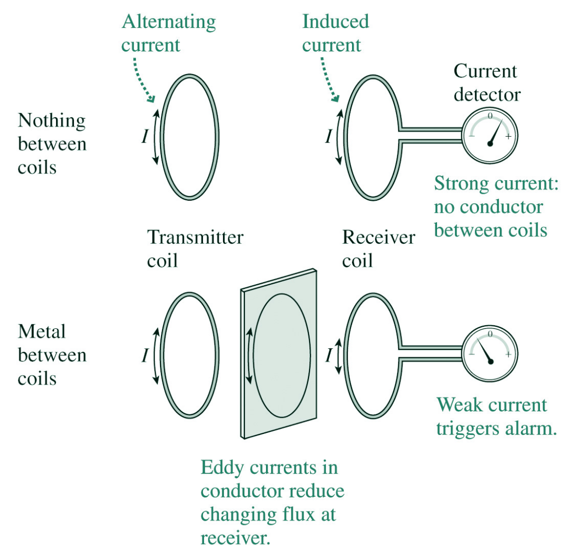


- A magnet is being pushed towards the right away from a solenoid wrapped as shown above.
- Which way does current flow through the resistor?
  - From a to b
  - From b to a
  - Neither

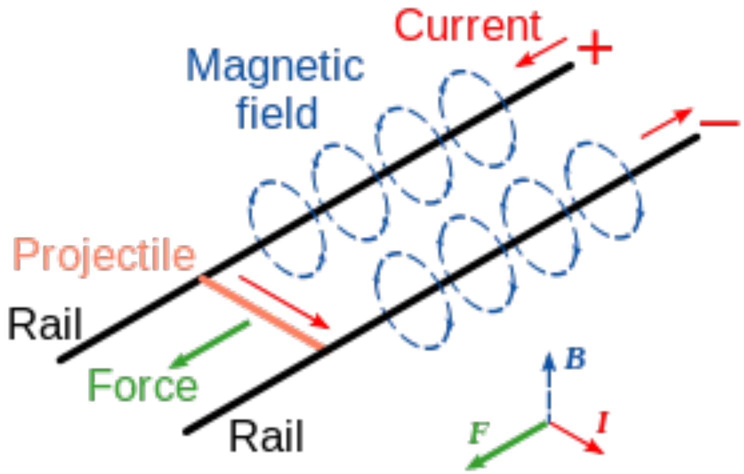
Other Uses of Induction

- Electromagnetic induction is used to retrieve information stored magnetically on audio and video tapes and credit cards.
- **Eddy currents** produced by induction in moving conductors act as a kind of electromagnetic friction.
  - This may be a nuisance, sapping energy.
  - Or it can be used to provide electromagnetic braking of spinning machinery.
  - Eddy currents play an important role in metal detectors.





Railguns

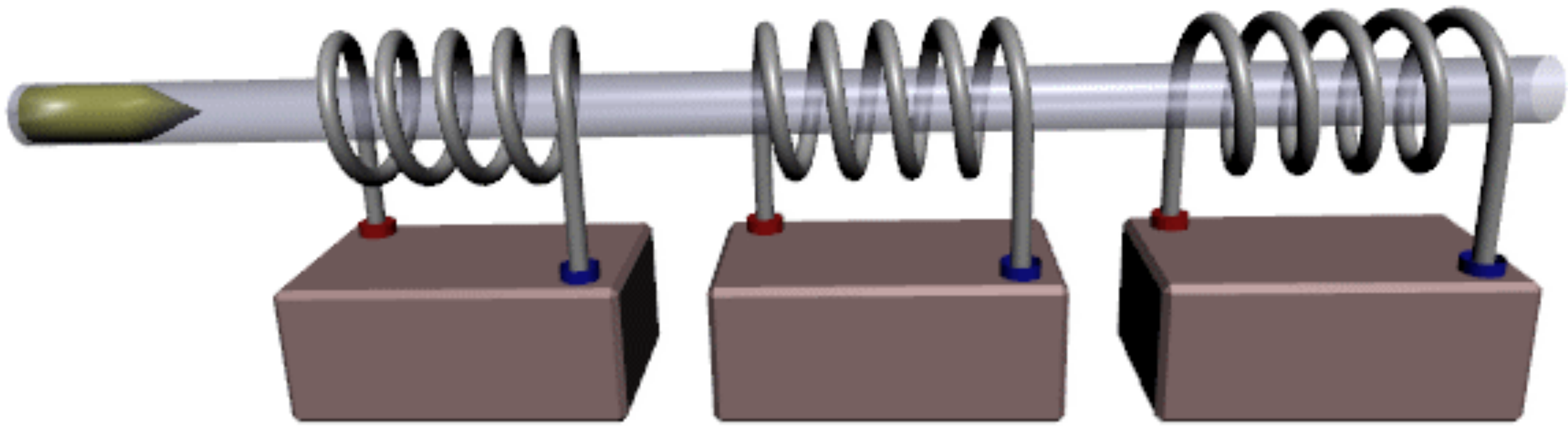


$$\vec{F} = I\vec{L} \times \vec{B}$$

$$a = \frac{I^2}{2m}\left(\frac{dL_{\text{ind}}}{dx}\right)$$

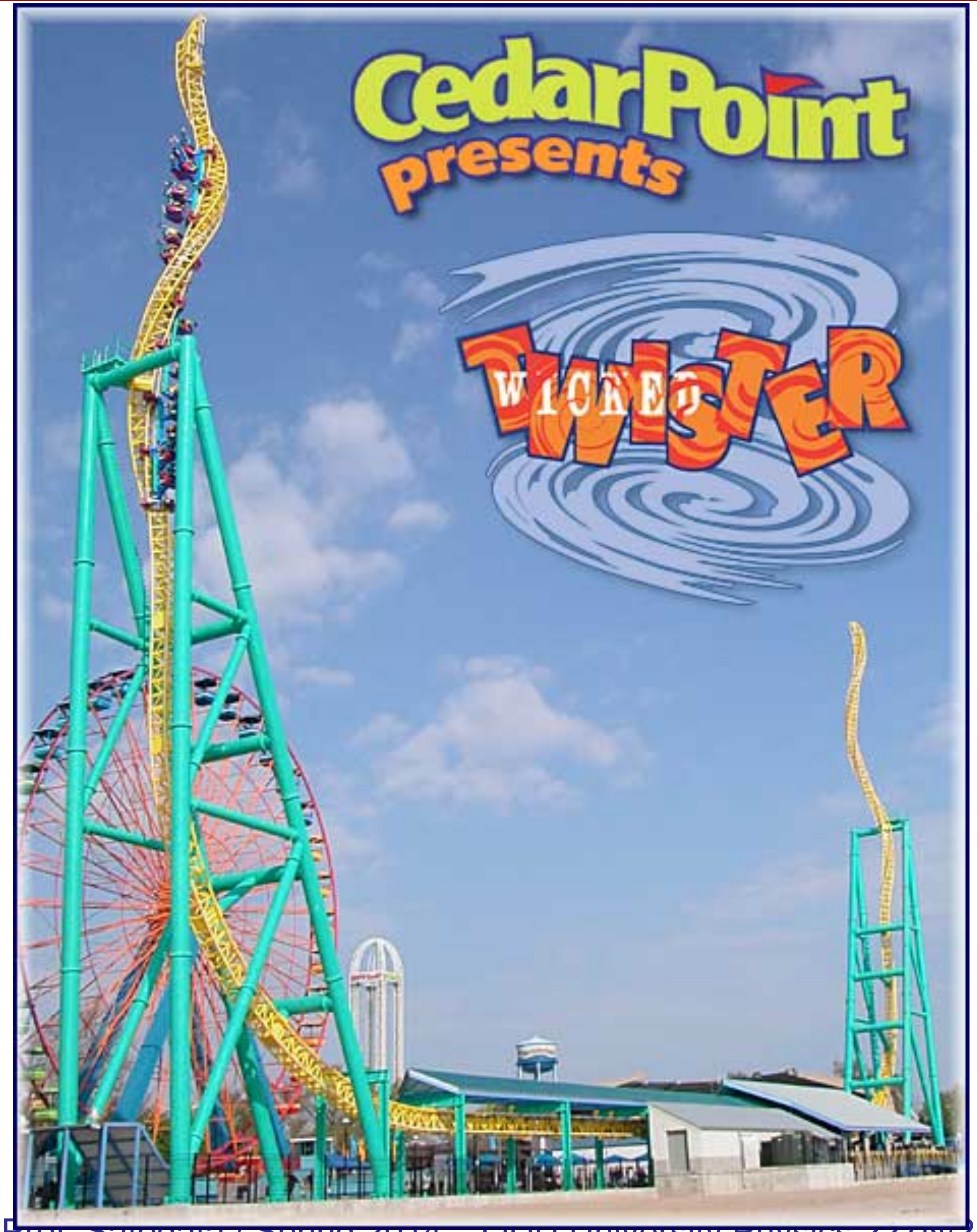
- Uses: military projectiles, possible space launch capability
  - Muzzle velocities up to 3000+ m/s
  - Magnetic fields up to 10 T
  - Instantaneous current: several million Amps!
  - Limited by
    - Current carrying capacity of rails
    - Self-inductance (roughly half of energy stored in B, half of energy goes to kinetic energy of projectile)

Coilguns



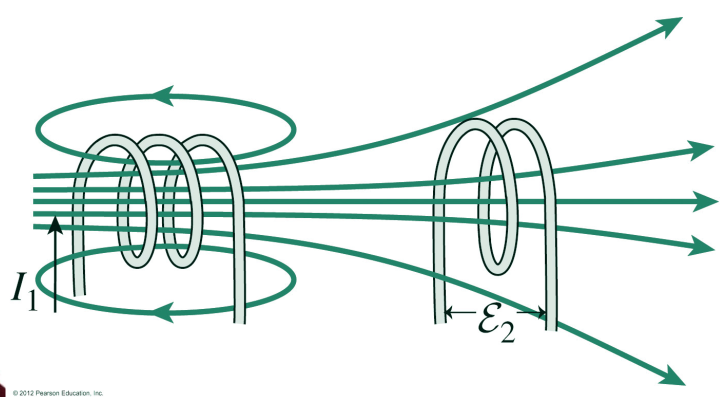
- Progressive solenoid fields create acceleration
  - Projectile must either be ferromagnetic, or have its own solenoidal coil that can be influenced through induction
  - Range from hobbyist single-coils to large military/NASA guns
  - Limited by
    - Magnetic reluctance (B field outside of center that rings back into electrical circuit: Lenz's Law in action)
    - Magnetic saturation of ferromagnetic projectile

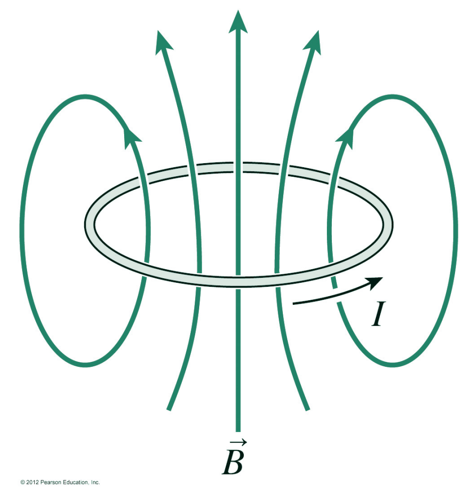
Linear Induction Motors



Inductance

- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit.
  - Mutual inductance occurs because some of the magnetic flux produced by one circuit passes through the other circuit.
- **Self-inductance** occurs when a changing current in a circuit results in an induced emf that opposes the change in the circuit itself.
  - Self-inductance occurs because some of the magnetic flux produced in a circuit passes through that same circuit.

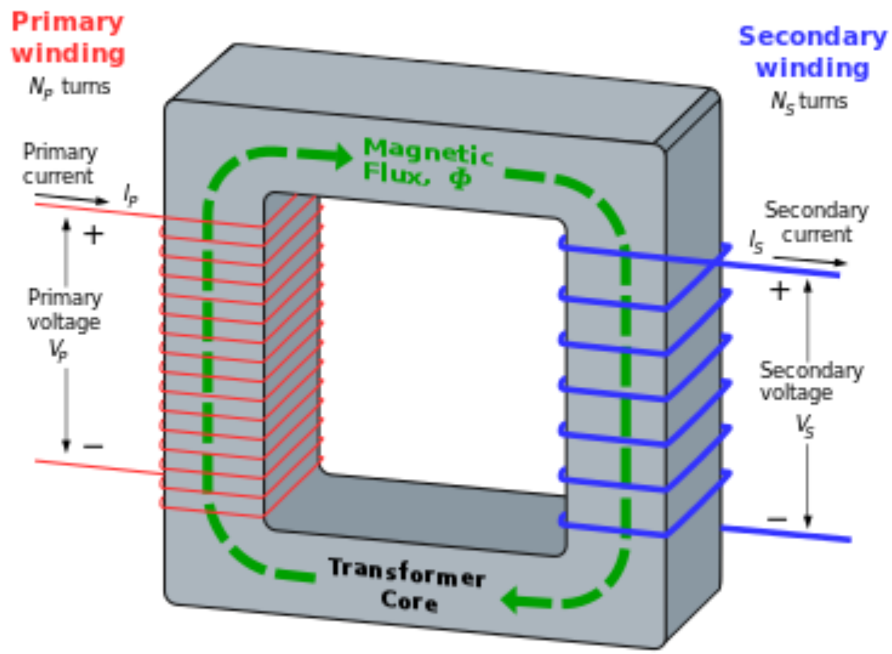




Mutual Inductance: Transformers

Mutual Inductance: Transformers

- An electrical transformer transmits electrical energy through mutual inductance
  - A purely AC device
    - Requires AC current to produce changing magnetic flux and induced current
    - AC frequency is same on both sides
  - Primary and secondary solenoid windings, usually around same core
  - Note opposite directions of windings!
  - Used to “step up” or “step down” AC voltage
    - Also used for AC circuit isolation

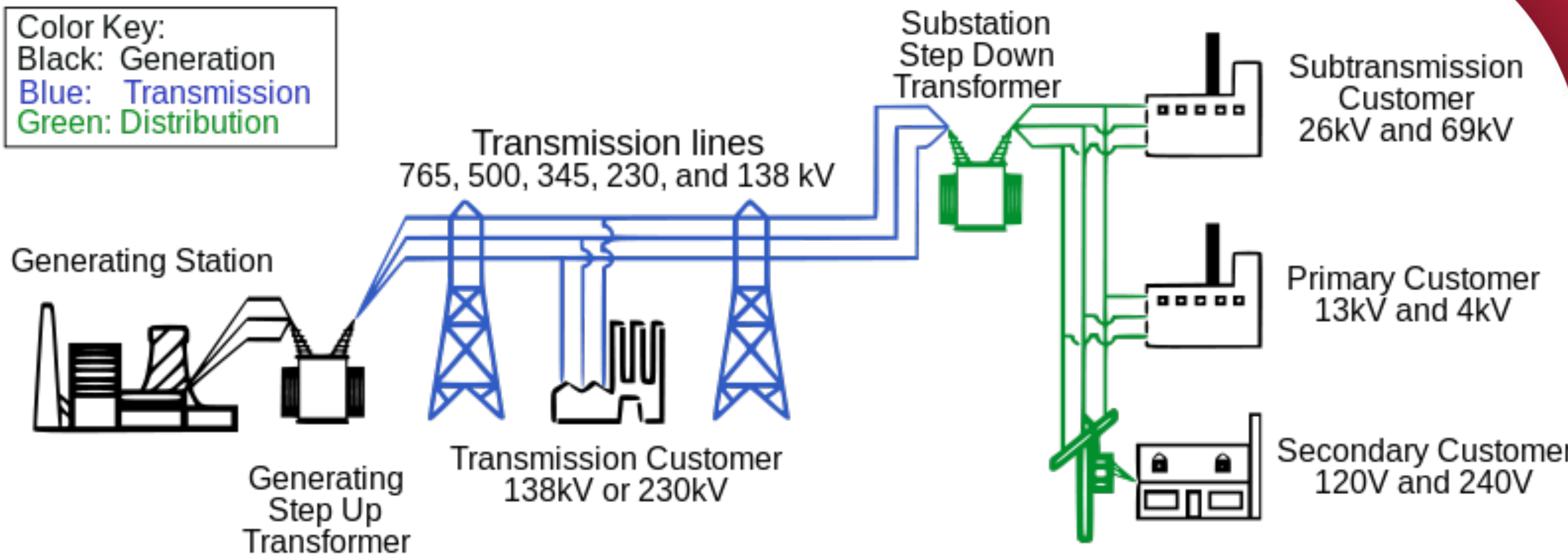


Flux is approximately the same on both sides

$$V_{\rm P} = N_{\rm P}\frac{d\Phi_{\rm B}}{dt} \qquad V_{\rm S} = N_{\rm S}\frac{d\Phi_{\rm B}}{dt}$$

$$\frac{V_{\rm S}}{V_{\rm P}} = \frac{N_{\rm P}}{N_{\rm S}}$$

Transformers and Electrical Power Distribution



- Transformers are crucial to electric power distribution
  - High-voltage AC wires lose less power over long distribution distances
  - Lower voltage is needed for use and electrical safety in homes
  - A transformer/diode combination can be used to produce DC voltage: a **rectifier**