University Physics 226N/231N
Old Dominion University

Kinematics in One Dimension Examples
(and starting Kinematics in Two Dimensions)

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Happy Birthday to Kevin Love, Evan Rachel Wood, Buddy Holly, Elizabeth I, David Packard (of HP), and James Van Allen (No Nobel though)

Happy National Beer Lover's Day and National Acorn Squash Day!

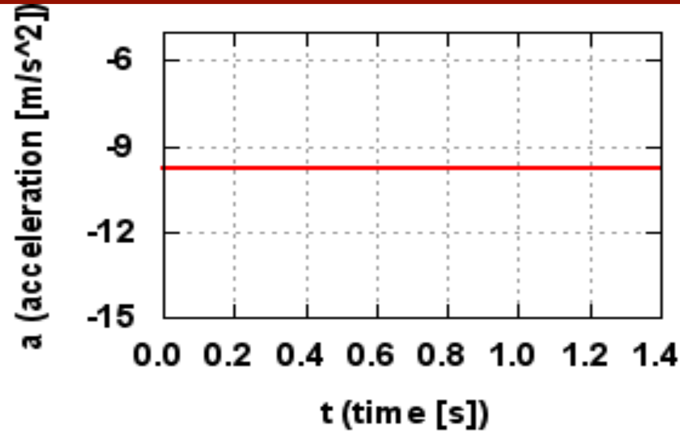
Please set your cell phones to "vibrate" or "silent" mode. Thanks!

Review: Constant Acceleration

- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t}$$

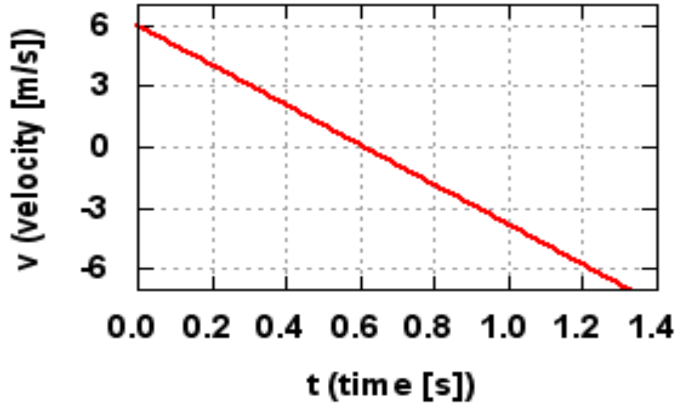
$$\Delta v = a\Delta t$$
$$(area!!)$$



- Velocity v(t)

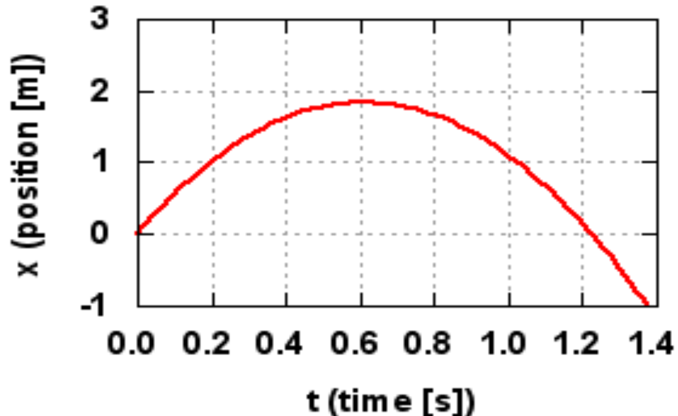
$$v(t) = \frac{\Delta x(t)}{\Delta t}$$

$$\Delta x = \text{area too!}$$



- Position x(t)

$$\Delta x(t) = v_0\ \Delta t + \frac{1}{2} a \Delta t^2$$



- **Magnitude** of grav. acceleration

$$|g| = 9.8\ \mathrm{m/s^2} = 32\ \mathrm{ft/s^2}$$

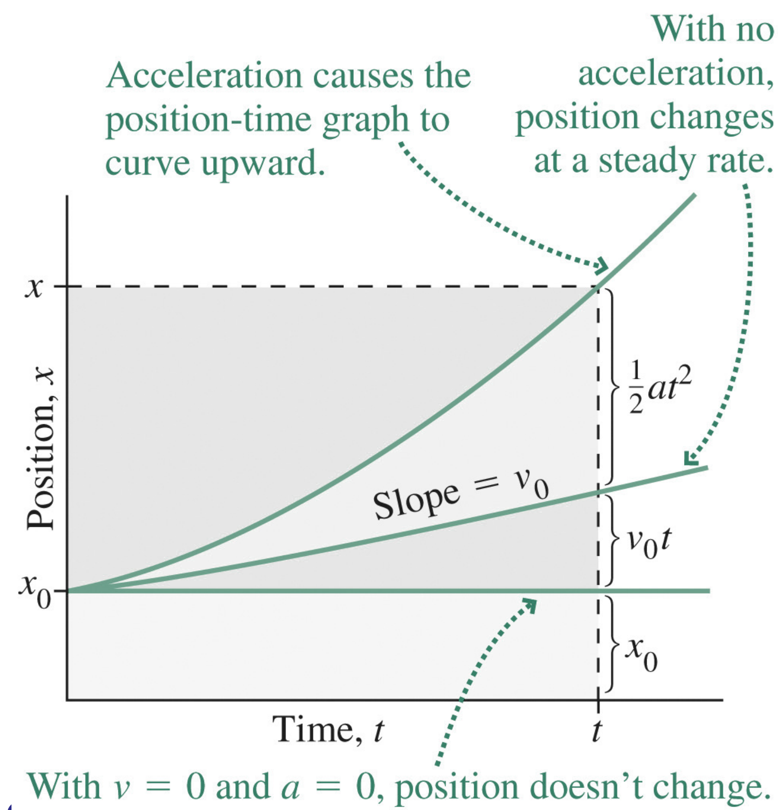
Constant Acceleration

- When **acceleration is constant**: position x, velocity v, acceleration a, and time t are related by

$$\Delta v(t) = at$$
$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$
$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$
$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

where x_0 and v_0 are **initial values** at time $t = 0$ and $x(t)$ and $v(t)$ are the values at an arbitrary time t.

- With **constant acceleration**
  - Velocity is a linear function of time
  - Position is a quadratic function of time



Review: Juggler Physics

$\Delta y > 0 \quad v_y > 0 \quad a_y < 0$

Position, velocity, and acceleration are **vectors**

Their signs depend on their orientations relative to the choice of coordinate axis direction

$\hat{y}$

v_y

$\Delta y > 0 \quad v_y \approx 0 \quad a_y < 0$

acceleration of gravity -9.8 m/s^2

$\Delta y = 0 \quad v_y \approx 0 \quad a_y \approx 0$

Review: Also Juggler Physics!

$\Delta y < 0 \quad v_y \approx 0 \quad a_y > 0$

$\Delta y < 0 \quad v_y < 0 \quad a_y > 0$

Position, velocity, and acceleration are **vectors**

Their signs depend on their orientations relative to the choice of coordinate axis direction

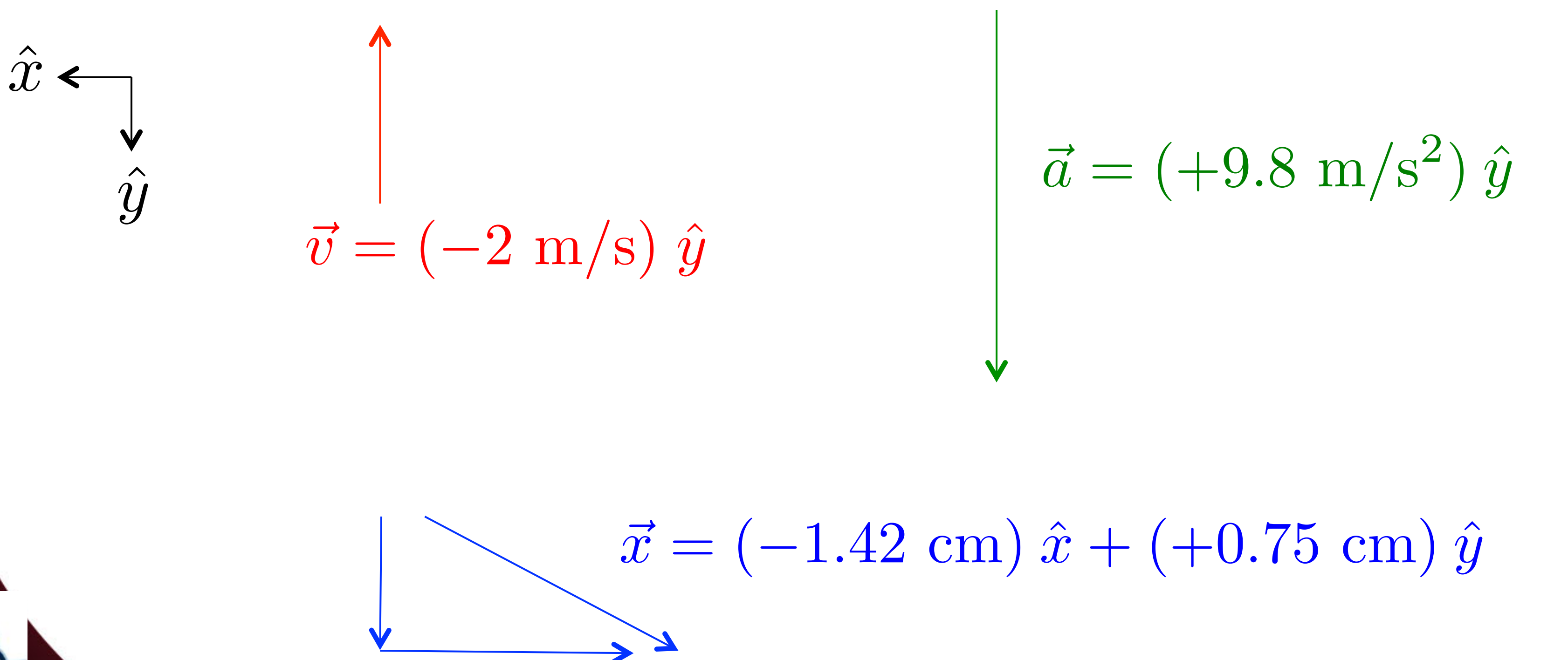
$\hat{y}$

acceleration of gravity
+9.8 m/s^2

$\Delta y = 0 \quad v_y \approx 0 \quad a_y \approx 0$

Simple Vector Signs and Directions

- Vector components pointing in the **same direction as the coordinate axes** are **positive**
- Vector components pointing in the **opposite direction of the coordinate axes** are **negative**



Review: One Dimensional Motion Problem

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, y_0=0.0 m).

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$v_y(\text{top}) = 0\ \text{m/s}$

$v_{0y} = -6.0\ \text{m/s}$

$y_0 = 0\ \text{m}$

$v_y(\text{end}) = +6.0\ \text{m/s}$

$\hat{y}$

$a_y = g = +9.8\ \text{m/s}^2$

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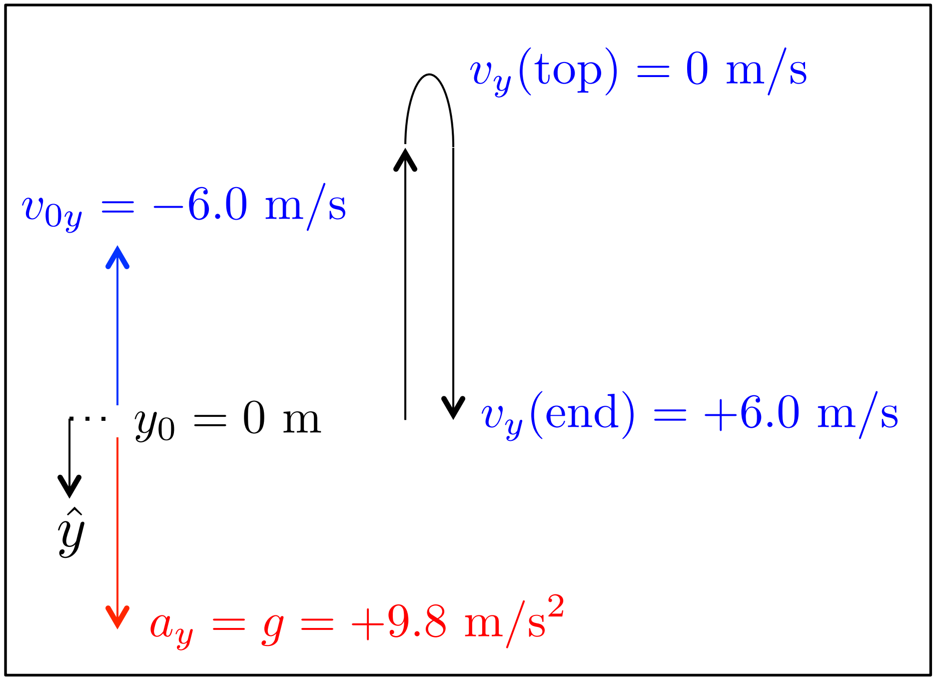

$$\Delta v(t) = at$$
$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$
$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$
$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

First part of question does not depend on time, so try the last equation:

$$v_y^2(\text{top}) - v_{0y}^2 = 2a_y\Delta y$$

$$\Delta y = \frac{(v_y^2(\text{top}) - v_{0y}^2)}{2a_y} = \frac{((0 \text{ m/s})^2 - (-6.0 \text{ m/s})^2)}{2(+9.8 \text{ m/s}^2)} = \boxed{-1.8 \text{ m} = \Delta y}$$

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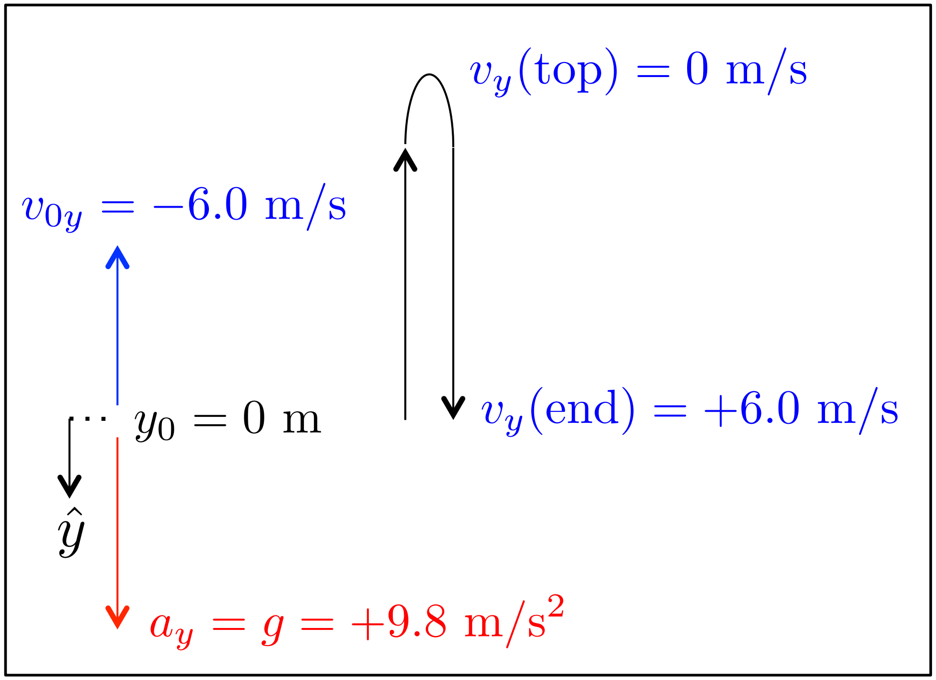

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Second part of question asks for time: use any of first three equations.

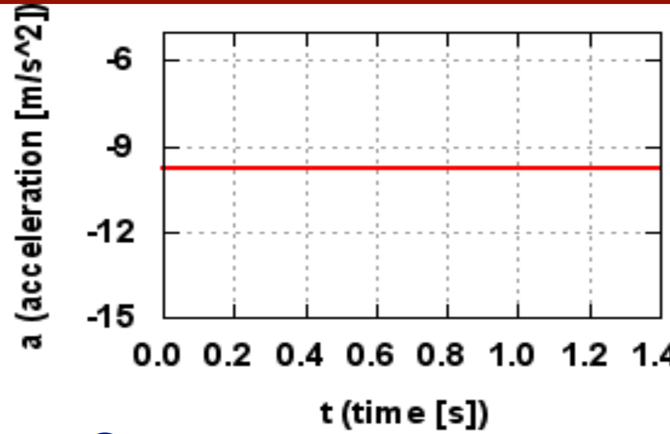
$$v_y(\text{end}) - v_{0y} = a_y t$$

$$t = \frac{v_y(\text{end}) - v_{0y}}{a_y} = \frac{(+6.0\text{ m/s}) - (-6.0\text{ m/s})}{+9.8\text{ m/s}^2} = \boxed{1.2\text{ s} = t}$$

Huh, that looks a bit familiar….

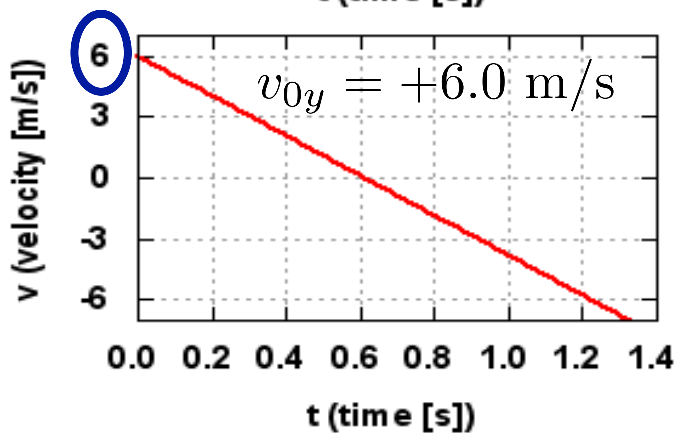
- Acceleration a (constant!)

$$a = \frac{\Delta v}{\Delta t} \qquad \Delta v = a \Delta t \quad (area!!)$$



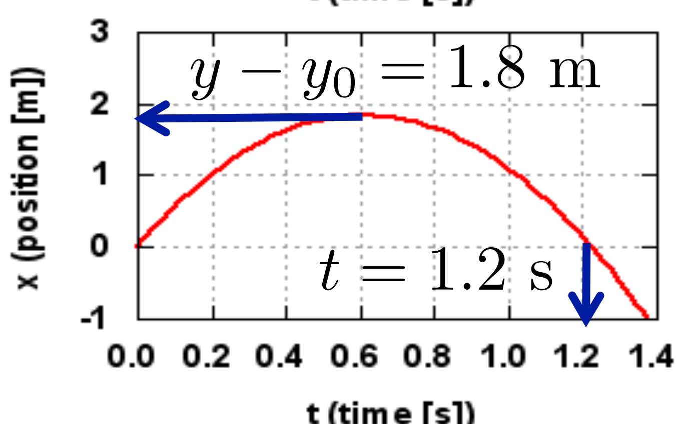
- Velocity v(t)

$$v(t) = \frac{\Delta x(t)}{\Delta t} \qquad \Delta x = \text{area too!}$$



- Position x(t)

$$\Delta x(t) = v_0 \, \Delta t + \frac{1}{2} a \Delta t^2$$



- **Magnitude** of grav. acceleration

$$|g| = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$$

Another One Dimensional Motion Problem

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_{0y} do I have to throw the beanbag?
  - If I toss the beanbag at t=0.0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, y_0=0.0 m).

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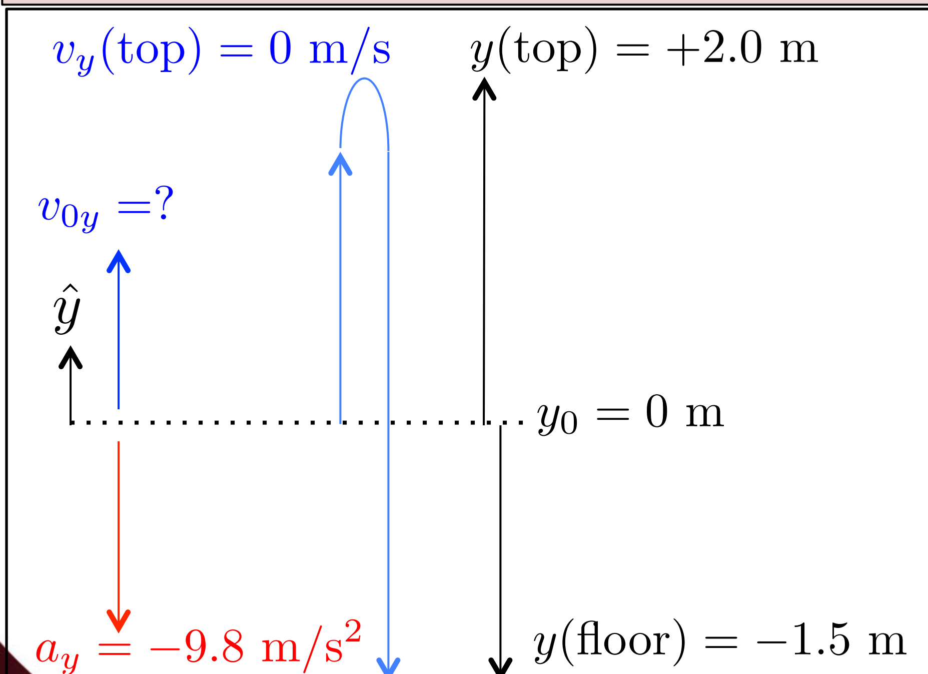

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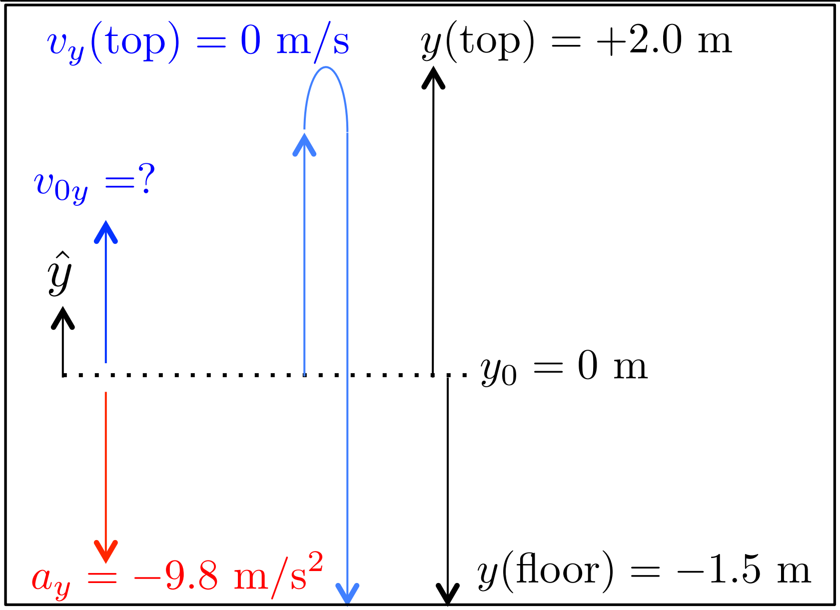

$$\Delta v(t) = at$$
$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$
$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$
$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

First part of question does not depend on time again…

$$v_y^2(\text{top}) - v_{0y}^2 = 2a_y\Delta y$$

$$v_{0y} = \sqrt{-2a_y\Delta y} = \sqrt{-2(-9.8\ \text{m/s}^2)(+2.0\ \text{m})} = \boxed{6.3\ \text{m/s} = v_{0y}}$$

Another One Dimensional Motion Problem

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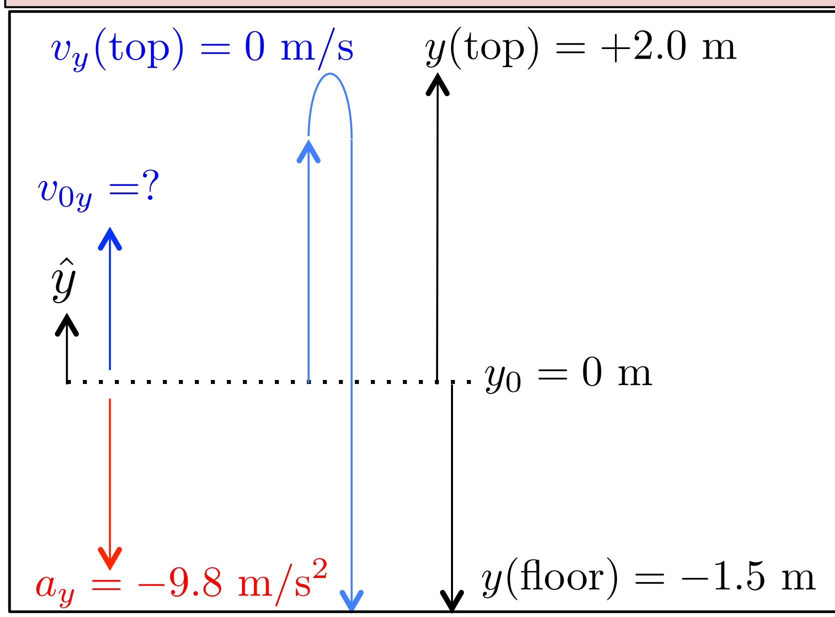

$$\Delta v(t) = at$$

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$

$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$

$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

Second part of question finds time, but don't know $v_y(\text{floor})$ or Δv_y

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$\frac{1}{2}(-9.8 \text{ m/s}^2)t^2 + (6.3 \text{ m/s})t - (-1.5 \text{ m}) = 0$$

Another One Dimensional Motion Problem

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand.
  - How fast v_{0y} do I have to throw the beanbag?
  - If I toss the beanbag at t=0.0s and miss the catch on the way back down, at what time does the beanbag hit the floor 1.5m below my hand?
    - (Assume my hand stays at the same height, y_0=0.0 m).

Use quadratic formula to solve for t

$$\underbrace{\frac{1}{2}(-9.8\ \mathrm{m/s^2})t^2}_{a\,t^2} + \underbrace{(6.3\ \mathrm{m/s})t}_{+\,b\,t} - \underbrace{(-1.5\ \mathrm{m})}_{+\,c} = 0$$

$$a = (-4.9\ \mathrm{m/s^2}) \qquad b = (6.3\ \mathrm{m/s}) \qquad c = (+1.5\ \mathrm{m})$$

$$\boxed{t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} = \frac{-(6.3\ \mathrm{m/s}) \pm \sqrt{(6.3\ \mathrm{m/s})^2 - 4(-4.9\ \mathrm{m/s^2})(+1.5\ \mathrm{m})}}{2(-4.9\ \mathrm{m/s^2})}$$

$$\boxed{t = 1.5\ \mathrm{s}\ \text{or}\ -0.21\ \mathrm{s}}$$

Two solutions!

Ponderable (10-15 minutes)

- I drop a bean bag from my hand to the floor 1.5 m below my hand.
  - How long does it take to reach the floor?
  - What is the beanbag's velocity at floor height?
    - (Assume floor height is y_0=0.0 m).

$$\Delta v(t) = at$$

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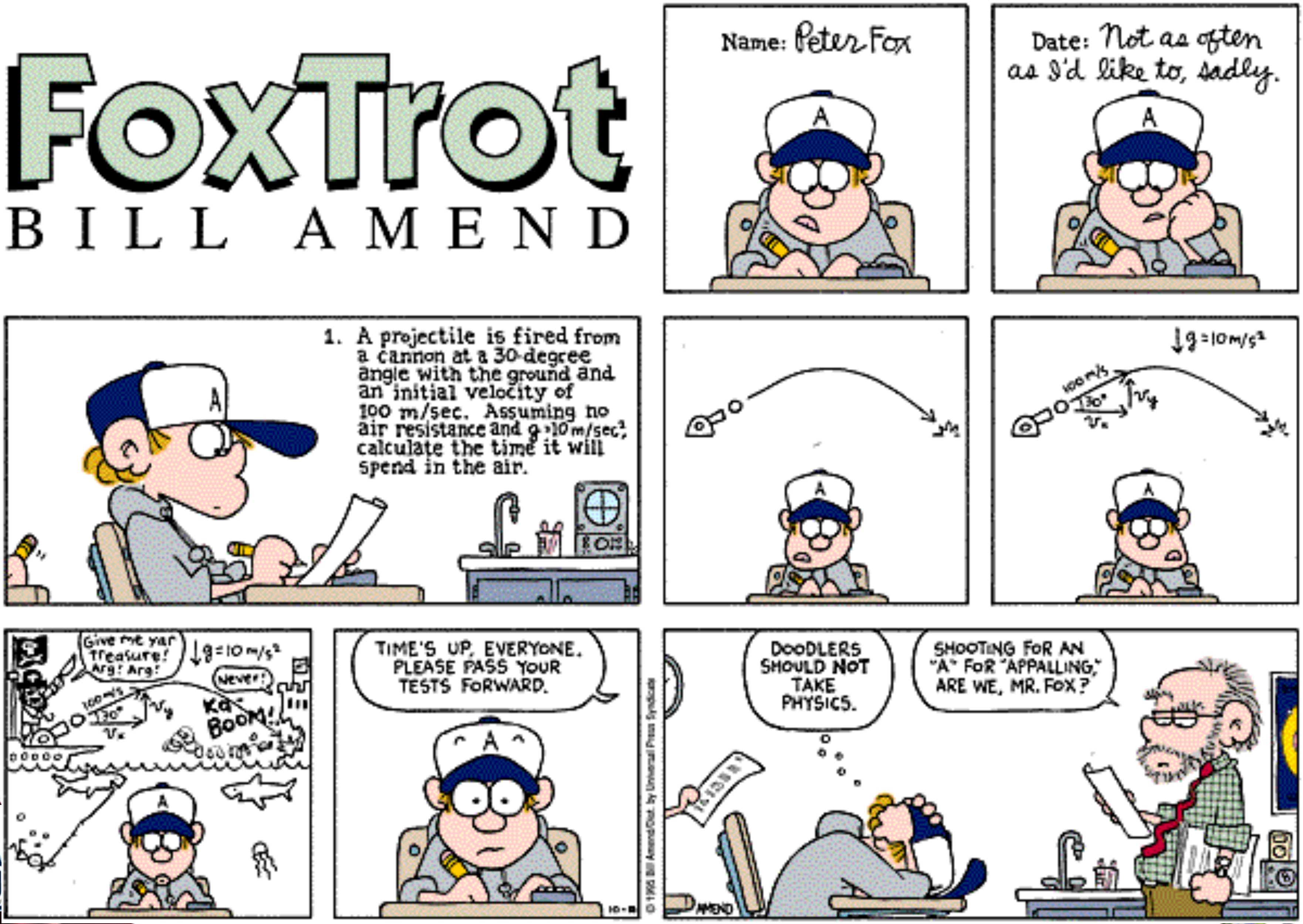
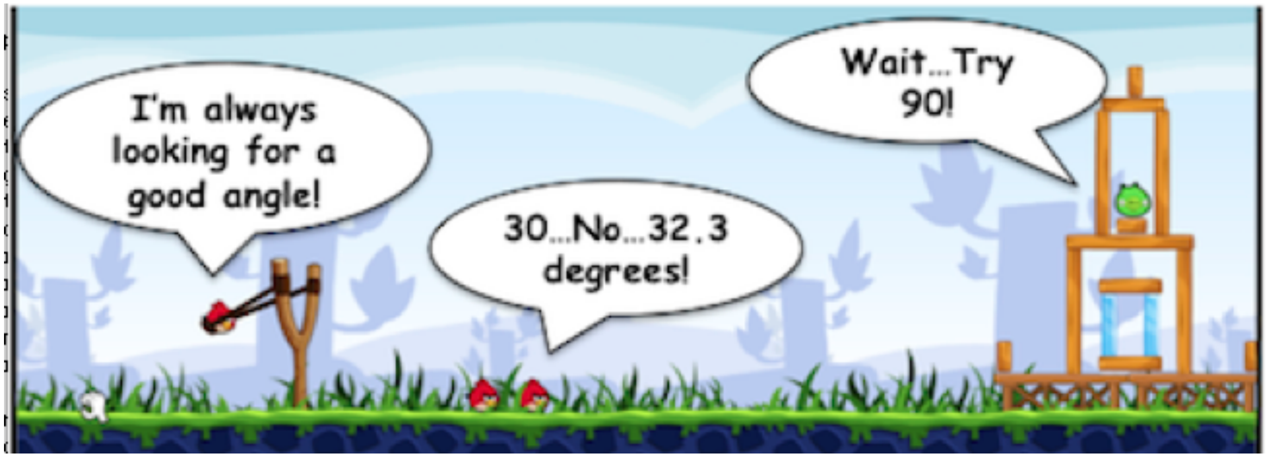
Break Time



FoxTrot
BILL AMEND
Name: Peter Fox
Date: Not as often as I'd like to, sadly.
1. A projectile is fired from a cannon at a 30-degree angle with the ground and an initial velocity of 100 m/sec. Assuming no air resistance and g=10 m/sec², calculate the time it will spend in the air.
g=10 m/s²
100 m/s
30°
Give me yer treasure! Arg! Arg!
Never!
Ka-BOOM!
TIME'S UP, EVERYONE. PLEASE PASS YOUR TESTS FORWARD.
DOODLERS SHOULD NOT TAKE PHYSICS.
SHOOTING FOR AN "A" FOR "APPALLING," ARE WE, MR. FOX?

Now With a Twist (~5 minutes)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Here, catch! (*thud*)
  - How is this different from just throwing the beanbag straight up to graze the ceiling?
    - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling?
    - Which one stays in the air a longer time?
    - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees?



Now With a Twist (Explanation)

- I want to toss a bean bag up in the air to just barely graze the ceiling that is 2.0m above my hand, AND I'm throwing it to some unfortunate student who is 5.0m away from me.
  - Do I have to throw the one to the student with a higher or lower velocity than the one straight up that grazes the ceiling? **Higher!**
  - Which one stays in the air a longer time? (Neglecting air resistance)
  - At what angle from horizontal do I have to throw the one to the student? More or less than 45 degrees? **Less, but how much less?**

The time thing is **tricky** and **nonintuitive.**

- The vertical portion of the motion is the same for both cases
  - Therefore the time the bag is in the air is the **SAME** for both too!
- Two objects that have the same vertical motion take the same time for that motion **regardless** of their horizontal motion!
  - Example: a bullet fired horizontally out of a gun and another bullet dropped from the same height take the same time to hit the ground. https://www.youtube.com/watch?v=D9wQVIEdKh8

Onwards to Two Dimensional Motion

- You already "know" two dimensional kinematics
  - Horizontal motion is easy: acceleration is zero!
  - You've already done the hard part, vertical motion with gravity

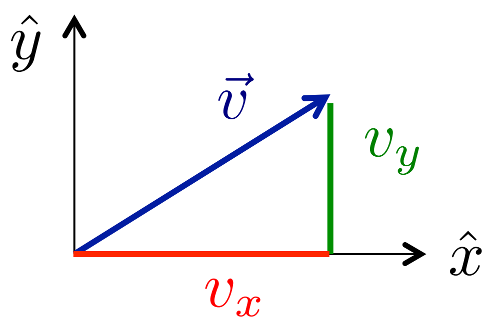
$$v_x = v_{x0}$$
$$x = x_0 + v_{x0} t$$

Horizontal motion (no acceleration)

$$v_y = v_{y0} + at$$
$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$

Vertical motion (gravitational acceleration)

Components of velocity, and vectors:

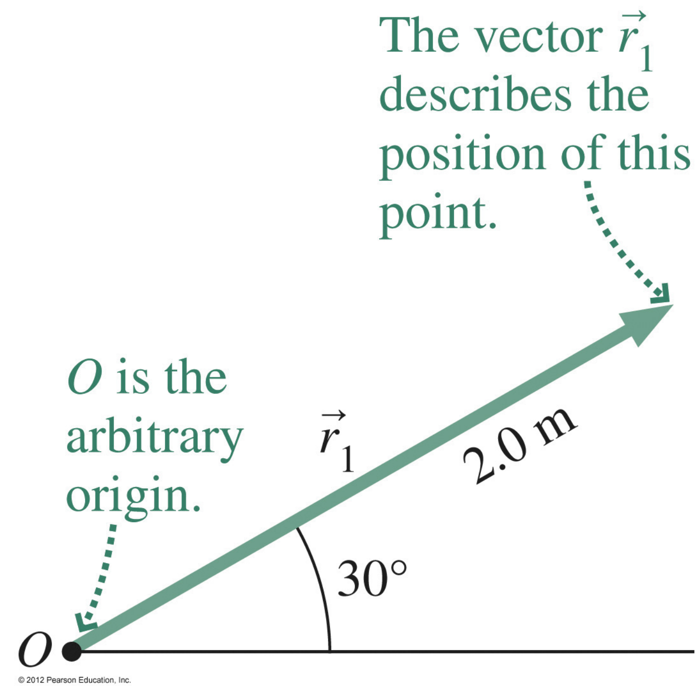


$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$
$$v = \sqrt{v_x^2 + v_y^2}$$

Vectors

- A **vector** is a quantity that has both magnitude and direction.
  - In two dimensions it takes two numbers to specify a vector.
  - In three dimensions it takes three numbers.
  - A vector can be represented by an arrow whose length corresponds to the vector's magnitude.

- Position is a vector quantity.
  - An object's position is specified by giving its distance from an origin and its direction relative to an axis.
  - Here $\vec{r}_1$ describes a point 2.0 m from the origin at a 30° angle to the axis.

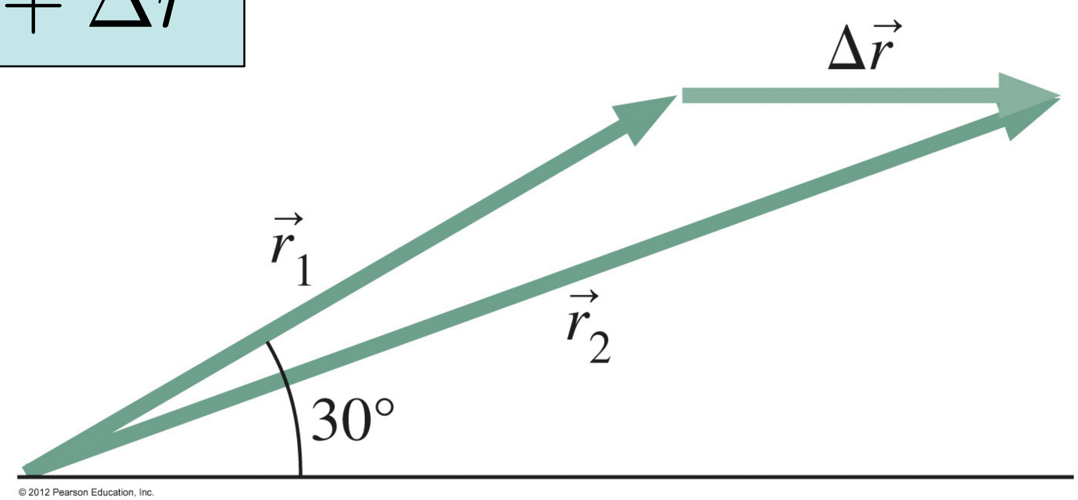


Adding Vectors

- To add vectors graphically, place the tail of the first vector at the head of the second.
  - Their sum is then the vector from the tail of the first vector to the head of the second.

– Here $\vec{r}_2$ is the sum of $\vec{r}_1$ and $\Delta\vec{r}$.

$$\vec{r}_2 = \vec{r}_1 + \Delta\vec{r}$$



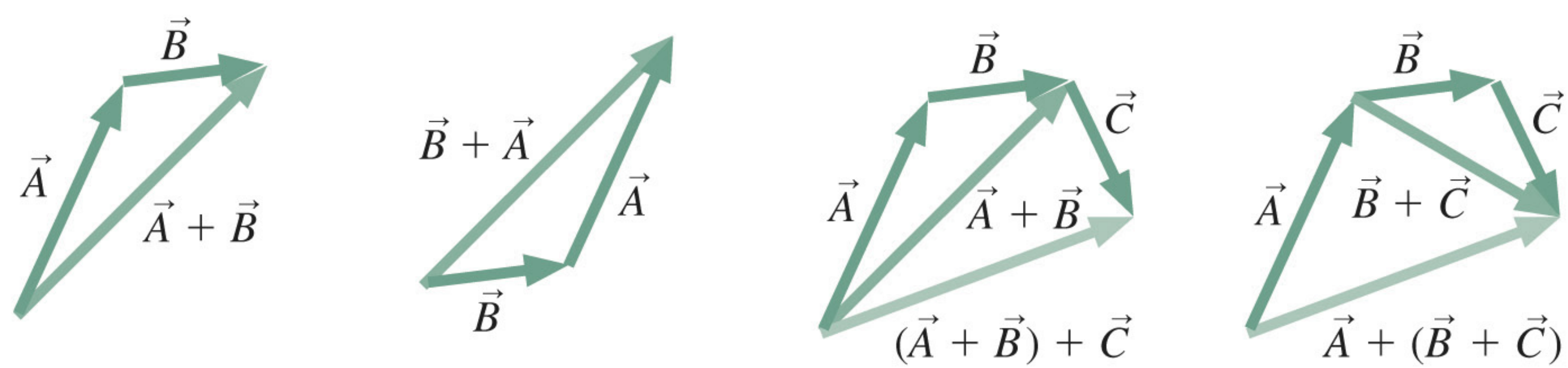
Vector Arithmetic

- To multiply a vector by a scalar, multiply the vector's magnitude by the scalar.
  - For a positive scalar the direction is unchanged.
  - For a negative scalar the direction reverses.

- To subtract vectors, add the negative of the second vector to the first: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
- Vector arithmetic is commutative and associative:

Vector addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

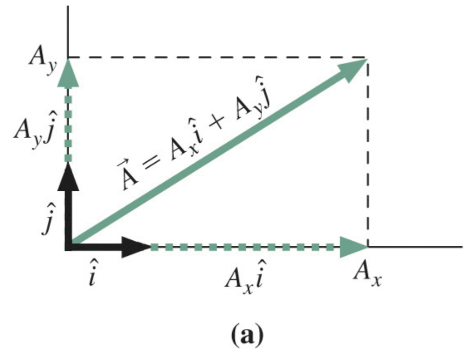
Vector addition is also associative: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$.



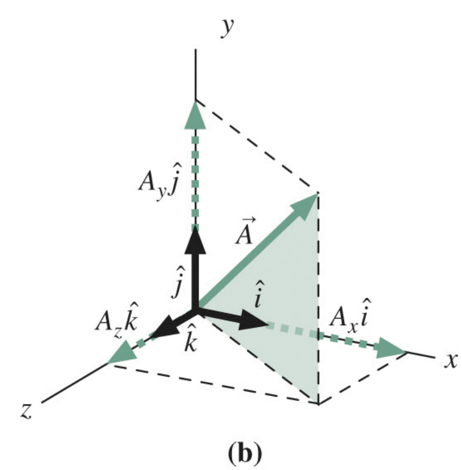
Unit Vectors

- Unit vectors have magnitude 1, no units, and point along the coordinate axes.
  - They're used to specify direction in compact mathematical representations of vectors.
  - Unit vectors in the *x*, *y*, and *z* directions are designated

$$\hat{i},\ \hat{j},\ \text{and}\ \hat{k}.$$



  - Any vector in two dimensions can be written as a linear combination of $\hat{i}$ and $\hat{j}$.
  - Any vector in three dimensions can be written as a linear combination of $\hat{i}$, $\hat{j}$, and $\hat{k}$.



I often use $\hat{x}$, $\hat{y}$, and $\hat{z}$ as well . . .

Vector Arithmetic with Unit Vectors

- To add vectors, add their individual components

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$

  - You can only add and subtract vectors that have the same units

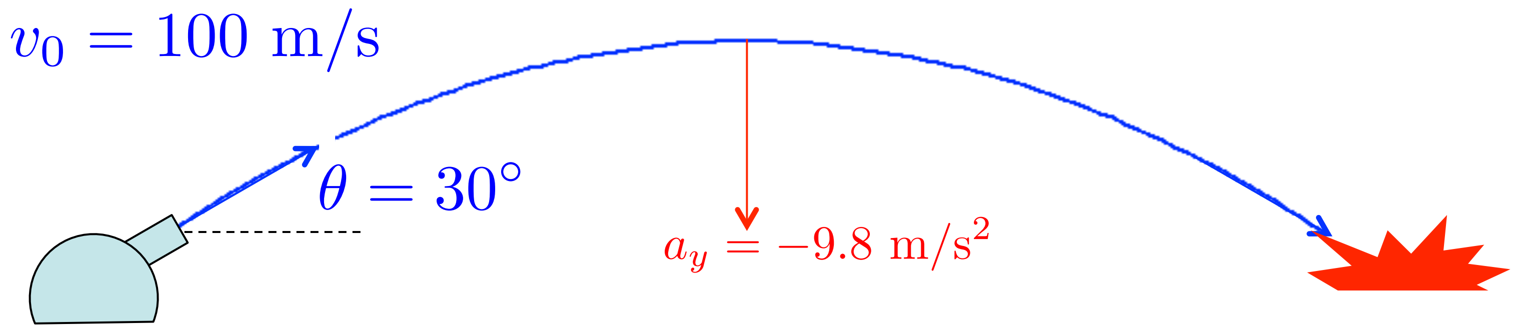
- To multiply a vector by a scalar, multiply all components by the scalar:

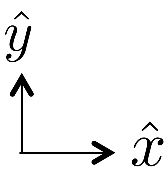
$$c\vec{A} = (cA_x)\hat{i} + (cA_y)\hat{j}$$

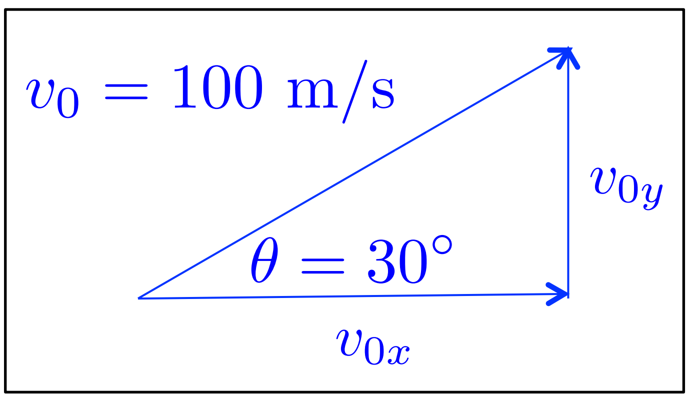
$$\text{where c is a scalar}$$

That Projectile Example

- A projectile is fired from a cannon at a 30 degree angle with the ground, and an initial velocity of 100 m/s. Assuming no air resistance, calculate the time it will spend in the air.







$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{v_{0y}}{v_0}$$

$$v_{0y} = v_0 \sin\theta = (100\ \mathrm{m/s}) \sin 30^\circ = (+50\ \mathrm{m/s})$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypoteneuse}} = \frac{v_{0x}}{v_0}$$

$$v_{0x} = v_0 \cos\theta = (100\ \mathrm{m/s}) \cos 30^\circ = (+87\ \mathrm{m/s})$$

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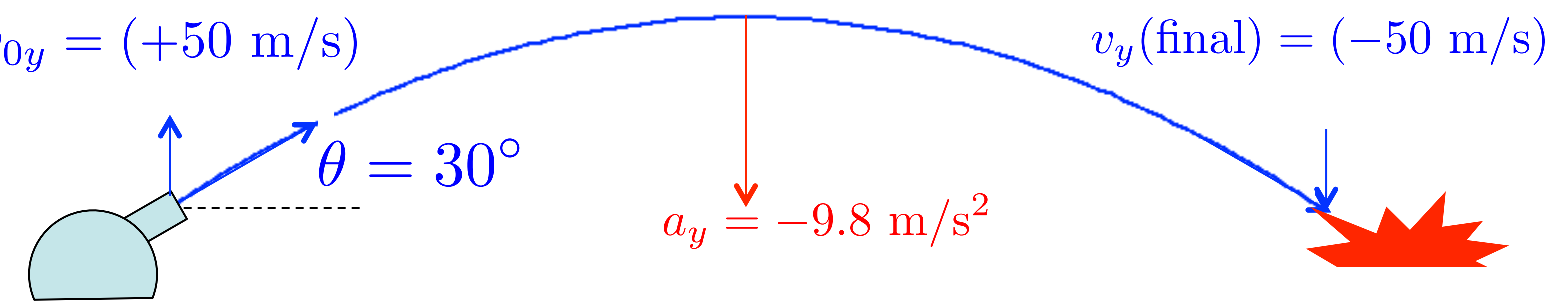
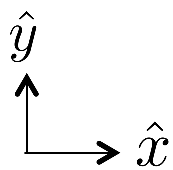


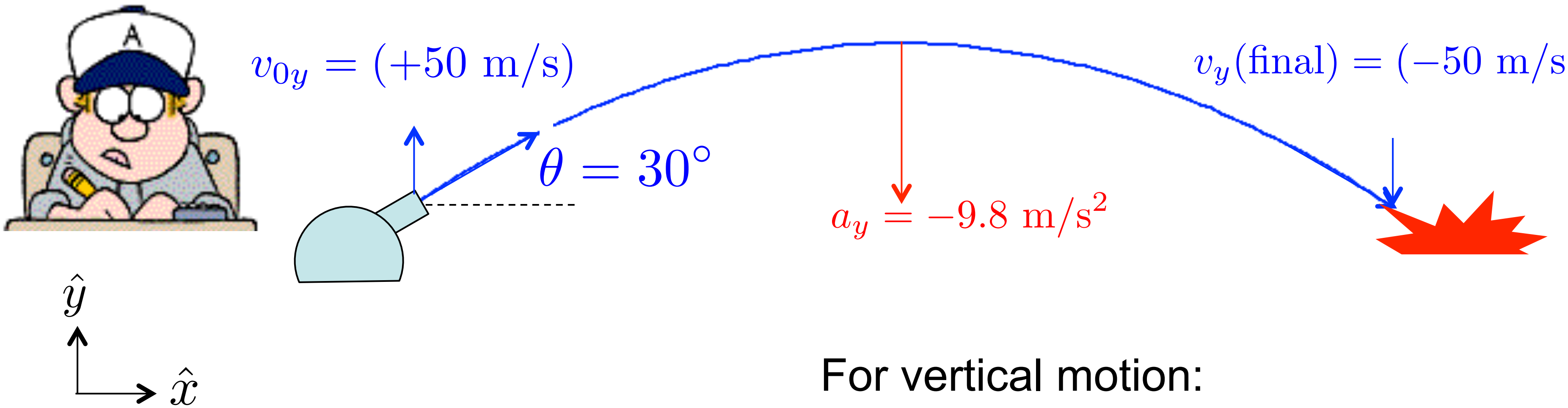


Now this problem is JUST what we did on slide 10 for one dimensional motion:

- I toss a bean bag up in the air at v_0=6.0 m/s.
  - How far up from my hand does it go?
  - How long does it take to come back to my hand?
    - (Assume my hand stays at the same height, y_0=0.0 m).

That Projectile Example

- A projectile is fired from a cannon at a 30 degree angle with the ground, and an initial velocity of 100 m/s. Assuming no air resistance, calculate the time it will spend in the air.



For vertical motion:

$$v_y(\mathrm{end}) - v_{0y} = a_y t$$

$$\Delta v(t) = at$$

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$

$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$

$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

$$t = \frac{v_y(\mathrm{end}) - v_{0y}}{a_y} = \frac{(-50\ \mathrm{m/s}) - (+50\ \mathrm{m/s})}{(-9.8\ \mathrm{m/s^2})}$$

$$\boxed{t = 10\ \mathrm{s}}$$

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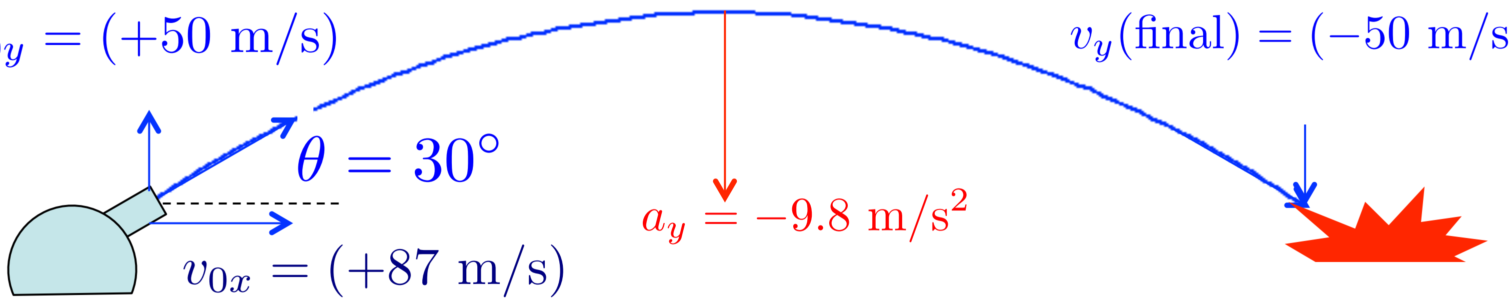
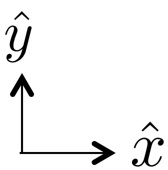




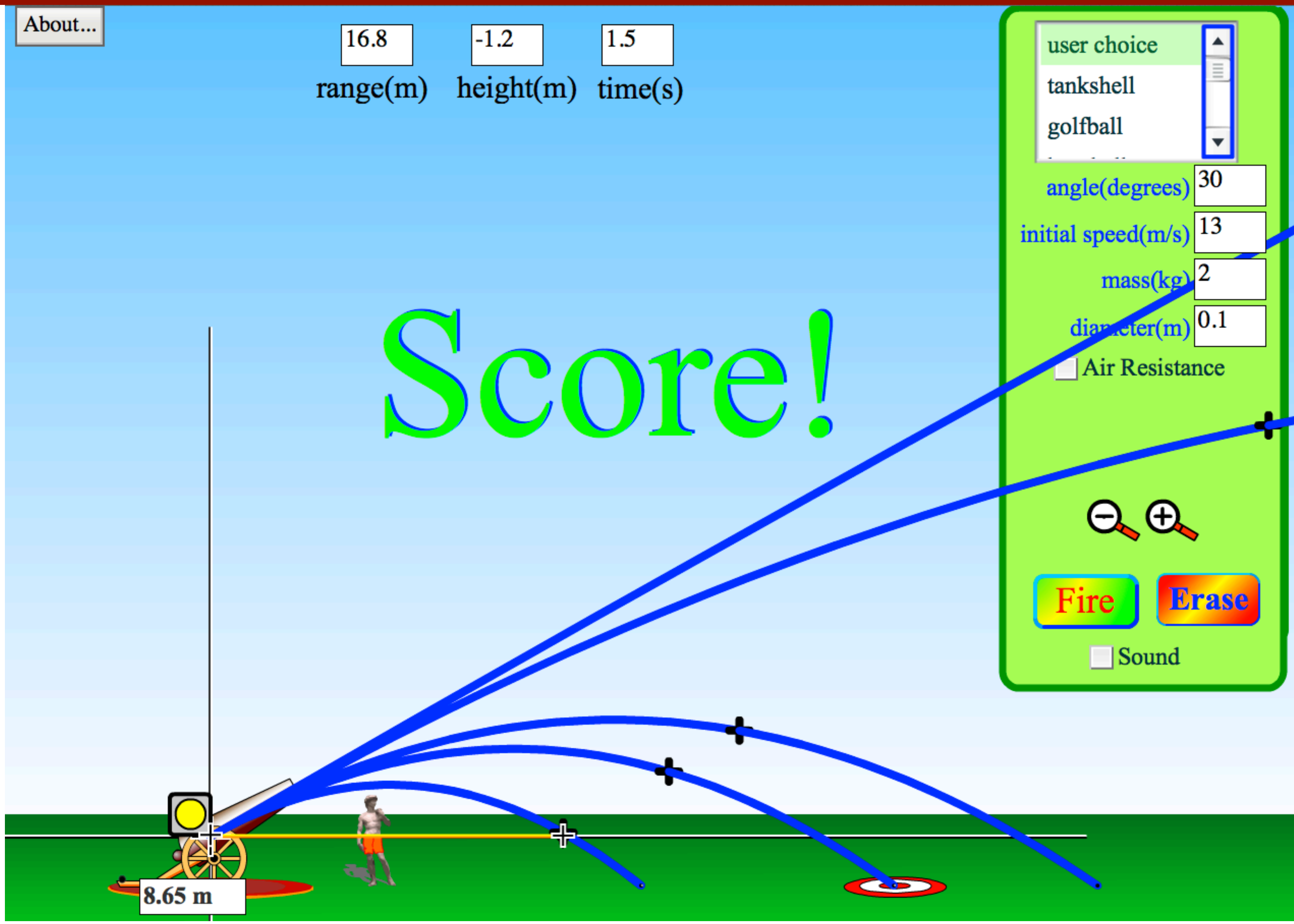
For horizontal motion: $a_x = 0 \text{ m/s}^2$

$$\Delta v(t) = at$$
$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$
$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$
$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

$$\Delta x = v_{0x}t = (+87 \text{ m/s})(10 \text{ s})$$

$$\boxed{\Delta x = 870 \text{ m}}$$

Next Time: More 2D Kinematics



https://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html