University Physics 227N/232N

Interference and Diffraction

Homework "Optics 2" due Friday AM
Quiz Friday
Optional review session next Monday (Apr 28)
Bring Homework Notebooks to Final for Grading

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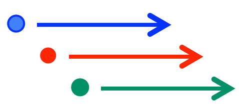
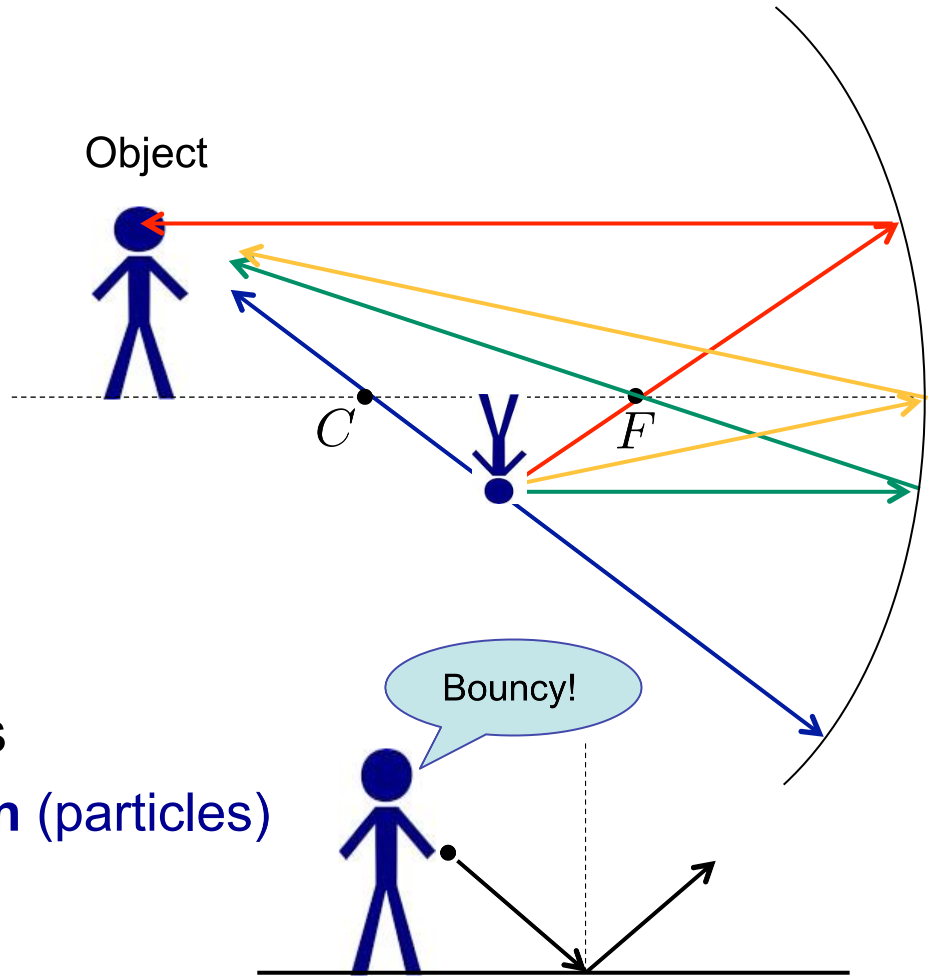
Wednesday, April 23 2014

Happy Birthday to George Lopez, Valerie Bertinelli, Roy Orbison, Shirley Temple, Sergei Prokofiev, and Max Planck (1918 Nobel)!

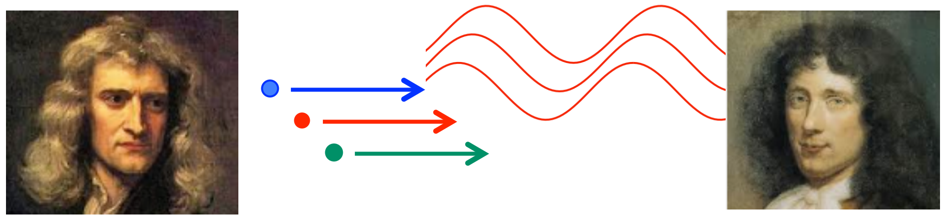
Light Rays = Particles?

- We've been learning about geometric optics by modeling light as a ray that travels in a straight line, except when reflecting or refracting
  - Light reflection looks a lot like how an object bounces off a surface.
  - Maybe light is made of little particles that can interact with surfaces…
  - The little particles can even be colored for different colors of light 
- This is not as crazy as it sounds
  - Lots of debate between **Newton** (particles) and **Huygens** (waves)



A Long Debate and a Surprising Resolution

- This debate lasted long after Newton and Huygens
  - Not fully resolved until the advent of quantum mechanics in the early 1900s

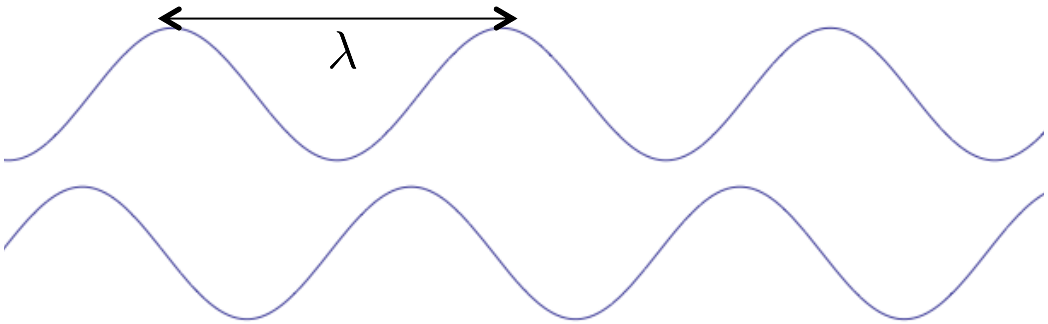


- The solution?
  - Newton and Huygens were both wrong… and both right!
  - Light acts like **both particles and waves**
  - It really depends on how you interpret various observations
  - (Also confusing: even though we now know of light as electromagnetic waves, the waves aren't actually vibrating anything. There is no vibrating medium for light -- no "luminiferous aether" – as demonstrated by Michelson and Morley in 1887.)

- We've seen a lot of geometric optics that Newton used to argue that light is made of particles
  - Today (and maybe Friday) we'll learn how light acts like waves, through phenomena like **interference** and **diffraction**

Review: Waves

- You hopefully learned about mechanical waves last semester
- A wave is like a sine wave in space that is also moving in time
- We can describe a single wave mathematically
  - For simplicity, assume it's moving in the z direction



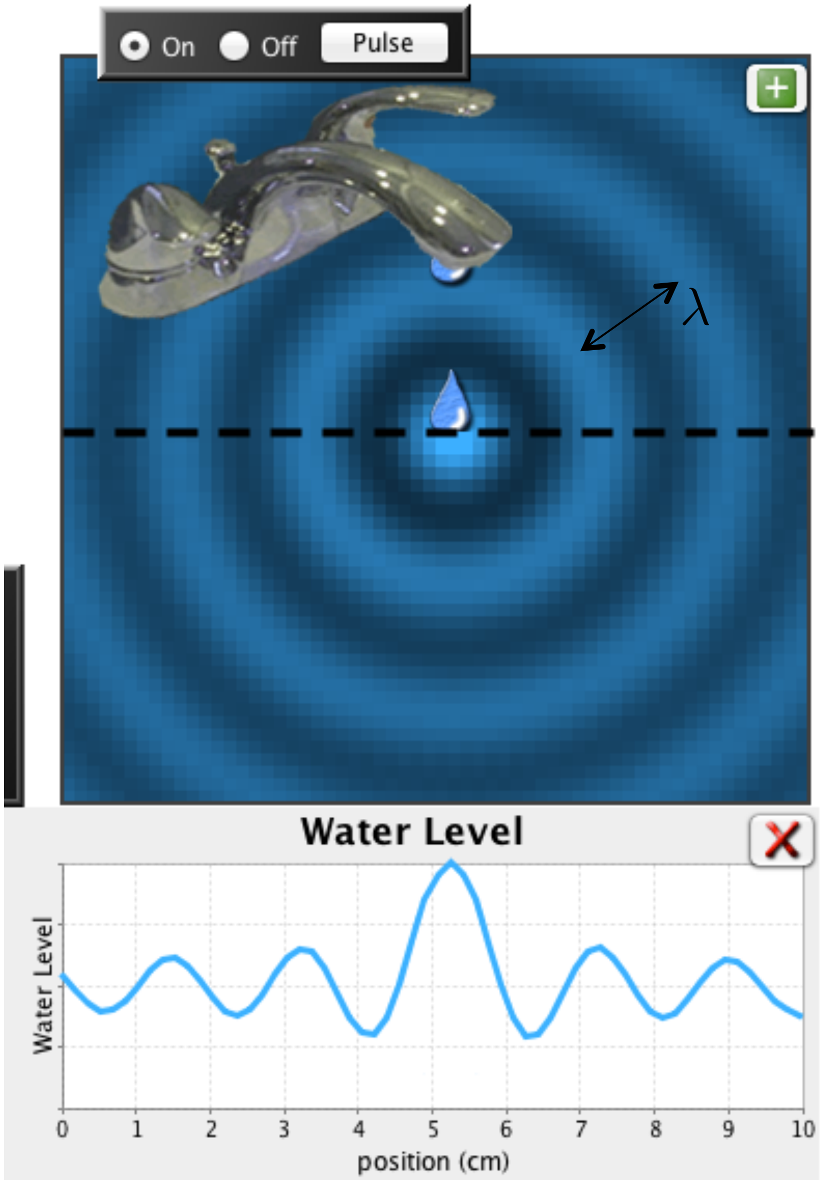
  - Then we have

$$a(z,t) = A \sin\left(\left(\frac{2\pi}{\lambda}\right)x - (2\pi f)t\right)$$

  - Where λ is the **wavelength**, f is the **frequency**, and A is the **amplitude** of the wave. Angles are in **radians**. (One wave: 0-2π)
  - More complicated waves can be made by adding many waves together – and waves can travel outwards in circles or spheres
    - Like water waves created by throwing a rock in a pool

Yes, It's One Last Appearance of Java App Wednesday

http://phet.colorado.edu/en/simulation/wave-interference

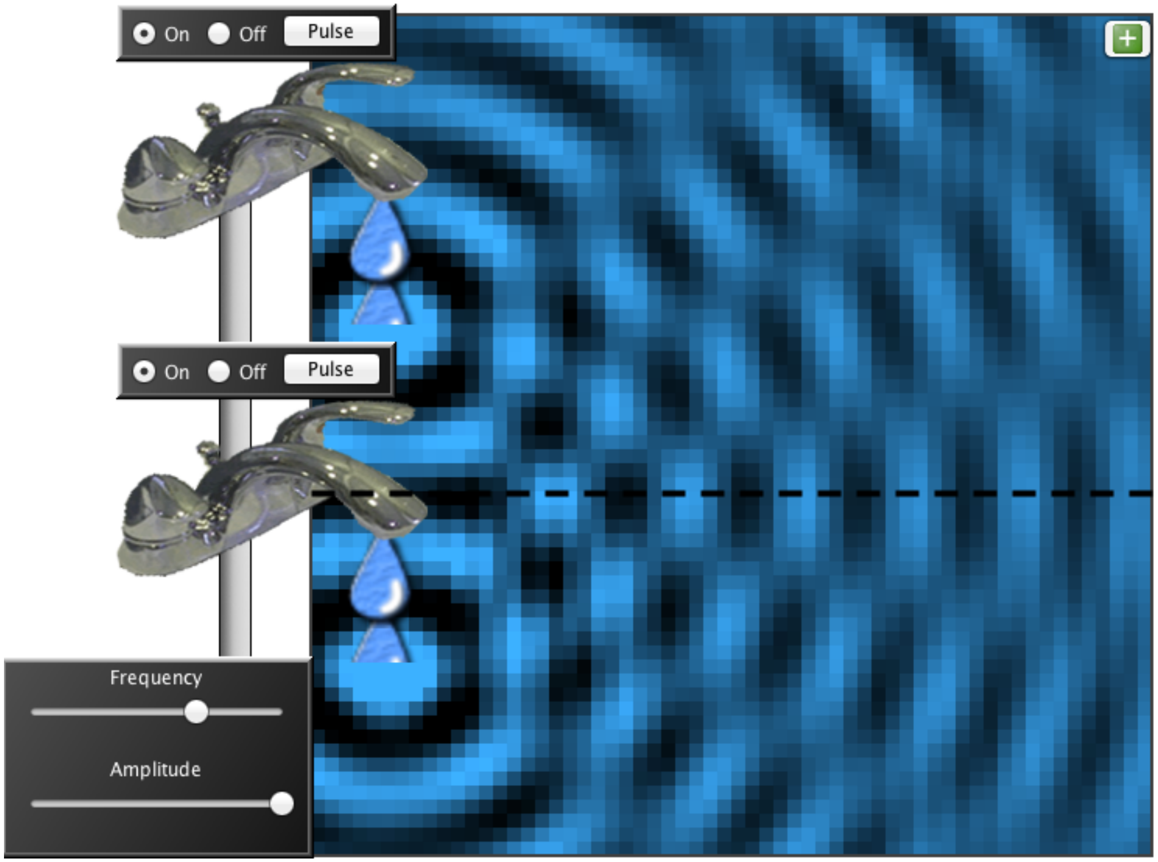


Brighter spots are peaks
Darker spots are valleys

Spacing between successive peaks or successive valleys is the wavelength

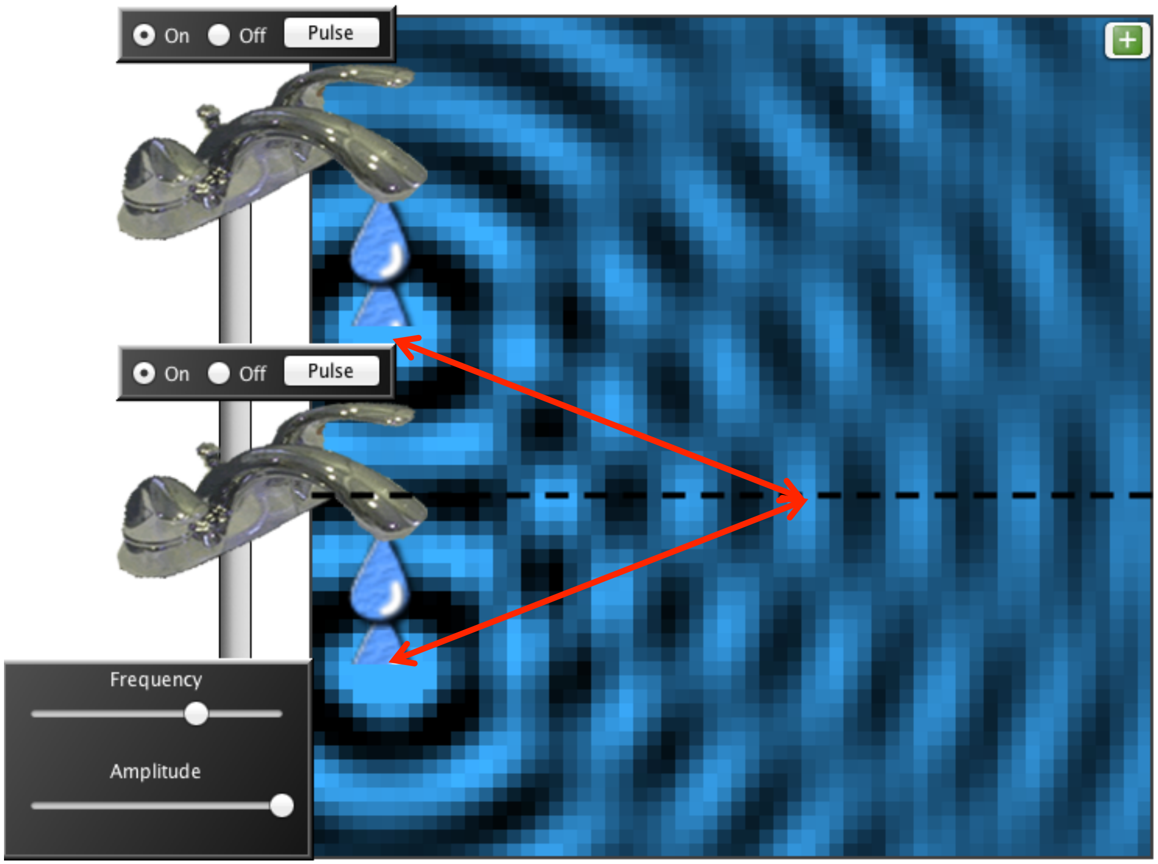
Water Wave Interference

- Multiple waves add together
  - The various crests and troughs move through each other, and add together depending on their mutual arrival times
  - Here we see two faucets in phase (drops drop at the same time)
  - Both waves expand out at the same speed, producing a pretty pattern



Constructive Interference (n=0)

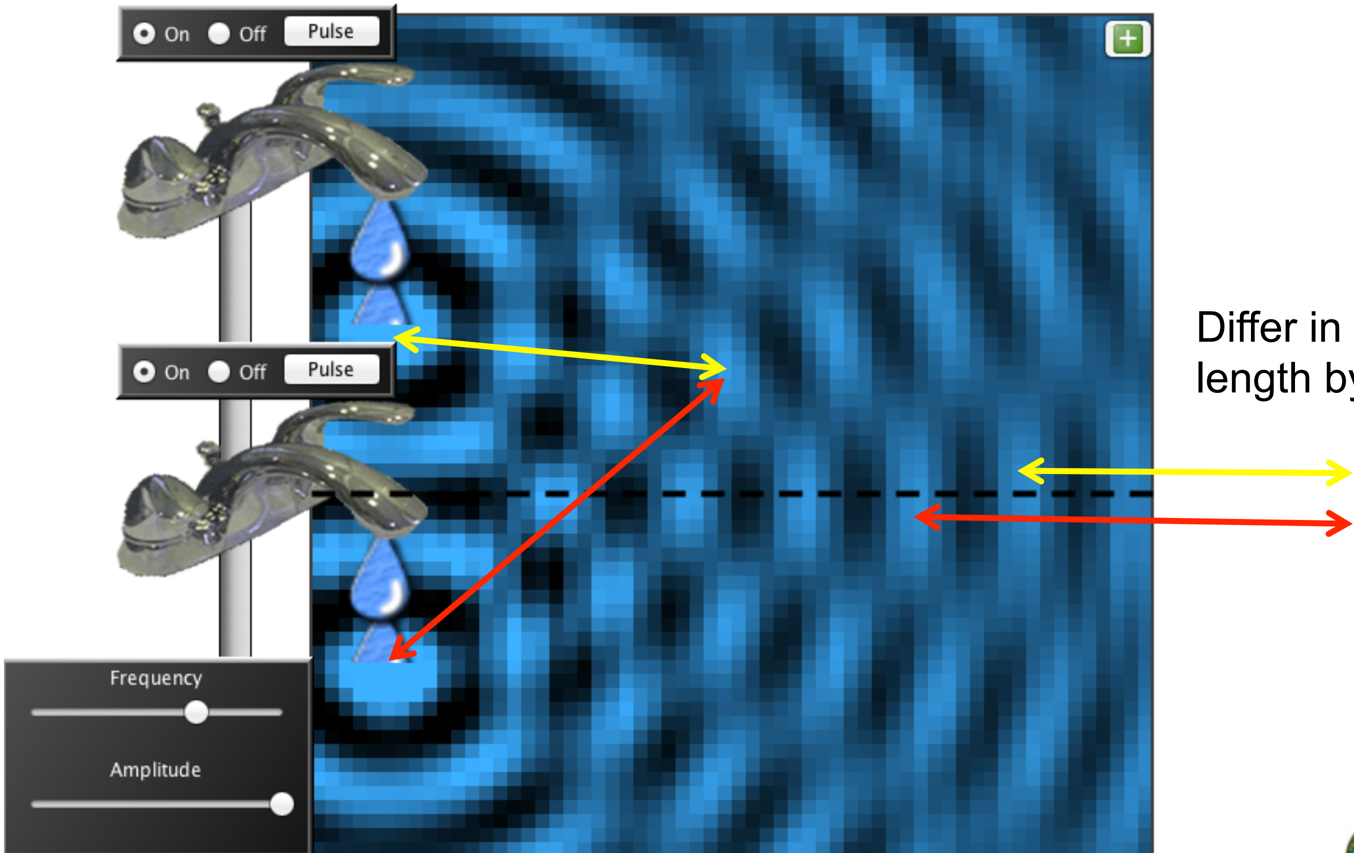
- For points where both waves travel the same distance (or travel distances that differ by nλ), both waves crest and trough together
  - These points move up and down like a wave with double the amplitude of the original waves
  - This is called **constructive interference**



Differ in path length by 0 (same path length)

Constructive Interference (n=1)

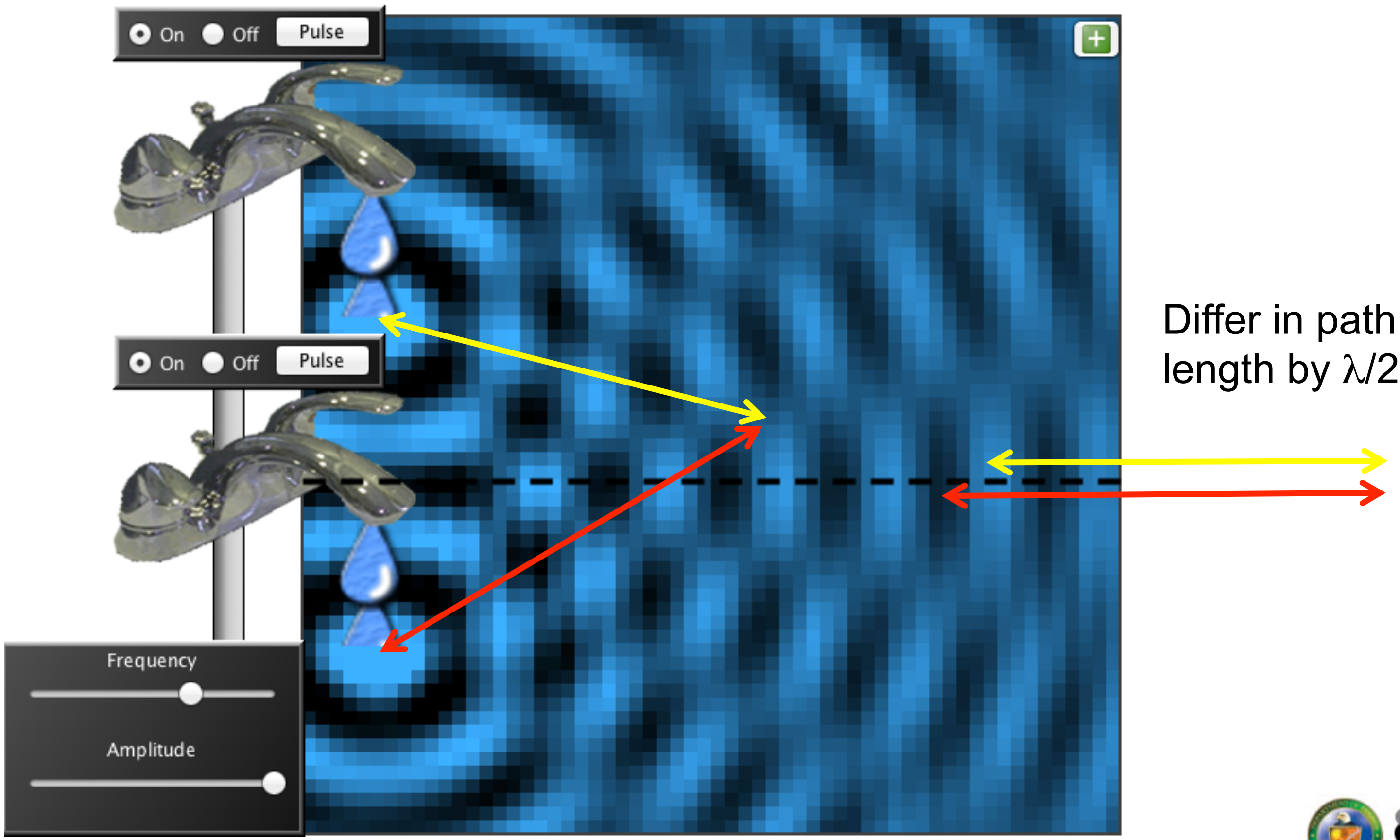
- For points where both waves travel the same distance (or travel distances that differ by nλ), both waves crest and trough together
  - These points move up and down like a wave with double the amplitude of the original waves
  - This is called **constructive interference**



Differ in path length by λ

Destructive Interference (n=0)

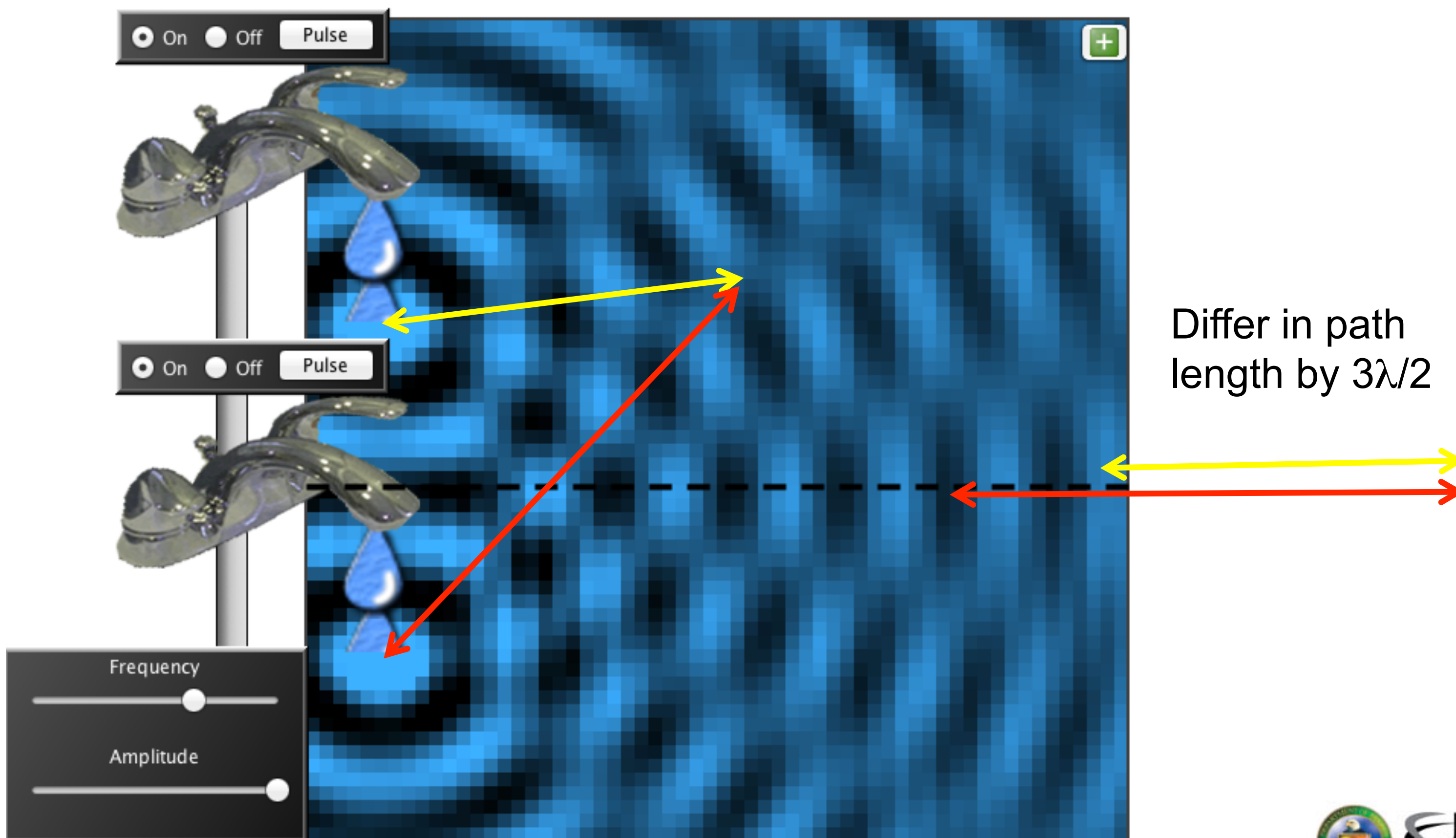
- For points where both waves travel distance that differ by λ/2 (or travel distances that differ by nλ+λ/2), both waves crest and trough exactly 180 degrees apart
  - The waves cancel each other out at these points for all times
  - This is called **destructive interference**



Differ in path length by λ/2

Destructive Interference (n=1)

- For points where both waves travel distance that differ by $\lambda/2$ (or travel distances that differ by $n\lambda+\lambda/2$), both waves crest and trough exactly 180 degrees apart
  - The waves cancel each other out at these points for all times
  - This is called **destructive interference**



An Incoherent Word About Coherence

- These waves produce pretty patterns because
  - …the drops that generate them stay synchronized
  - …the waves that are generated have the same wavelength

- It's easier to produce this sychronization (**coherence**) with water waves than with light
  - Sunlight is composed of many different colors: different wavelengths
  - All those colors blur together
  - Interference patterns also get blurry if the sources are far apart compared to the wavelength of the waves
    - This creates many close areas of constructive and destructive interference that are hard to see
    - The separation ends up being comparable to the wavelength of the waves
    - In light's case, that is awfully short (390-700 nm), even for Newton

Quick Question

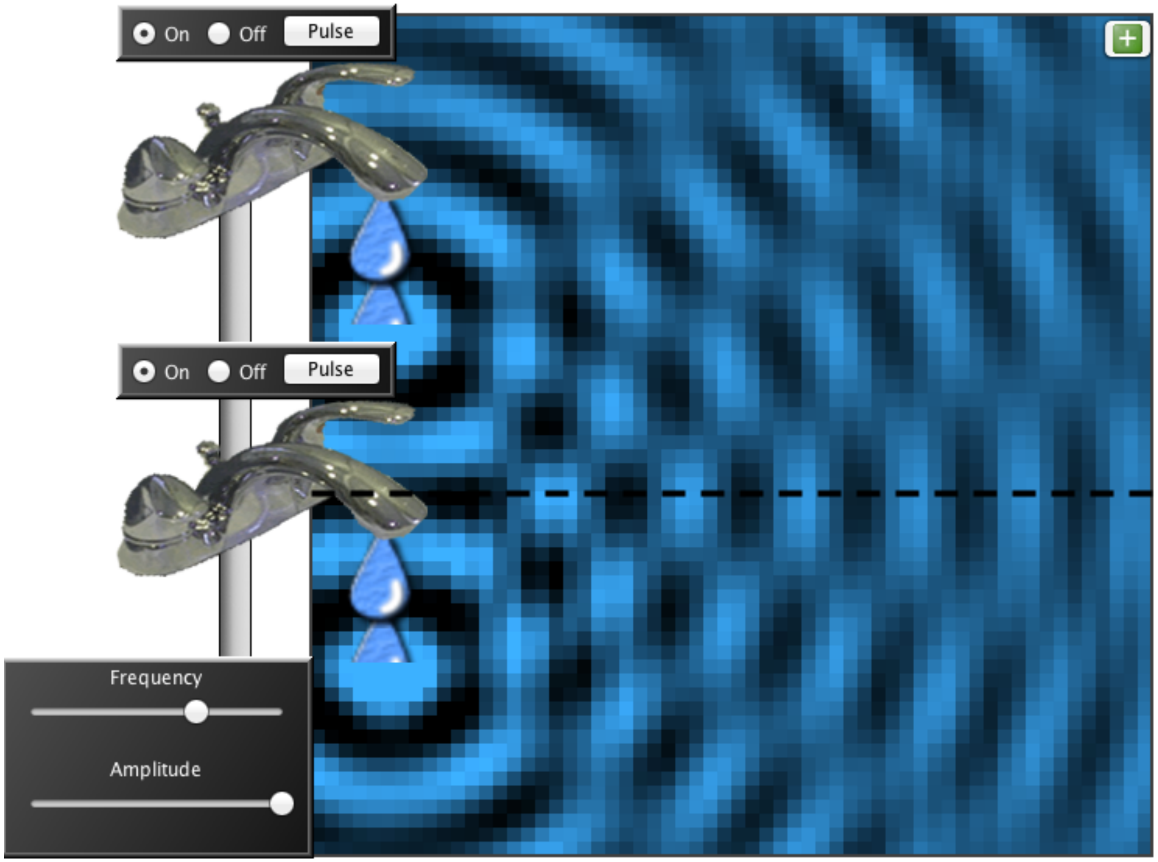
- Constructive interference of two coherent waves of wavelength λ will occur if their path length difference is

A. 2λ

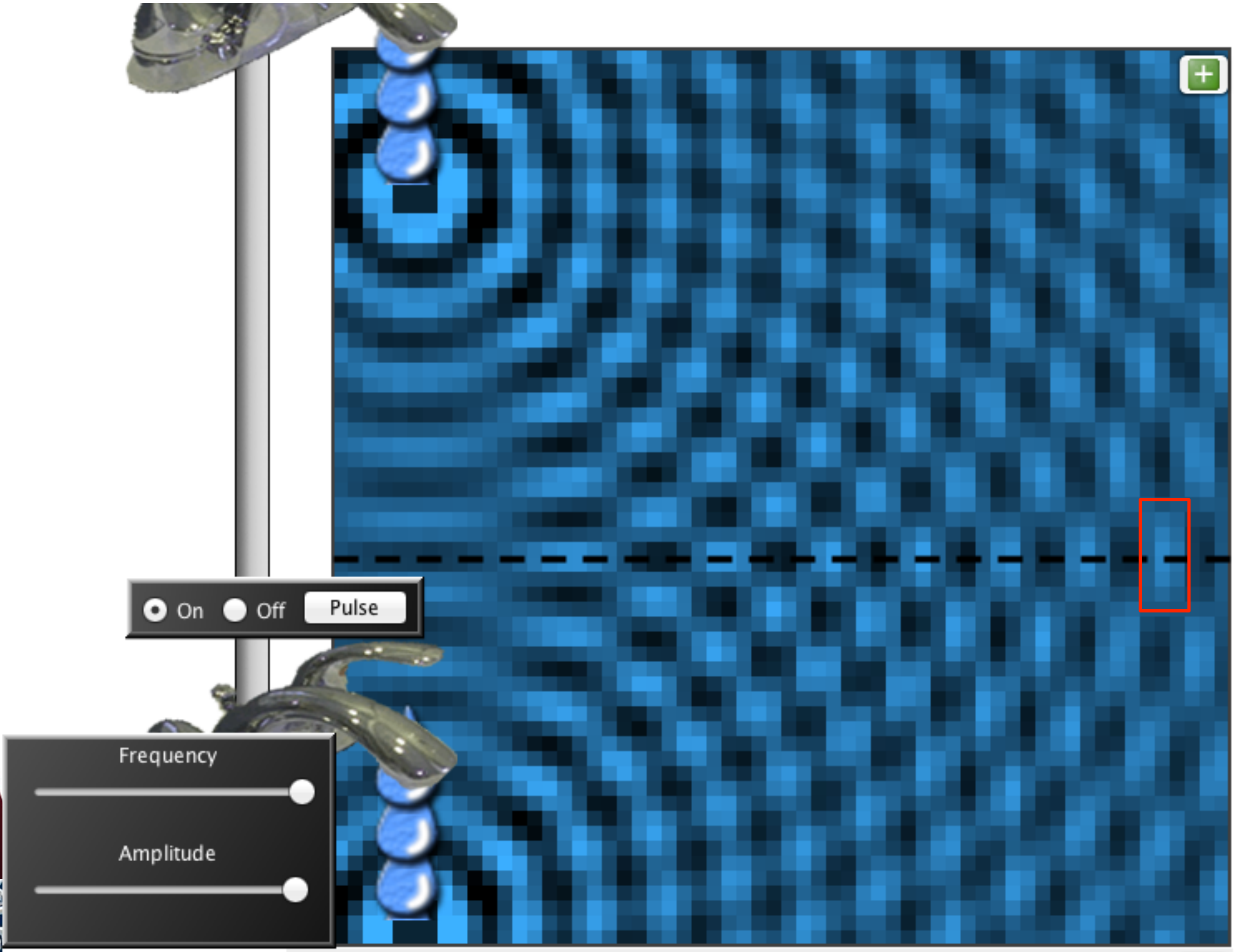
B. $3\lambda/2$

C. $5\lambda/2$

D. $\lambda/3$



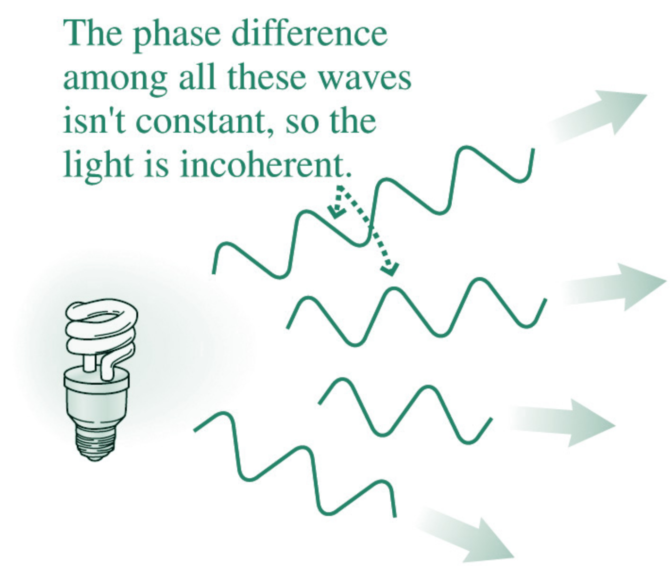
Widely Separated Sources (compared to λ)



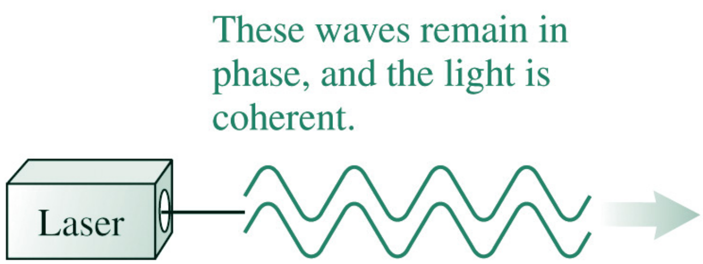
Similar width and height

Interference and Coherence

- Wave interference is best observed with **coherent** light.
  - Coherence means that two interfering waves have a phase relationship that stays fixed for many wave cycles.
  - Coherence also requires that interfering light beams have exactly the same frequency and therefore wavelength or color.
  - Lasers are nearly ideal sources of coherent light.
    - Their light is very nearly monochromatic, consisting of a narrow band of wavelengths.



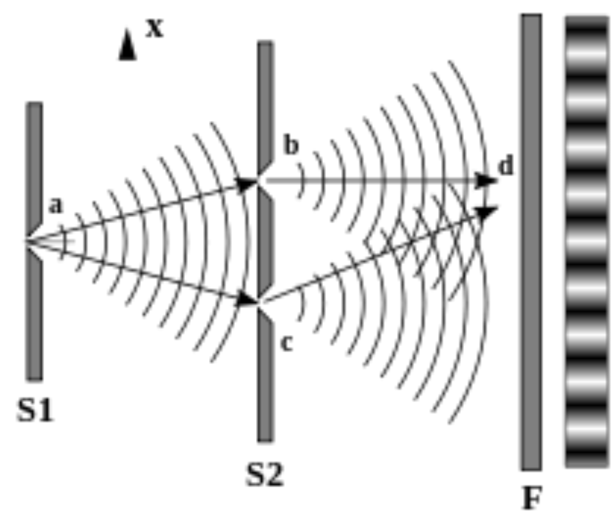
(a)



(b)

How To Produce Two Coherent "Drops" of Light?

- We can use a laser and split it with reflection and refraction
  - But that's a pretty modern approach
- Huygens didn't have the experimental techniques necessary to see light interference directly
- Thomas Young in 1801: "On the Theory of Light and Colours"
  - Available on the interwebz! http://www.jstor.org/stable/107113
  - Clever idea: pass light through a single pinhole, then two small slits
  - Young already knew the approximate wavelength of light
  - Interference appears as light (constructive) and dark (destructive) projected bands on a dark wall
  - The pinhole and slit size should be comparable to (or not very much larger than) the wavelength of the light being investigated



Double-Slit Interference

- Interference from two coherent sources produces a pattern of light and dark **interference fringes**
- A convenient way to produce the two sources is to pass light from a single source through two narrow slits.

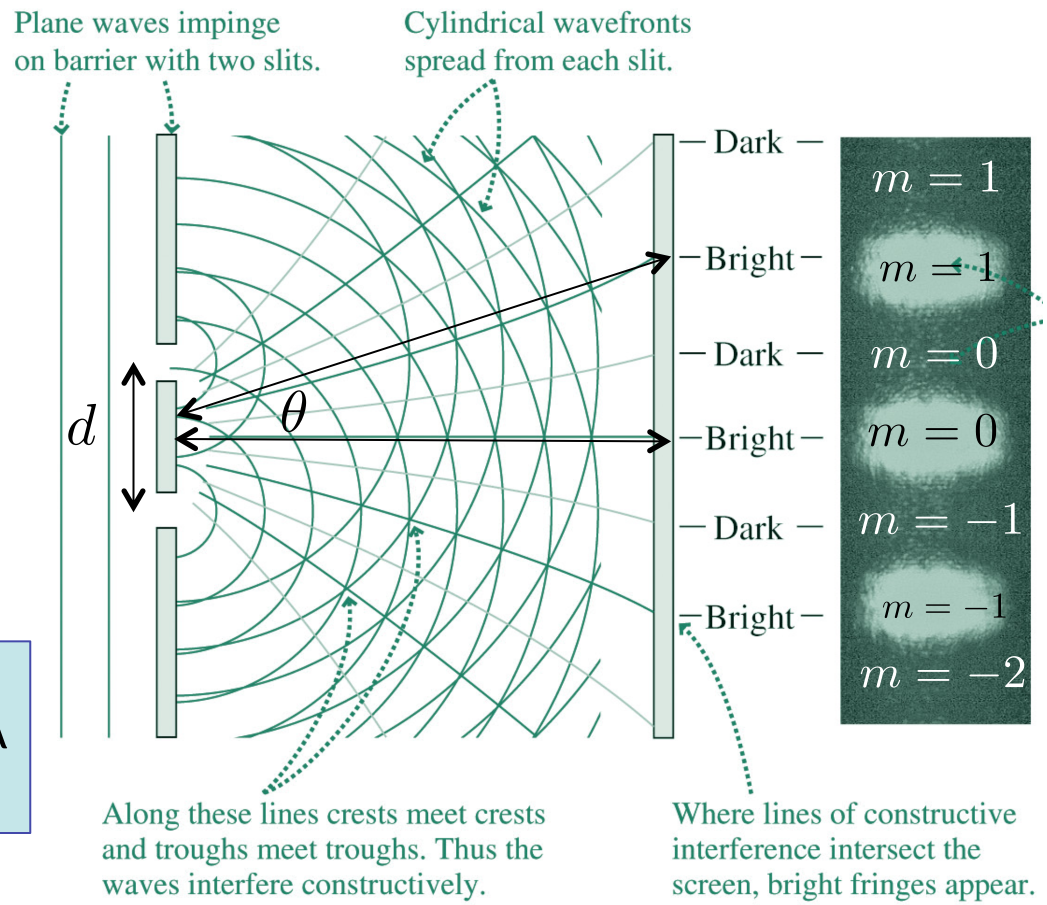
- Positions of bright fringes are given by

$$d \sin \theta_{\text{bright}} = m\lambda$$

where m is the m$^{\text{th}}$ fringe, and λ is the wavelength.

- Dark fringes occur where

$$d \sin \theta_{\text{dark}} = \left(m + \frac{1}{2}\right) \lambda$$



Trig Happy

- We can also calculate the distances between fringes as measured on the screen

$$\tan\theta = \frac{x}{L} \quad \Rightarrow \quad \sin\theta \approx \frac{x}{L}$$

Small-angle approximation for x<<L!!

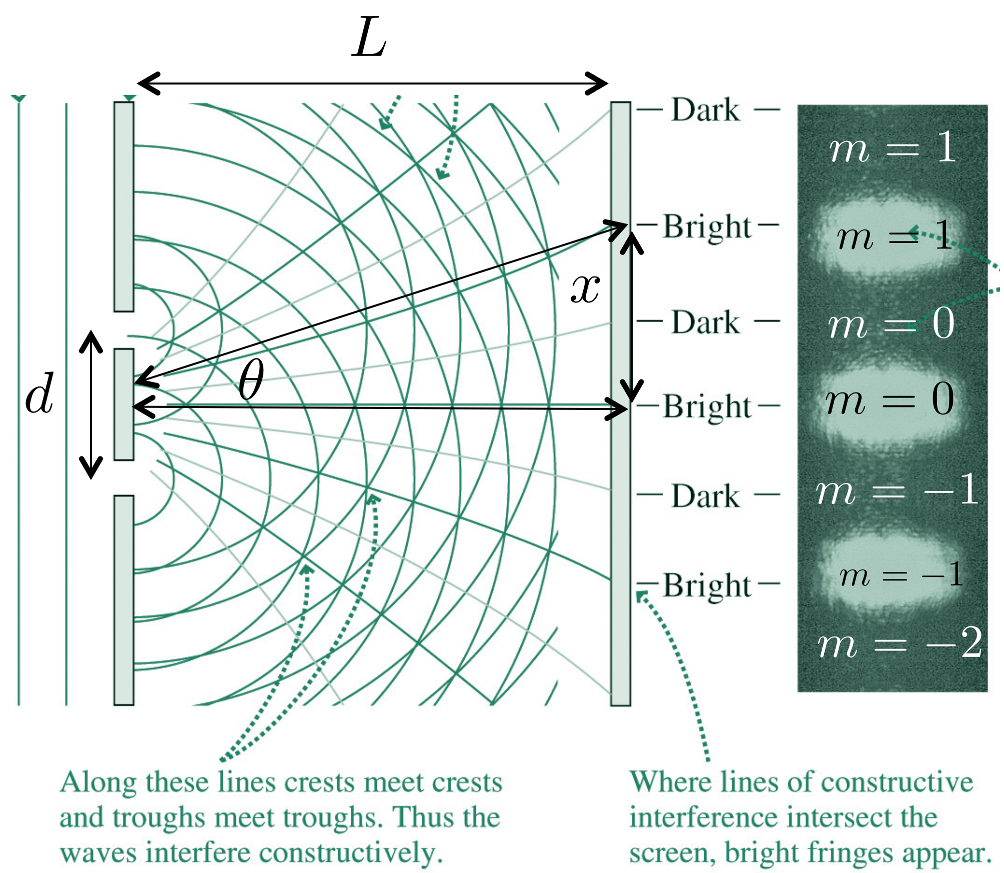
$$\sin\theta = \frac{x}{\sqrt{x^2+L^2}}$$

$$d\sin\theta_{\text{bright}} = m\lambda$$

$$x_{\text{bright}} = \frac{m\lambda L}{d}$$

$$d\sin\theta_{\text{dark}} = \left(m+\frac{1}{2}\right)\lambda$$

$$x_{\text{dark}} = \frac{\left(m+\frac{1}{2}\right)\lambda L}{d}$$



Quick Question

- If you increase the slit separation in a two-slit system, how does the spacing of interference fringes change?

A. They become farther apart.
B. Their spacing does not change.
C. They become closer together.

Example

- Coherent red light (λ=650 nm) is shining on a double-slit apparatus, where the slits are separated by d=1.0 mm. The image is projected onto a screen positioned L=2.0 m away from the slits.
  - How far apart are the first dark bands to either side of the central bright spot?
  - How far apart are the second dark bands to either side of the central bright spot?

$$x_{\mathrm{bright}} = \frac{m\lambda L}{d}$$

$$x_{\mathrm{dark}} = \frac{\left(m + \frac{1}{2}\right) \lambda L}{d}$$

Example: Solution

- Coherent red light (λ=650 nm) is shining on a double-slit apparatus, where the slits are separated by d=0.1 mm. The image is projected onto a screen positioned L=3.0 m away from the slits.
  - How far apart are the first dark bands to either side of the central bright spot? m=0 and m=-1, dark bands $\boxed{\Delta x = 3.9\ \mathrm{mm}}$
  - How far apart are the second dark bands to either side of the central bright spot? m=1 and m=-2, dark bands $\boxed{\Delta x = 11.7\ \mathrm{mm}}$

$$x_{\mathrm{bright}} = \frac{m\lambda L}{d}$$

$$x_{\mathrm{dark}} = \frac{\left(m + \frac{1}{2}\right)\lambda L}{d}$$

$$\frac{\lambda L}{d} = \frac{(6.5 \times 10^{-7}\ \mathrm{m})(3.0\ \mathrm{m})}{5.0 \times 10^{-4}\ \mathrm{m}} = 3.9\ \mathrm{mm}$$

$$x_{\mathrm{dark}}(m=0) = 1.95\ \mathrm{mm} \qquad x_{\mathrm{dark}}(m=-1) = -1.95\ \mathrm{mm}$$

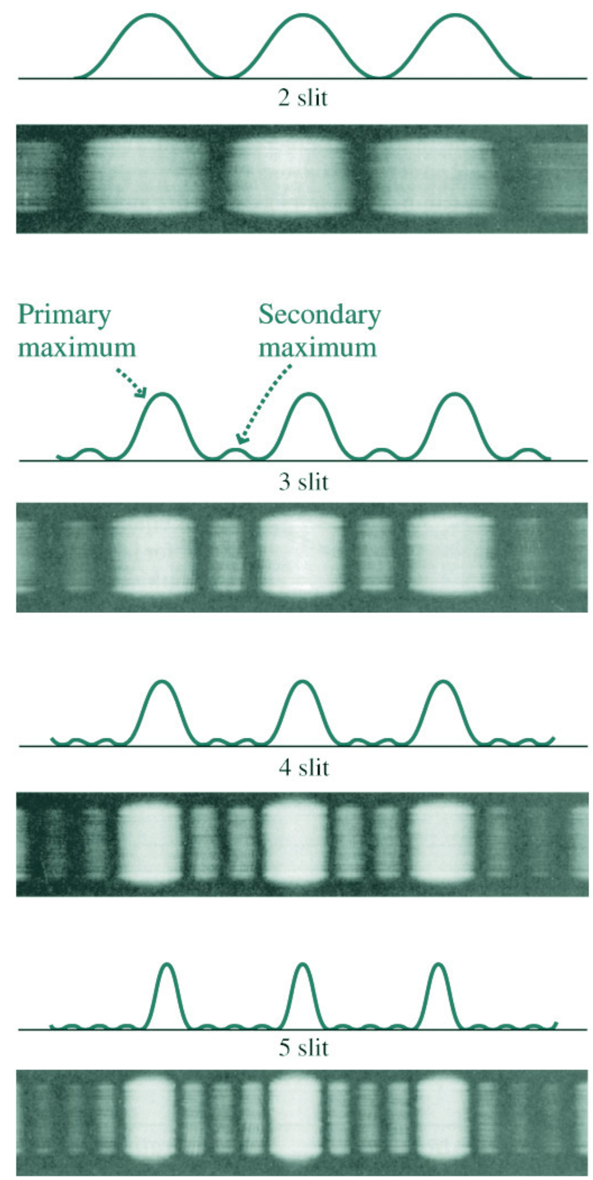
$$x_{\mathrm{dark}}(m=1) = 5.85\ \mathrm{mm} \qquad x_{\mathrm{dark}}(m=-2) = -5.85\ \mathrm{mm}$$

Multiple-Slit Interference

- For three or more slits, the condition for constructive interference remains the same as with two slits, namely

$$d \sin \theta_{\text{bright}} = m\lambda$$

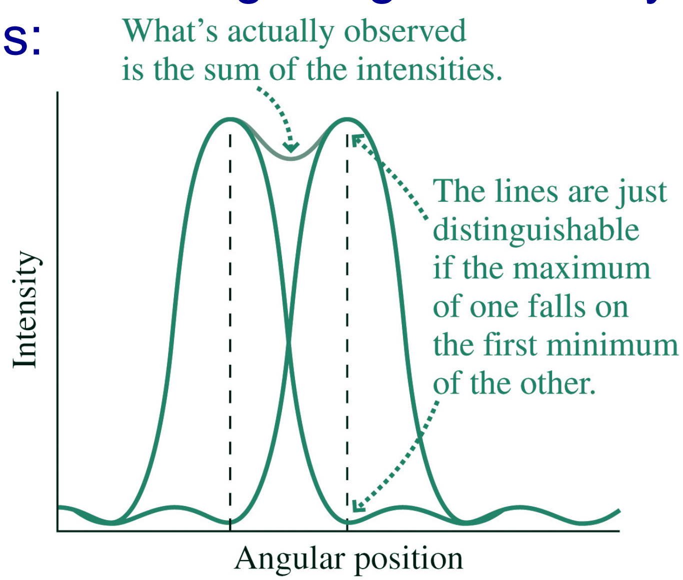
  - As the number of slits increases, the intensity maxima become higher and narrower.
  - The intervening regions, which consist of minima interspersed with secondary maxima, become more uniformly dark in contrast with the bright maxima.
  - With a very large number of slits, the interference pattern becomes a set of very bright, narrow lines at the primary maxima, with dark regions between.

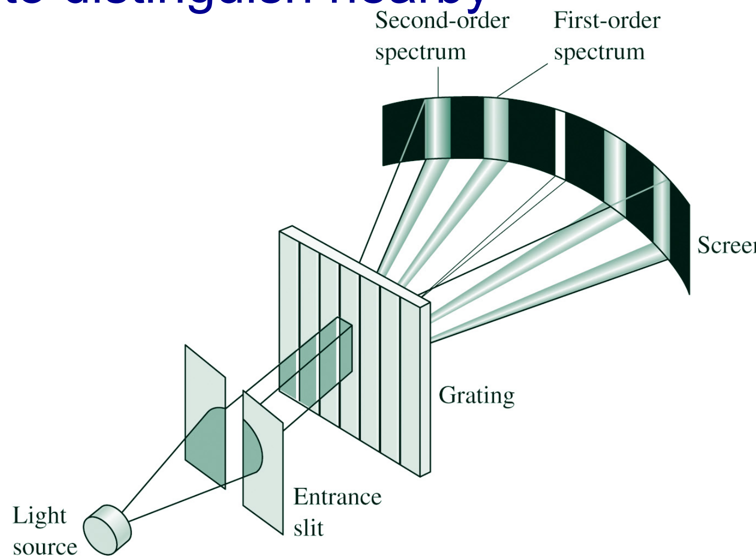


Diffraction Gratings and Spectroscopy

- A system with many closely, evenly spaced slits (typically thousands) is a **diffraction grating.**
  - Gratings work by transmission through actual slits or by reflection off closely spaced parallel lines ruled in reflective material.
  - Because the intensity maxima of a grating occur where $d\sin\theta = m\lambda$ maxima occur at different places for different wavelengths.
  - A grating spectrometer uses this to separate light into different wavelengths, very much like dispersion in a prism.
  - The resolution of the grating is its ability to distinguish nearby wavelengths:

$$\frac{\lambda}{\Delta\lambda} = mN$$



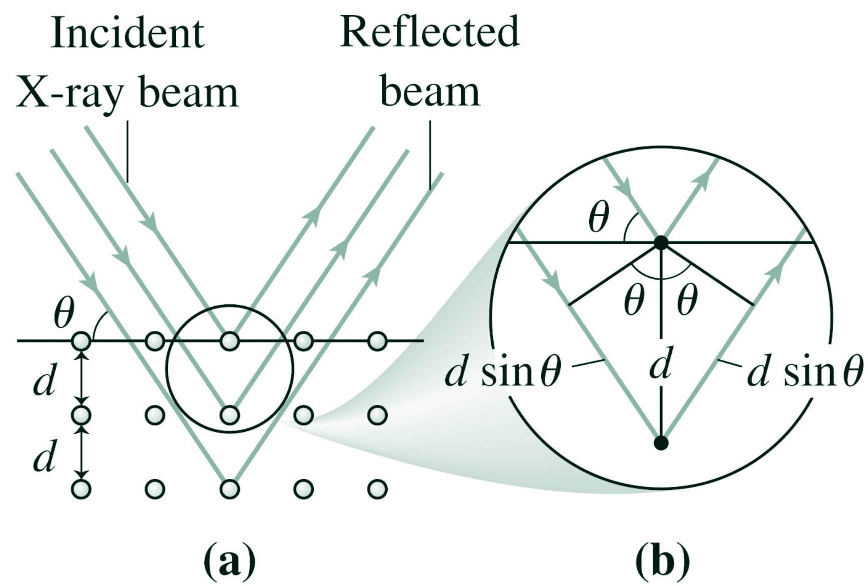


X-ray Diffraction

- The regularly spaced atoms of a crystal can serve as a diffraction grating for X rays.
  - That's because the X-ray wavelength (~0.1 nm) is comparable to the atomic spacing.
  - The diagram shows that constructive interference occurs when the **Bragg condition** is met:

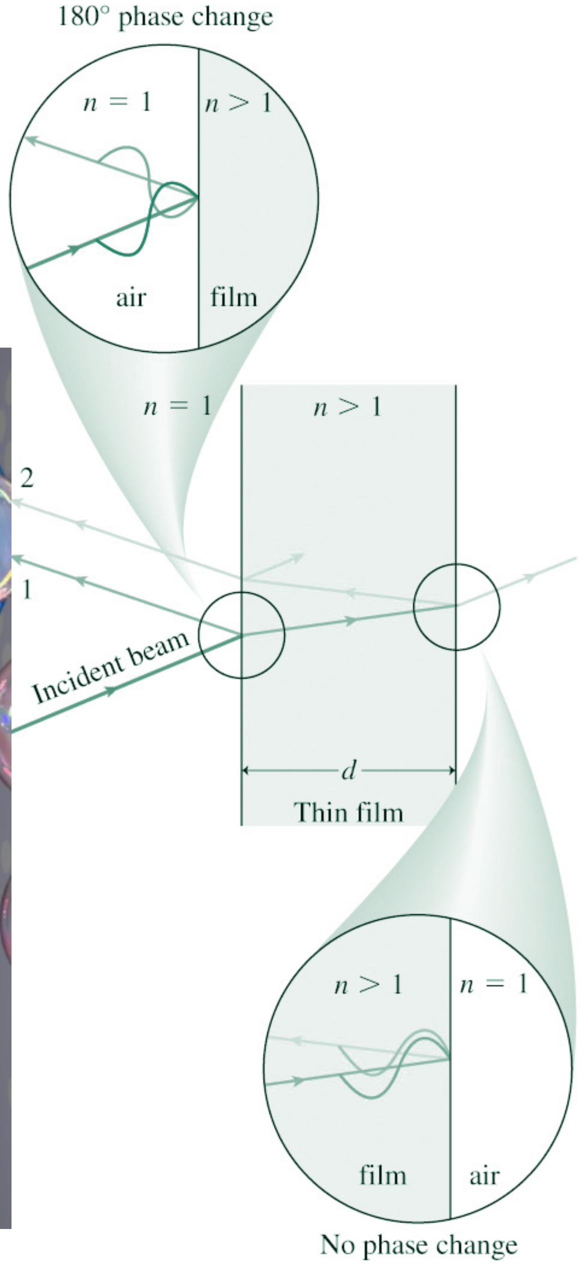
$$2d\sin\theta = m\lambda$$

– X-ray diffraction is used to study crystal structure.
  - X-ray diffraction was crucial in establishing the structure of DNA.



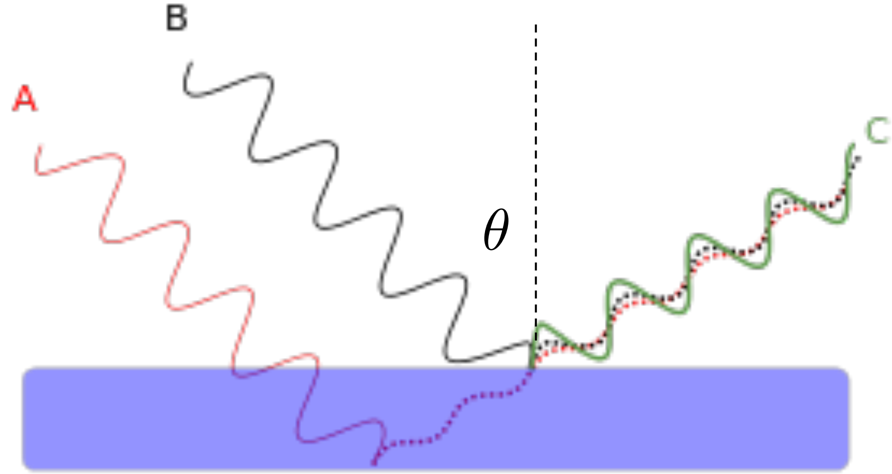
Interference in Thin Films

- Reflection at both surfaces of a thin film results in interfering light beams that produce wavelength-dependent interference patterns.

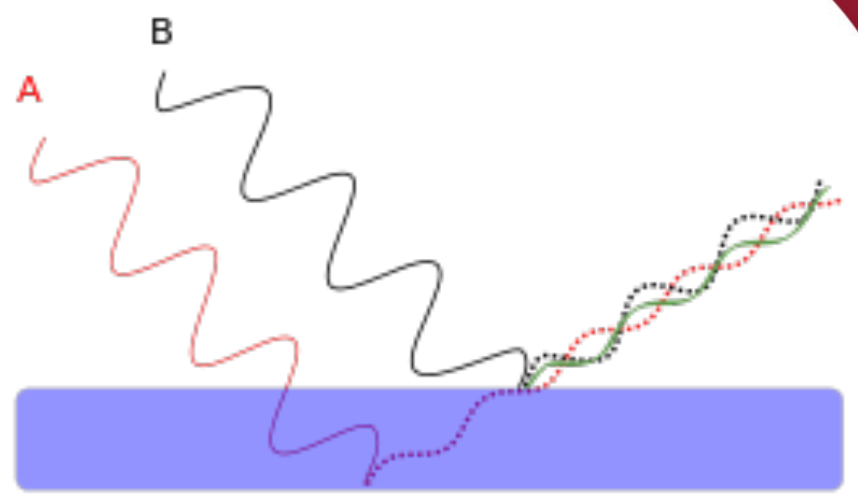


- Reflecting from lower to higher index of refraction n, the light wave flips sign (inverts)
  - Front surface of a bubble
  - Destructive interference
- Reflecting from higher to lower index of refraction n, the light wave does not flip sign
  - Back surface of a bubble
  - Constructive interference

More Thin Films



Constructive interference



Destructive interference

- For thin films in air, the first reflection flips 180 degrees but the second does not
  - The difference in path length for both waves must equal an even number of wavelengths (plus another 180 degrees to compensate for the difference in reflections) for them to constructively interfere
  - For a film of thickness d of refractive index n, this requirement is

$$2nd\cos\theta = \left(m - \frac{1}{2}\right)\lambda$$

m is an integer, the number of extra wavelengths traveled in the thin film

http://micro.magnet.fsu.edu/primer/java/interference/soapbubbles/

Example

- Find the maximum wavelength of light that will constructively interfere with itself when bouncing off the back end of an oil slick (n=1.7) that is d=100 nm thick.

$$2nd\cos\theta = \left(m - \frac{1}{2}\right)\lambda$$

  - Maximum wavelength is where cos=1 and m-1/2 is smallest (=1/2)
  - Then we have

$$2nd = \left(\frac{1}{2}\right)\lambda$$

$$\lambda = 4nd = 4(1.7)(100\,\text{nm}) = \boxed{680\,\text{nm} = \lambda}$$

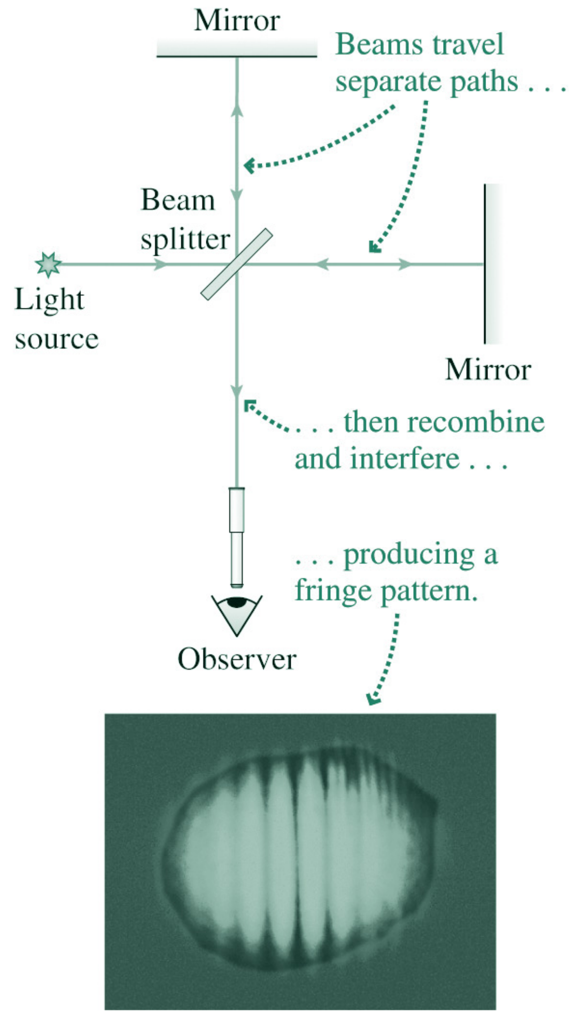
Quick Question

- What phenomena cause the colors on an oil slick?

A. Reflection and diffraction
B. Reflection and interference
C. Refraction and diffraction
D. Refraction and interference
E. Polarization and diffraction

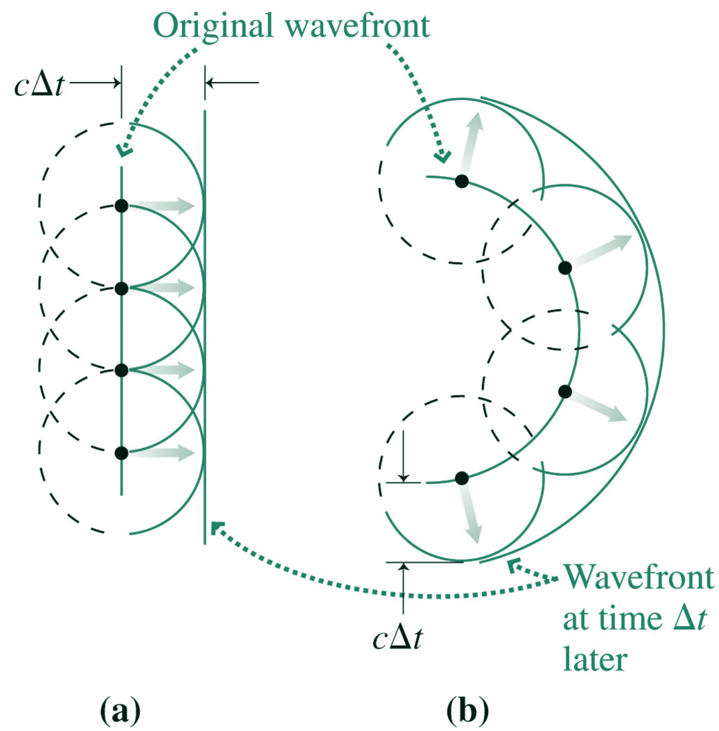
The Michelson Interferometer

- The **Michelson interferometer** uses interference to make very precise measurements of distance, wavelength, and other quantities.
  - The device was developed by A. A. Michelson in the 1880s for the famous Michelson-Morley experiment (more in the next chapter).
  - Michelson's design is still in widespread use for precision measurements in science and technology.
  - The interferometer splits a beam of light, sends it traveling on two perpendicular paths, and recombines the beams to produce an interference pattern.
  - Details of the pattern depend on the difference in travel times for light on the two paths.



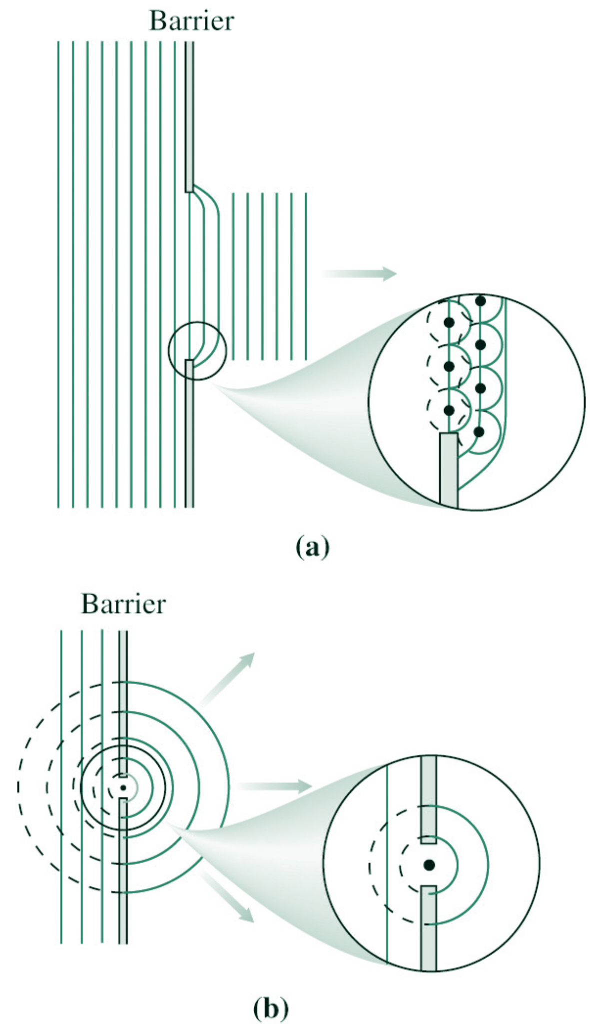
Huygens' Principle

- **Huygens' principle** states that all points on a wavefront act as point sources of spherically propagating "wavelets" that travel at the speed of light appropriate to the medium. At a short time Δ*t* later, the new wavefront is the unique surface tangent to all the forward-propagating wavelets.



Diffraction

- **Diffraction** is the bending of waves as they pass around objects or through apertures.
  - Huygens' wavelets produced near each barrier edge cause the wavefronts to diffract, or bend at the barrier.
  - Diffraction is most notable when the size of objects is comparable to or smaller than the wavelength, as suggested by comparing (a) and (b) in the diagram at right.

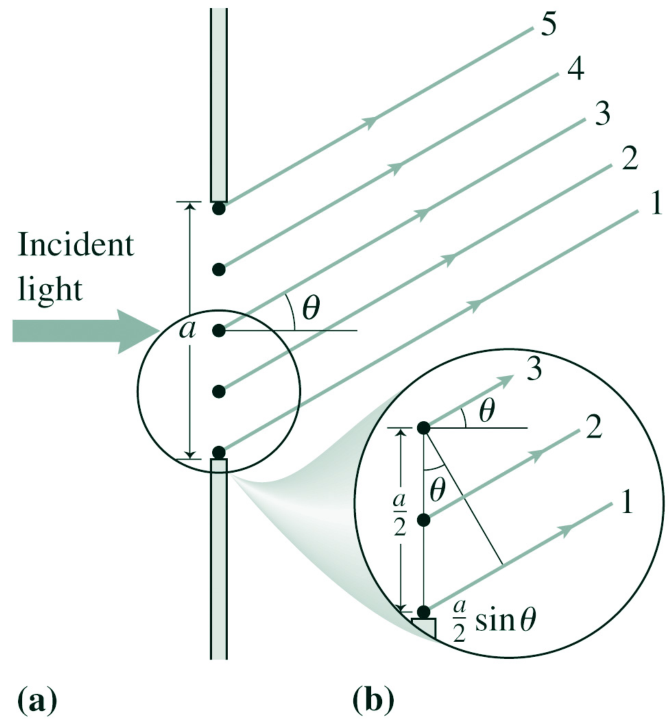


Single-Slit Diffraction

- Each point within a slit acts like a source of circular waves.
  - These waves interfere to produce a **diffraction pattern** from a single slit.
  - Intensity minima occur where

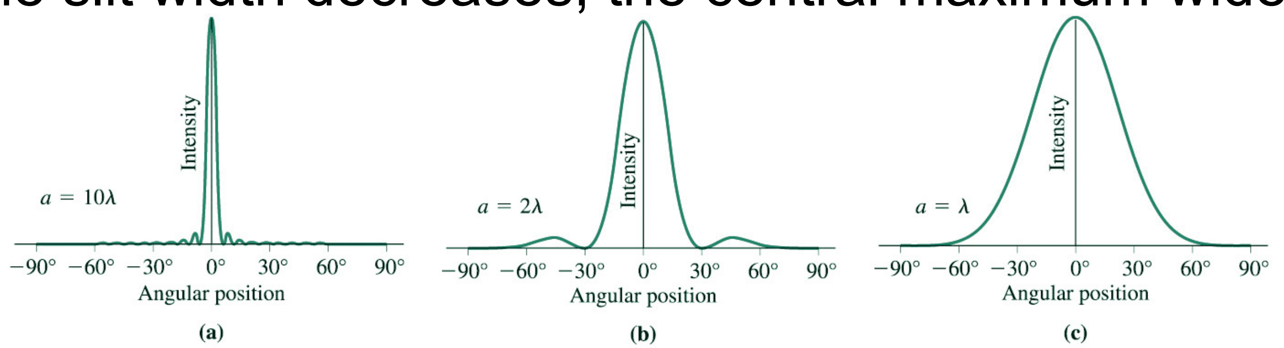
$$a\sin\theta = m\lambda$$

  - The intensity as a function of angle in single-slit diffraction is



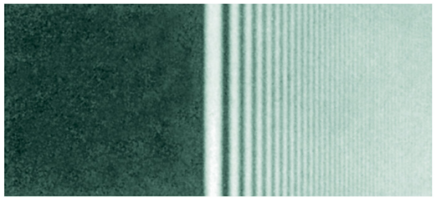
$$\bar{S} = \bar{S}_0 \left[\frac{\sin(\phi/2)}{\phi/2} \right]^2, \quad \phi = \frac{2\pi}{\lambda} a\sin\theta$$

– As the slit width decreases, the central maximum widens.

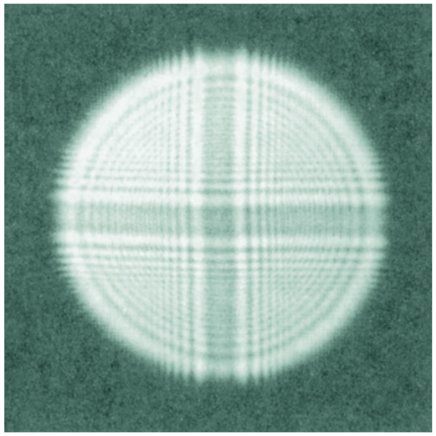


Examples of Diffraction Patterns

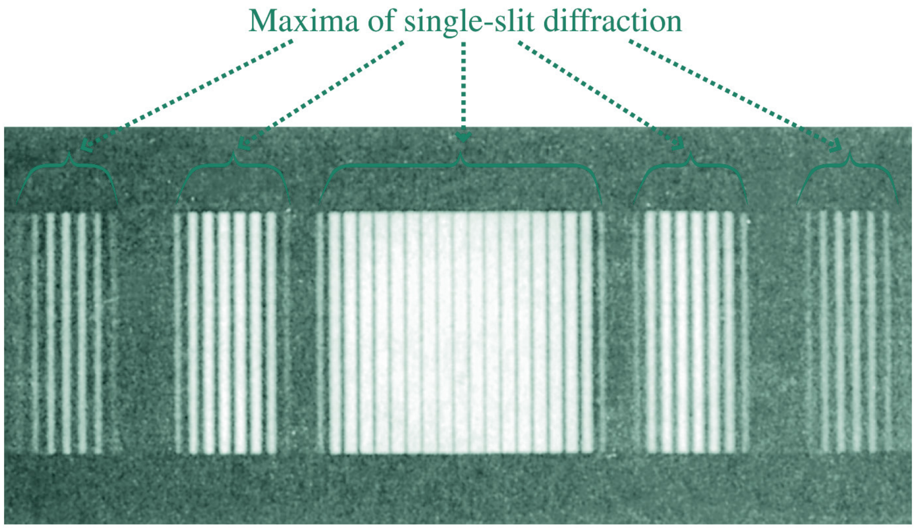
- Diffraction at a sharp edge:



- Diffraction through a circular aperture with crosshairs:



- Two-slit interference combined with diffraction through the individual slits:



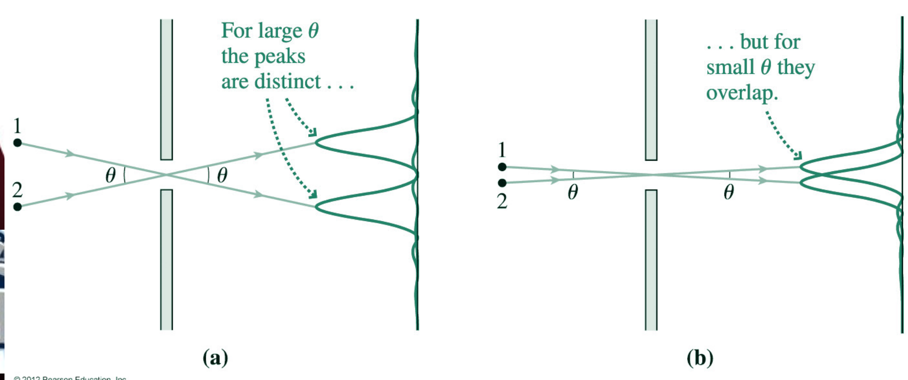
Quick Question

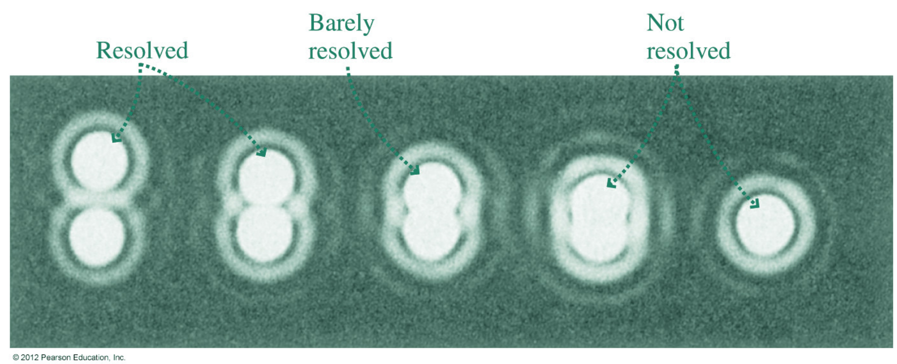
- Which of the following statements about single-slit diffraction is TRUE?

A. The distance between adjacent bright fringes is independent of wavelength.
B. The angular separation of fringes decreases as the order (m) increases.
C. The central maximum becomes narrower as slit width is decreased.
D. The intensity of the fringes decreases as the order (m) increases.

The Diffraction Limit – Rayleigh Criterion

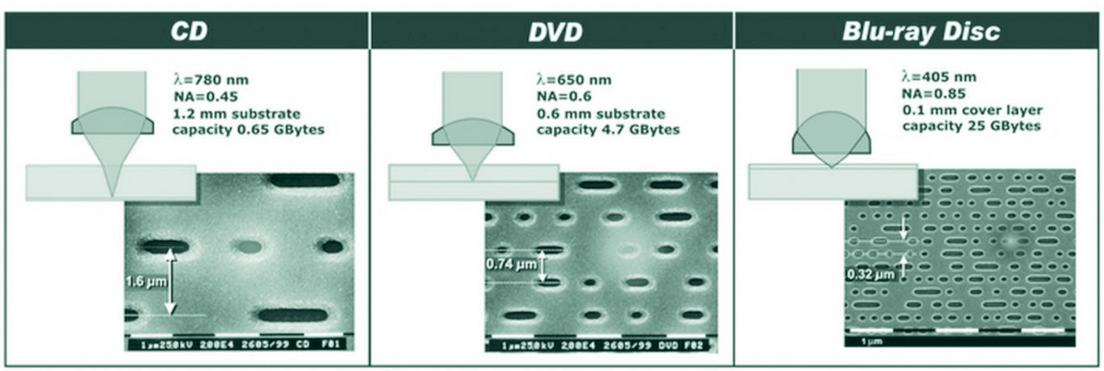
- Diffraction means the images of small objects are no smaller than the size of the central maximum of the interference pattern.
  - As the object size becomes comparable to the wavelength or smaller, this makes it impossible to form sharp images.
  - Two objects become indistinguishable when their central maxima overlap.
  - For a single slit, the diffraction limit is $\theta_{min} = \lambda/a$.
  - For a circular aperture of diameter *D*, we have $\theta_{min} = 1.22\lambda/D$.





Applications of the Diffraction Limit

- The diffraction limit makes it impossible for microscopes to resolve objects smaller than the wavelength of the light used.
  - Ultraviolet light or high-energy electrons have short wavelengths, allowing microscopy of smaller objects than is possible with visible light.
- The diffraction limit makes it impossible for telescopes to resolve closely spaced objects, or to see details of distant objects.
  - The larger the telescope aperture, the better the resolution.
  - Atmospheric turbulence, not diffraction, limits most ground-based telescopes.
- The diffraction limits the "pit" size and therefore determines the amount of information storage on optical discs.



- CDs use infrared lasers.
- DVDs use red lasers, have smaller "pits," and hold more information.
- HD-DVDs and Blu-ray discs use violet lasers and hold still more.