University Physics 226N/231N
Old Dominion University

Kinematics in One Dimension

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Happy Birthday to Richard Gere, Van Morrison, Frank Robinson, Hermann Helmholtz, and David Politzer (Nobel Prize, 2004)

Happy Eat Outside Day and National Trail Mix Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

What We Covered Last Time…

- We will usually have one slide of “previous concepts review” at the start of every new class
- For today, the most important concepts from Monday are:
  - Nearly all physical quantities have **units** (m, s, kg, …)
    - Units can be treated algebraically, e.g. canceled in ratios
  - Pay attention to **significant figures** when writing results
    - Carry all digits through calculations then round only final result
    - There is a nice Wikipedia article on significance arithmetic
  - Example: $\Delta x = \bar{v}\Delta t = (3.1\ \mathrm{m/s})(3\ \mathrm{s}) = 9\ \mathrm{m}$
  - **Draw pictures** and **tell stories** when solving problems
    - This keeps you organized and helps find a path to a solution
    - This also helps “future you” remember what you did
  - We’ll return to vector algebra when we get into 2D motion
    - We don’t need it much for 1D motion to be discussed today.

Kinematics

- **Kinematics**

  - the branch of mechanics concerned with the **motion of objects** without reference to the forces that cause the motion.

- Primary concepts of kinematics
  - **Time** Δt
    - Things don't move without time passing
  - **Position** $\Delta x(t)$
    - Where have we been? Where are we going?
      (Are we there yet? Are we there yet? Are we there yet?)
  - **Velocity** $v(t)$
    - How are changes in position and time related to each other?
  - **Acceleration** $a(t)$
    - How are changes in *velocity* and time related to each other?

Kinematics

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  - the branch of mechanics concerned with the **motion of objects** without reference to the forces that cause the motion.

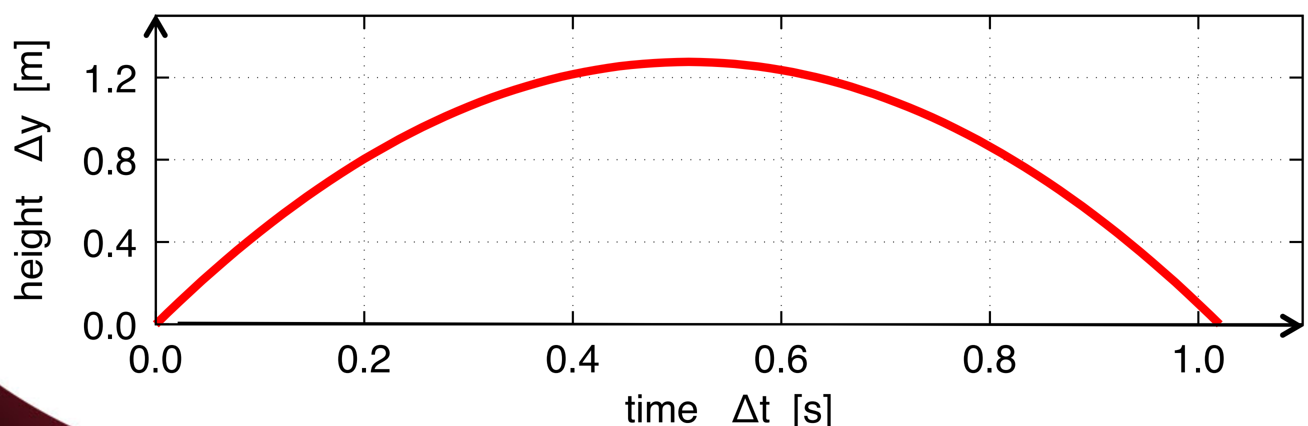
- Primary concepts of kinematics
  - **Time** Δt
    - Things don't move without time passing
  - **Position** $\Delta x(t) \quad \Delta y(t) \quad \Delta z(t)$
    - Where have we been? Where are we going?
      (Are we there yet? Are we there yet? Are we there yet?)
  - **Velocity** $v_x(t) \quad v_y(t) \quad v_z(t)$
    - How are changes in position and time related to each other?
  - **Acceleration** $a_x(t) \quad a_y(t) \quad a_z(t)$
    - How are changes in velocity and time related to each other?

Vector functions of time

- It turns out we don't need to go beyond these – these are enough

Motion in One Dimension: Position

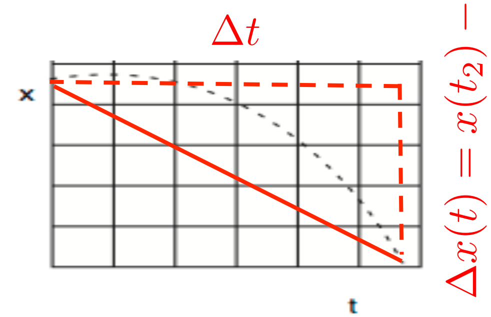
- **Position** x **:** linear distance from a "zero" reference point
  - To be general, we only discuss position differences or changes
  - For example $\Delta x = x_{\text{final}} - x_{\text{initial}} = x(t_2) - x(t_1) = x_2 - x_1$
  - I generally try to use Δx instead of x wherever I can
    - Δx is a positive or negative **distance between** two points
  - In these discussions we will be talking about falling objects a lot, so you will see both Δx and Δy
    - Try to get in the habit of drawing and labeling axes
    - This helps make positive and negative signs obvious



Motion in One Dimension: Velocity

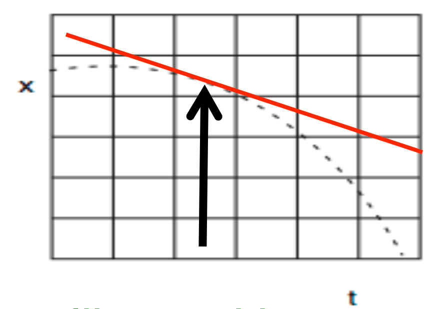
- **Velocity:** How far an object moves Δx in a given time interval Δt : $v = \Delta x / \Delta t$
  - Velocity, like position and displacement, is a **vector** with **magnitude** and **direction**
    - We use **speed** for just the magnitude in common language
  - Velocity, like position, is a **function** of time. Over a finite time period, we call this **average velocity**:

$$\bar{v}(t) = \Delta x(t) / \Delta t$$



  - The logical extension from calc is a derivative
  - **Instantaneous velocity** is the **slope** of position over a very small time, as

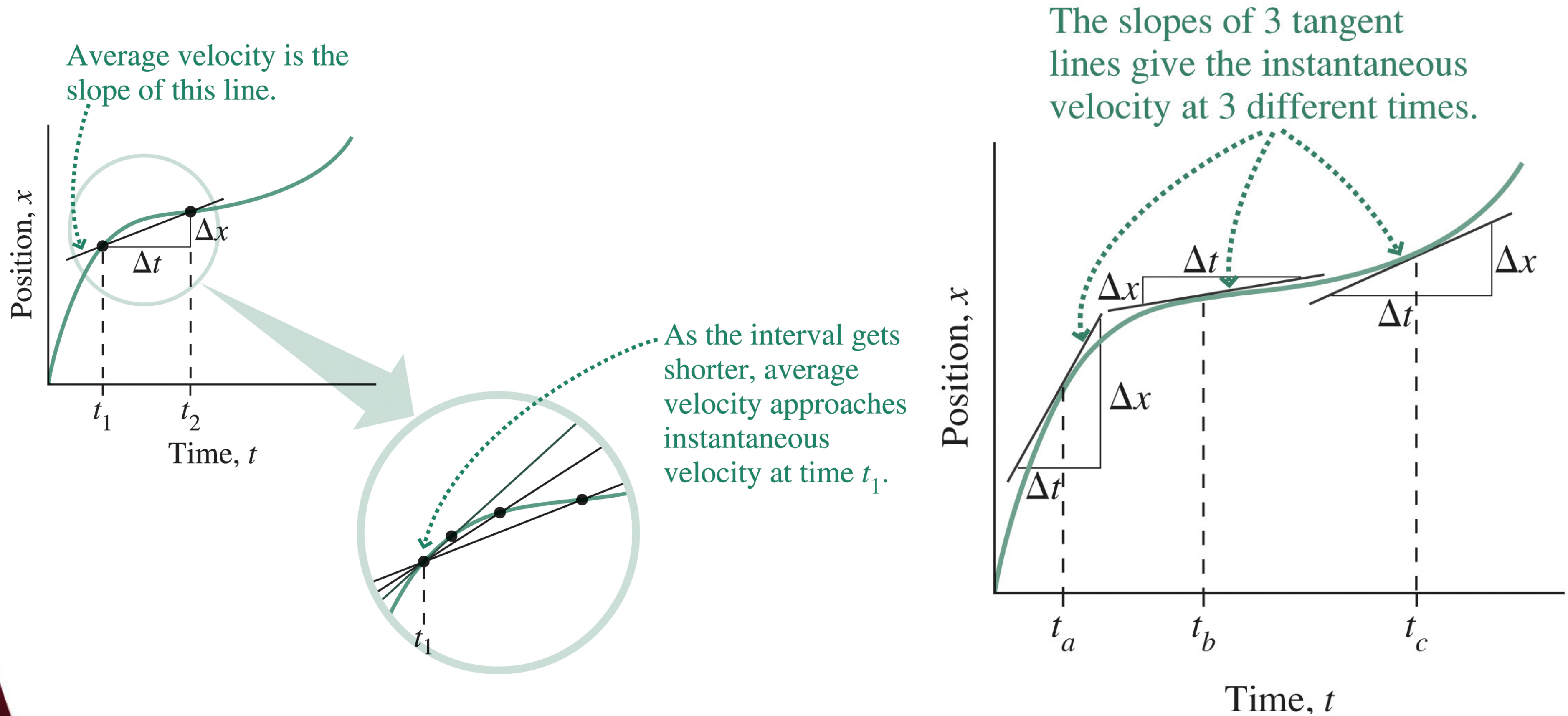
$$v(t) = \Delta x(t) / \Delta t \quad \text{(for very small } \Delta \text{t)}$$



We can figure this out for any time t, and it's continuous like position

Velocity is a Slope of Position vs Time

– Velocity is the slope of the curve of x(t): how fast position is changing with time. Note that it can be positive or negative!



- That means that velocity is **also** a function and we can plot velocity v(t) like we plot position x(t)
  - And then we can figure out how the velocity is changing (take slopes) too!

Acceleration is a Slope of Velocity vs Time

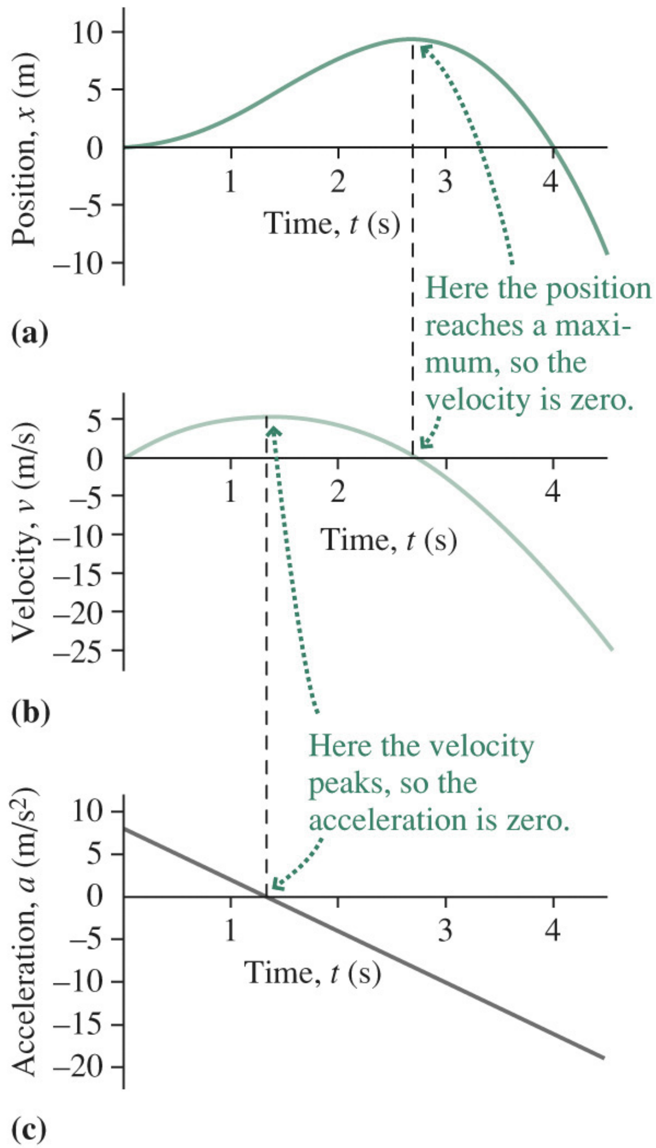
- **Acceleration** is the rate of change of velocity.
  - Exactly like velocity was the rate of change of position!
  - **Average velocity** over a time interval Δt is defined as the change in velocity divided by the time:

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

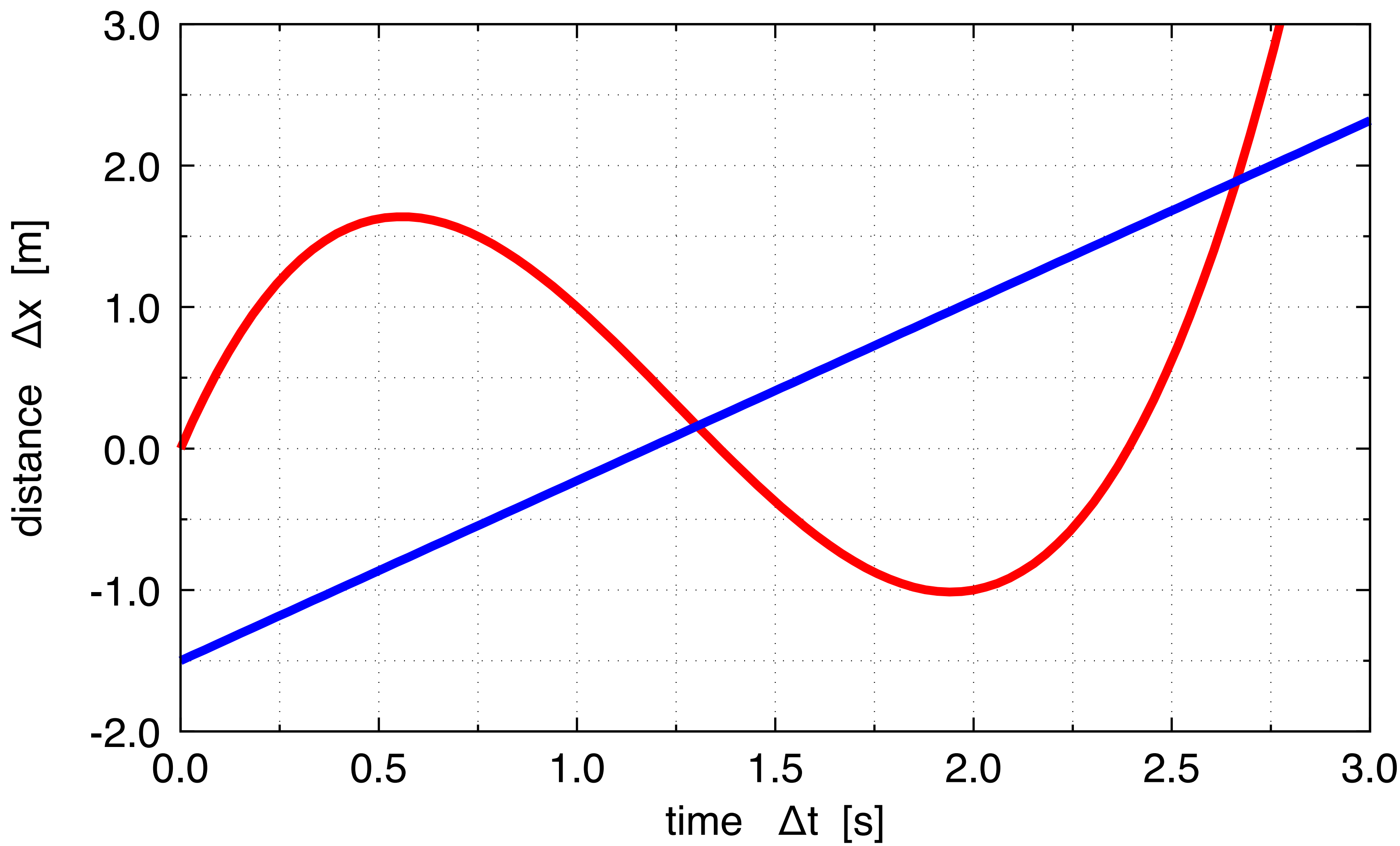
  - **Instantaneous acceleration** is the limit of the average acceleration as the time interval becomes arbitrarily short:

$$a = \frac{\Delta v}{\Delta t} \quad (\text{for very small } \Delta t)$$

– Acceleration is the slope of v(t)



Ponderable Example



What are the velocity and acceleration of the red trajectory at t=0 s? Blue trajectory at t=1.5 s?

Position, Velocity, and Acceleration

- Individual or absolute values of position, velocity, and acceleration are not related.
  - Instead, velocity depends on the *rate of change* of position.
  - Acceleration depends on the *rate of change* of velocity.
    - These are all **relative** quantities, and **not** based on absolute position or position of the origin
    - This makes our description of this motion **universal**
  - An object can be at position x = 0 and still be *moving.*
  - An object can have zero velocity and still be *accelerating*.
- At the peak of its trajectory, a juggling thud has
  - Maximum vertical displacement from my hand
  - Zero vertical velocity
  - **Constant** (negative?) acceleration due to the force of gravity

Juggler Physics

v_y

$\Delta y > 0 \quad v_y \approx 0 \quad a_y < 0$

$\Delta y > 0 \quad v_y > 0 \quad a_y < 0$

acceleration of gravity -9.8 m/s^2

$\hat{y}$

$\Delta y = 0 \quad v_y \approx 0 \quad a_y \approx 0$

Also Juggler Physics!

v_y

$\Delta y < 0 \quad v_y \approx 0 \quad a_y > 0$

$\Delta y < 0 \quad v_y < 0 \quad a_y > 0$

acceleration of gravity
+9.8 m/s^2

$\hat{y}$

$\Delta y = 0 \quad v_y \approx 0 \quad a_y \approx 0$

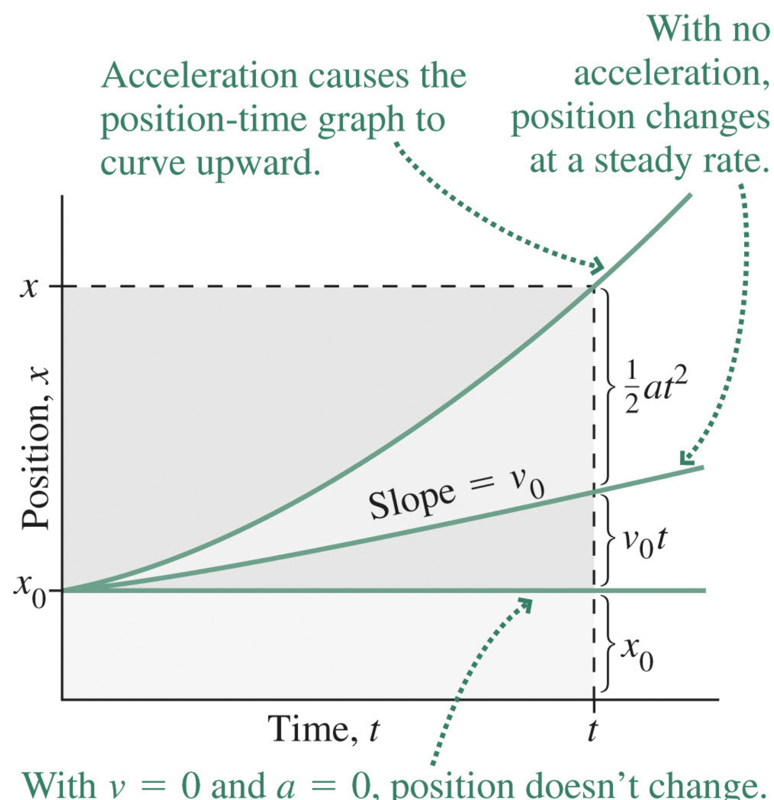
Constant Acceleration

- When **acceleration is constant**: position x, velocity v, acceleration a, and time t are related by

$$\Delta v(t) = at$$
$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$
$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$
$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

where x_0 and v_0 are **initial values** at time t = 0 and $x(t)$ and $v(t)$ are the values at an arbitrary time t.

- With **constant acceleration**
  - Velocity is a linear function of time
  - Position is a quadratic function of time



The Acceleration of Gravity

- The acceleration of gravity at any point is (basically) the same for all objects, regardless of mass.
- Near Earth's surface, the **magnitude** of the acceleration is essentially constant at $g = 9.8\ \mathrm{m/s}^2 = 32\ \mathrm{ft/s}^2$
- Therefore the equations for constant acceleration apply:
  - In a coordinate system **with *y* axis upward** so $\vec{a} = -g\hat{y}$, they read

$$\Delta v(t) = -gt$$

$$\Delta y(t) = \frac{1}{2}[v_0 + v(t)]t$$

$$\Delta y(t) = v_0 t - \frac{1}{2}gt^2$$

$$v^2(t) - v_0^2 = -2g\Delta y(t)$$

This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

(Honors Ponderable)

- Assume that the 13 ball in the photo is a "standard" 2.25 inch diameter billiard ball
- How fast is the strobe flashing between images of the falling ball?
- What is the ball's approximate instantaneous velocity in the first image? In the last?

This strobe photo of a falling ball shows increasing spacing resulting from the acceleration of gravity.

Example: The Acceleration of Gravity

- A ball is thrown straight up at 7.3 m/s, leaving your hand 1.5 m above the ground. Find its maximum height and when it hits the floor.
  - At the maximum height the ball is instantaneously at rest (even though it's still *accelerating*). Solving the last equation with $v = 0$ gives the maximum height:

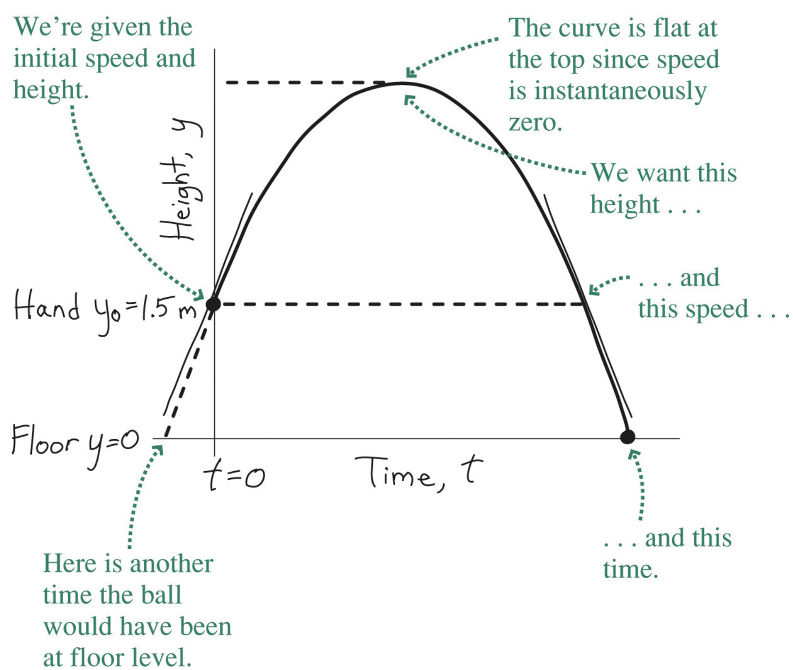
$$0 = v_0^2 - 2g(y - y_0)$$

2 Significant Figures!

or

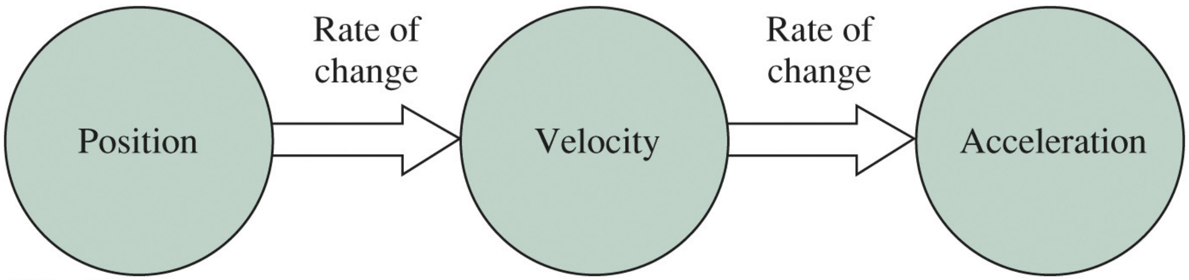
$$y = y_0 + \frac{v_0^2}{2g} = 1.5\text{ m} + \frac{(7.3\text{ m/s})^2}{(2)(9.8\text{ m/s}^2)} = 4.2\text{ m}$$

- Setting $y = 0$ in the third equation gives a quadratic in time; the result is the two values for the time when the ball is on the floor: $t = -0.18$ s and $t = 1.7$ s
- The first answer tells when the ball *would have been* on the floor if it had always been on this trajectory; the second is the answer we want.



Summary

- Position, velocity, and acceleration are the fundamental quantities describing motion.
  - Velocity is the rate of change of position.
  - Acceleration is the rate of change of velocity.



- When acceleration is constant, simple equations relate position, velocity, acceleration, and time.
  - An important case is the acceleration due to gravity near Earth's surface.
  - The **magnitude** of Earth's gravitational acceleration is g = 9.8 m/s^2 = 32 ft/s^2.

$$\Delta v(t) = at$$

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$

$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$

$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

Break Time


DO YOU KNOW HOW DEEP YOUR WELL IS?
NOT REALLY.
..13,..14,..15,..16,...
SPLASH
YOUR WELL IS 16 SECONDS DEEP.

Constant Acceleration and Graph Areas

- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a\Delta t$$
$$(area!!)$$

- Velocity v

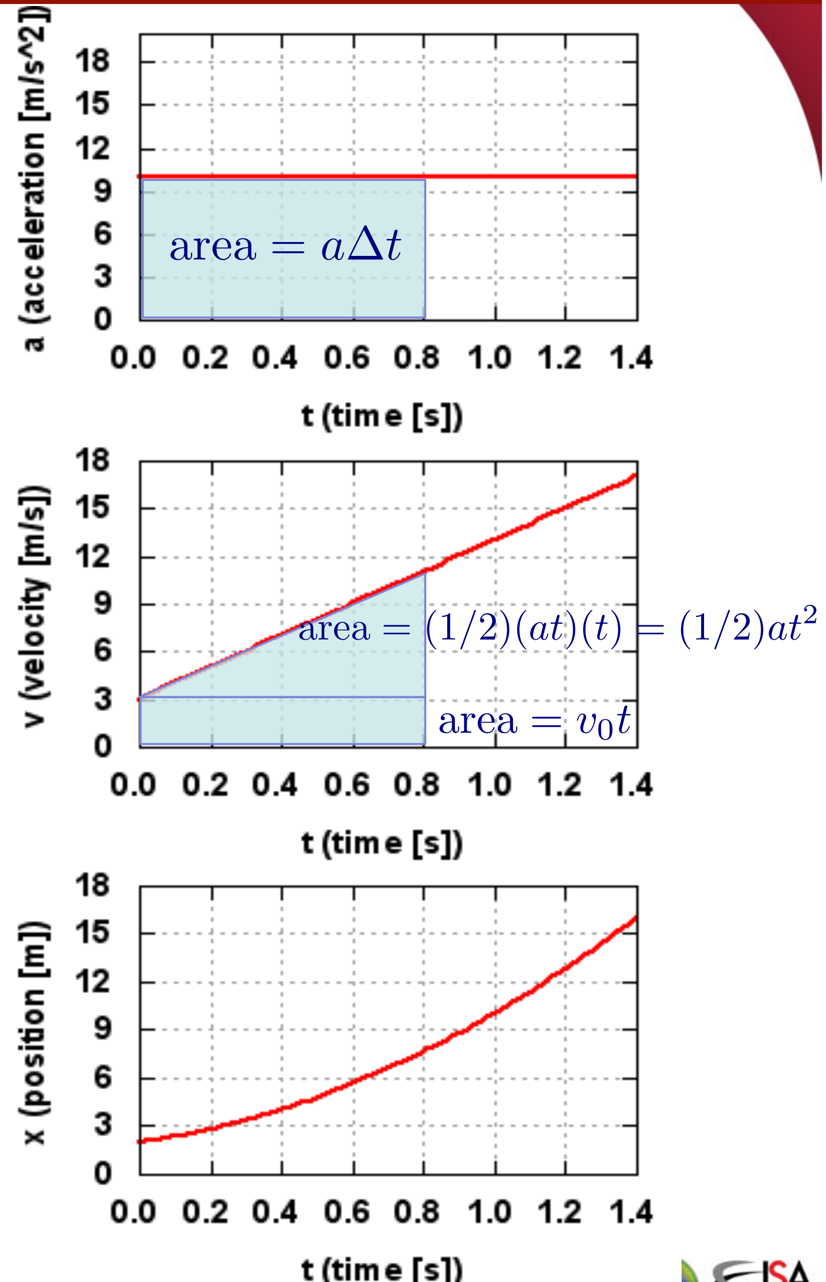
$$v = v_0 + at$$
$$v = \frac{\Delta x}{\Delta t}$$

$$t = \frac{v - v_0}{a}$$

$$\Delta x = \text{area too!}$$

- Position x

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$



Using the Equations: Example 1 (5 minutes)

- At the moment a traffic light turns green, a car that has been waiting at the intersection accelerates with constant acceleration of a_c=3.2 m/s^2. At that same moment, a truck whizzes by at a constant velocity of v_t=20.0 m/s.
  - How far beyond the traffic light does the car catch the truck?
  - How fast is the car moving when it catches the truck?
- Say the traffic light is x=0, so initial positions of both car and truck are $x_{c0}=x_{t0}=0$ m.

car

$$v_{c0} = 0 \text{ m/s}$$

$$a_c = 3.2 \text{ m/s}^2$$

$$x_c = v_{c0}t + \frac{1}{2}a_c t^2 = \frac{1}{2}a_c t^2$$

truck

$$v_{t0} = 20.0 \text{ m/s}$$

$$a_t = 0.0 \text{ m/s}^2$$

$$x_t = v_{t0}t + \frac{1}{2}a_t t^2 = v_{t0}t$$

$$x_c = x_t \quad \text{when} \quad \frac{1}{2}a_c t^2 = v_{t0}t$$

Using the Equations: Example 1 (cont)

- At the moment a traffic light turns green, a car that has been waiting at the intersection accelerates with constant acceleration of a_c=3.2 m/s^2. At that same moment, a truck whizzes by at a constant velocity of v_t=20.0 m/s.
  - How far beyond the traffic light does the car catch the truck?
  - How fast is the car moving when it catches the truck?

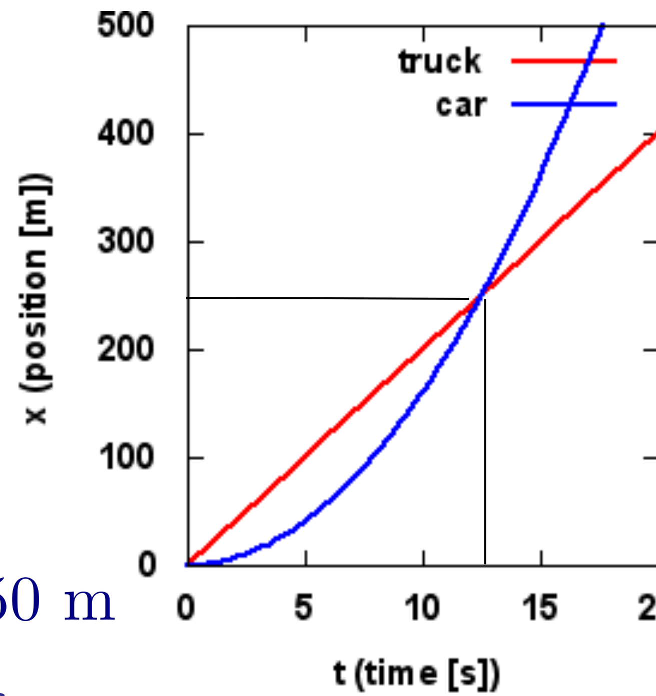
$$x_c = x_t \quad \text{when} \quad \frac{1}{2} a_c t^2 = v_{t0} t$$

$$t = \frac{2v_{t0}}{a_c} = \frac{2(20.0 \text{ m/s})}{3.2 \text{ m/s}^2} = \boxed{12.5 \text{ s}}$$

$$v_c = v_{c0} + a_c t = a_c t$$

$$v_c = (3.2 \text{ m/s}^2)(12.5 \text{ s}) = \boxed{40 \text{ m/s}}$$

$$x = v_{t0} t = (20.0 \text{ m/s})(12.5 \text{ s}) = 250 \text{ m}$$



Using the Equations: Example 2

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$x = x_0 + v_0 t + \frac{1}{2} a t^2 \qquad x_0 = 0 \text{ m} \qquad v_0 = 6.0 \text{ m/s}$$

  - We want to know x… but we don't know t to "plug and chug"!
    - (This is typical of a frustration with the homework)

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
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  - We want to know x… but we don't know t to "plug and chug"!
    - (This is typical of a frustration with the homework)
- You have another equation, and you know v=0 m/s at top.

$$v = v_0 + at \qquad \Rightarrow \qquad t = \frac{v - v_0}{a} = \frac{-v_0}{a} \text{ (here)}$$

  - If you have trouble going from the equation on the left to the equation on the right, please see me after class or email me
    - Calculus may not be a prerequisite for this class, but algebra is!

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
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    - (This is typical of a frustration with the homework)
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$$v = v_0 + at \qquad \Rightarrow \qquad t = \frac{v - v_0}{a} = \frac{-v_0}{a} \text{ (here)}$$

  - Now substitute into the first equation…

$$x - x_0 = v_0 \left(\frac{-v_0}{a} \right) + \frac{1}{2} a \left(\frac{-v_0}{a} \right)^2 = \boxed{-\frac{1}{2} \frac{v_0^2}{a} = x - x_0}$$

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$x - x_0 = -\frac{1}{2}\frac{v_0^2}{a}$$

$$x - x_0 = -\frac{1}{2}\frac{v_0^2}{a} = -\frac{1}{2}\left[\frac{(6.0\ \mathrm{m/s})^2}{(-9.8\ \mathrm{m/s^2})}\right] = \frac{18.0\ \mathrm{m^2/s^2}}{9.8\ \mathrm{m/s^2}} = \boxed{1.8\ \mathrm{m}}$$

- On the previous page we'd also figured out t in terms of v_0

$$t = \frac{-v_0}{a} = \frac{-6.0\ \mathrm{m/s}}{(-9.8\ \mathrm{m/s^2})} = \boxed{0.61\ \mathrm{s}}$$

Using the Equations: Example 2 (cont)

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
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- On the previous page we'd also figured out t in terms of v_0

$$t = \frac{-v_0}{a} = \frac{-6.0\ \mathrm{m/s}}{(-9.8\ \mathrm{m/s^2})} = \boxed{0.61\ \mathrm{s}}$$

- Be careful! The time back to my hand is TWICE this! (up/down)

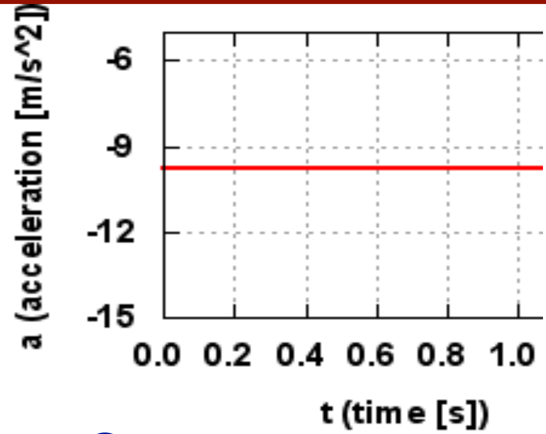
$$t = \text{time back to hand} = \boxed{1.21\ \mathrm{s}}$$

Huh, that looks a bit familiar…

- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t}$$

$$\Delta v = a\Delta t$$
$$(area!!)$$



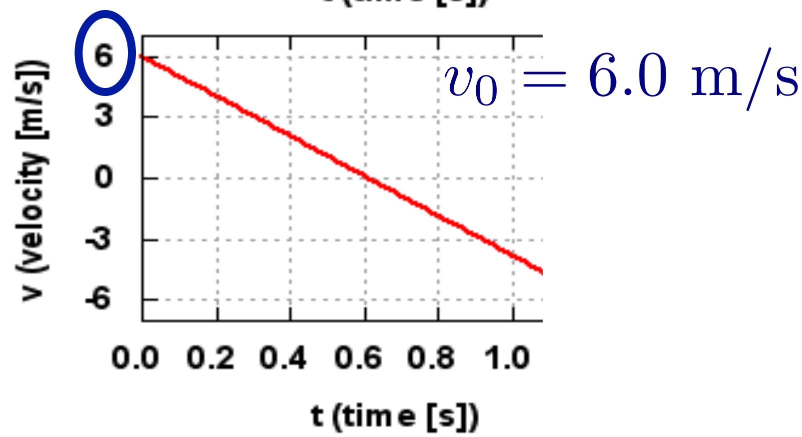
- Velocity v

$$v = v_0 + at$$

$$v = \frac{\Delta x}{\Delta t}$$

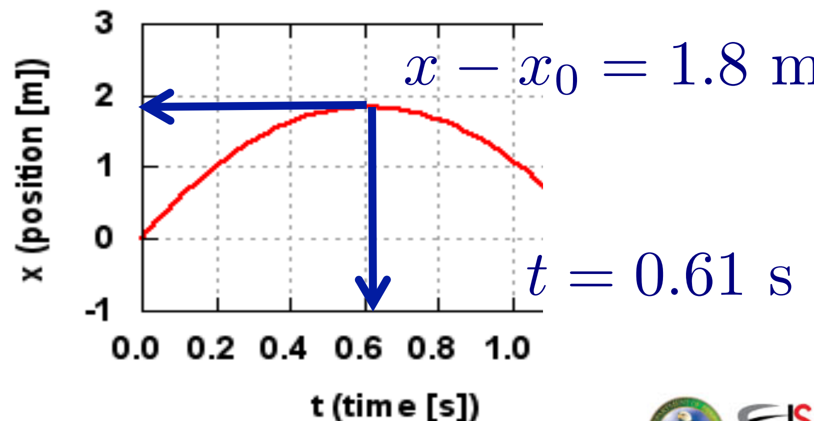
$$t = \frac{v - v_0}{a}$$

$$\Delta x = \text{area too!}$$



- Position x

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$



Using the Equations: Example 3

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$v = v_0 + at \qquad \Rightarrow \qquad v_0 = v - at = -at \text{ (here)}$$

- We know v=0 at the ceiling, and a=g, and want to know v_0… but we don't know t to "plug and chug"!
  - (This is typical of a frustration with the homework again)

Using the Equations: Example 3 (cont)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$v = v_0 + at \qquad \Rightarrow \qquad v_0 = v - at = -at \text{ (here)}$$

- In the previous problem we figured out an equation that eliminated time (**only** when v_f=0 m/s):

$$x - x_0 = v_0\left(\frac{-v_0}{a}\right) + \frac{1}{2}a\left(\frac{-v_0}{a}\right)^2 = -\frac{1}{2}\frac{v_0^2}{a} = x - x_0$$

$$\Rightarrow \quad v_0 = \sqrt{-2a(x - x_0)}$$

$$v_0 = \sqrt{-2(-9.8\ \text{m/s}^2)(2.0\ \text{m})} = \boxed{6.3\ \text{m/s} = v_0}$$

Using the Equations: Example 3 (cont)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

- For the second part, $x - x_0 = -1.5 \text{ m}$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$(-1.5 \text{ m}) = (6.3 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$$

- Uh oh… This is a quadratic equation. You can't solve this using simple algebra – you have to remember the equation for the solutions of a quadratic.

Quadratic Equation Refresher

- For any quadratic equation of the form

This is general math:
a is NOT acceleration
x is NOT position

$$ax^2 + bx + c = 0$$

where a,b,c are known values, the TWO values of x that "solve" this equation can be found with the equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note that you should usually find that $b^2 - 4ac > 0$ for physics problems otherwise the square root produces imaginary numbers (which don't have good physical meanings!)

Using the Equations: Example 3 (cont)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_0 do I have to throw the beanbag?
  - If I toss the beanbag at t=0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, x_0=0.0 m).

$$(-1.5\text{ m}) = (6.3\text{ m/s})t - (4.9\text{ m/s}^2)t^2$$

$$(-4.9\text{ m/s}^2)t^2 + (6.3\text{ m/s})t + 1.5\text{ m} = 0$$

$$a = (-4.9\text{ m/s}^2) \quad b = (6.3\text{ m/s}) \quad c = 1.5\text{ m}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6.3\text{ m/s} \pm 8.3\text{ m/s}}{-9.8\text{ m/s}^2} = 1.5\text{ s}, -0.20\text{ s}$$

Two solutions!

Ponderable (10 minutes)

- I drop a bean bag from my hand to the floor 1.5 m below my hand.
  - How long does it take to reach the floor?
  - What is the beanbag's velocity at floor height?
    - (Assume floor height is x_0=0.0 m).

$$v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2}at^2$$

Ponderable (10 minutes)

- I drop a bean bag from my hand to the floor 1.5 m below my hand.
  - How long does it take to reach the floor?
  - What is the beanbag's velocity at floor height?
    - (Assume floor height is x_0=0.0 m).

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad -1.5\ \text{m} = \frac{1}{2}at^2$$

$x-x_0$=-1.5m v_0=0 m/s

$$t = \sqrt{\frac{2(-1.5\ \text{m})}{(-9.8\ \text{m/s}^2)}} = \boxed{\pm 0.55\ \text{s}}$$

$$v = v_0 + at \quad v = (0\ \text{m/s}) + (-9.8\ \text{m/s}^2)(\pm 0.55\ \text{s}) = \boxed{\pm 5.4\ \text{m/s}}$$

- Note that the equations don't care about past or future!

Example 3 With a Twist (~5 minutes)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Here, catch! (*thud*)
  - How is this different from just throwing the beanbag straight up to graze the ceiling?
    - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?
    - Which one stays in the air a longer time?
    - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?

Example 3 With a Twist (Explanation)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?

Higher!

  - Which one stays in the air a longer time?
  - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?

Less, but how much less?

- The time thing is **tricky** and **nonintuitive.**
- The vertical portion of the motion is the same for both cases
  - Therefore the time the bag is in the air is the **SAME** for both too!
- Two objects that have the same vertical motion take the same time for that motion **regardless** of their horizontal motion!
  - Example: a cannonball fired horizontally out of a cannon and another cannonball dropped from that cannon take the same time to hit the ground.

Onwards to Two Dimensional Motion

- We'll get further into examples of these equations next class
  - Horizontal motion is easy: acceleration is zero!
  - You've already done the hard part, vertical motion with gravity

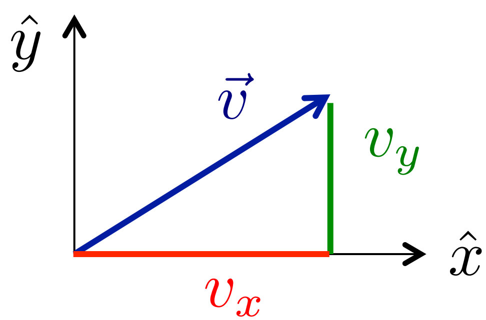
$$v_x = v_{x0}$$
$$x = x_0 + v_{x0} t$$

Horizontal motion (no acceleration)

$$v_y = v_{y0} + at$$
$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$

Vertical motion (gravitational acceleration)

Components of velocity, and vectors:



$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$
$$v = \sqrt{v_x^2 + v_y^2}$$