University Physics 226N/231N
Old Dominion University

More Kinematics in Two Dimensions

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FIRST MIDTERM WED SEP 21 (9 days!)

Monday, September 12, 2016

Happy Birthday to Andrew Luck, Jennifer Hudson, Paul Walker,
Louis C.K, Irene Joliot-Curie (Nobel Prize, 1897), and Makenzie Bilby!
Happy National Video Games Day, National Programmers Day (kind of),
and National Day of Encouragement (you are *awesome*)!

Please set your cell phones to "vibrate" or "silent" mode. Thanks!

Review: Moving Into Kinematics in Two Dimensions

- You already "know" two dimensional kinematics
  - Horizontal motion is easy: acceleration is zero!
  - You've already done the hard part, vertical motion with gravity

$$v_x = v_{x0}$$
$$x = x_0 + v_{x0} t$$

Horizontal motion (no acceleration)

$$v_y = v_{y0} + at$$
$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$

Vertical motion (gravitational acceleration)

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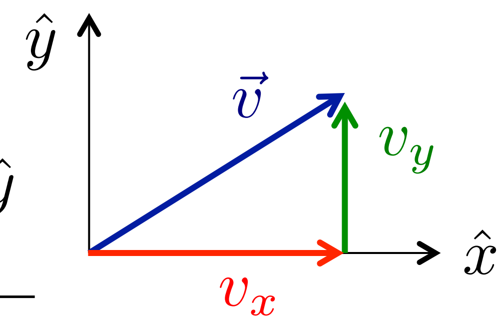
Horizontal motion (no acceleration)

$$v_y = v_{y0} + at$$
$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$
Vertical motion (gravitational acceleration)

Components of velocity, and vectors:

$$\vec{v} = v_x \hat{x} + v_y \hat{y}$$

$$v = \sqrt{{v_x}^2 + {v_y}^2}$$

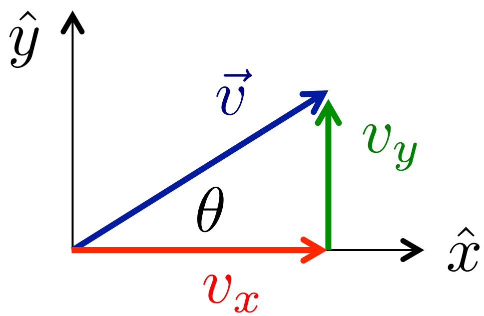


Signs of vector components are given by their relationship to the coordinate system that you use for a problem

Always draw your coordinate system!

Review: Vector Components

- Relationships between vector components and angles are found using trigonometry
  - Signs of vector components are given by their relationship to the coordinate system that you use for a problem



$$\cos\theta = \frac{\text{adjacent}}{\text{hypoteneuse}} = \frac{v_x}{v} \qquad \Rightarrow \qquad v_x = (\text{sign})\, v\cos\theta$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{v_y}{v} \qquad \Rightarrow \qquad v_y = (\text{sign})\, v\sin\theta$$

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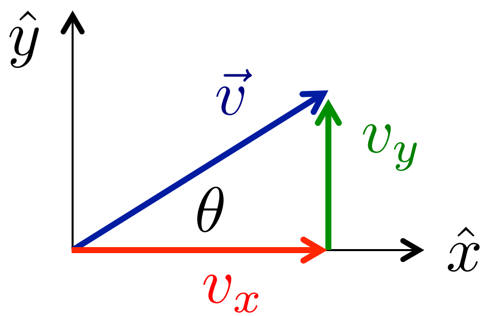

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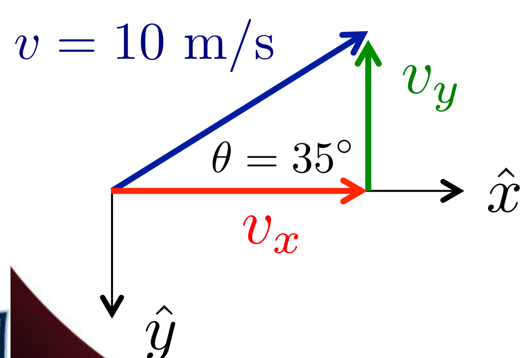

$$v_x = +v\cos\theta = +(10\ \text{m/s})\cos 35° = +8.2\ \text{m/s}$$

$$v_y = -v\sin\theta = -(10\ \text{m/s})\sin 35° = -5.7\ \text{m/s}$$

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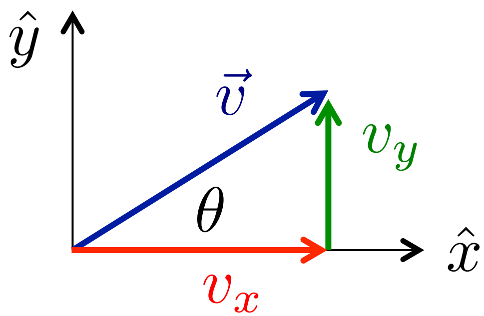

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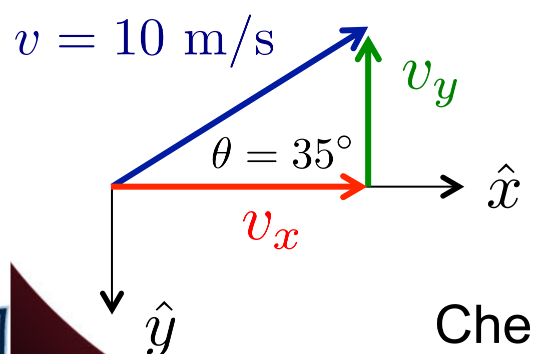

Notice signs!!

$$v_x = +v\cos\theta = +(10\ \text{m/s})\cos 35^\circ = +8.2\ \text{m/s}$$

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Check: $v = \sqrt{{v_x}^2 + {v_y}^2} = 10\ \text{m/s}$

Review: Vector Arithmetic with Unit Vectors

- To add vectors, add their individual components

$$\vec{A} = A_x \hat{x} + A_y \hat{y} \qquad \vec{B} = B_x \hat{x} + B_y \hat{y}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y}$$

$$\vec{A} - \vec{B} = (A_x - B_x)\hat{x} + (A_y - B_y)\hat{y}$$

  - You can only add and subtract vectors that have the same units
  - Remember that components can be positive or negative

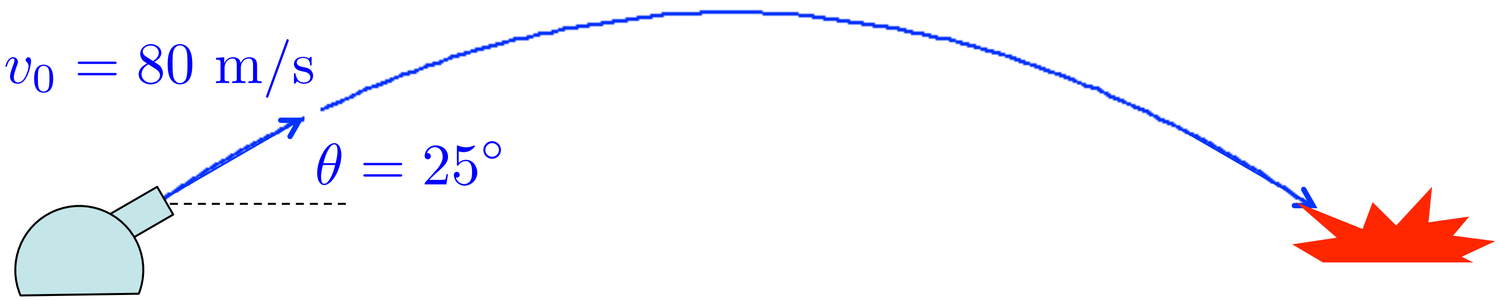
- To multiply a vector by a scalar, multiply all components by the scalar:

$$c\vec{A} = (c\ A_x)\,\vec{x} + (c\ A_y)\,\vec{y}$$

Where c is a scalar

Review: That Projectile Example (different numbers!)

- A projectile is fired from a cannon at a 25 degree angle with the ground, and an initial velocity of 80 m/s. Assuming no air resistance, calculate the time it will spend in the air.



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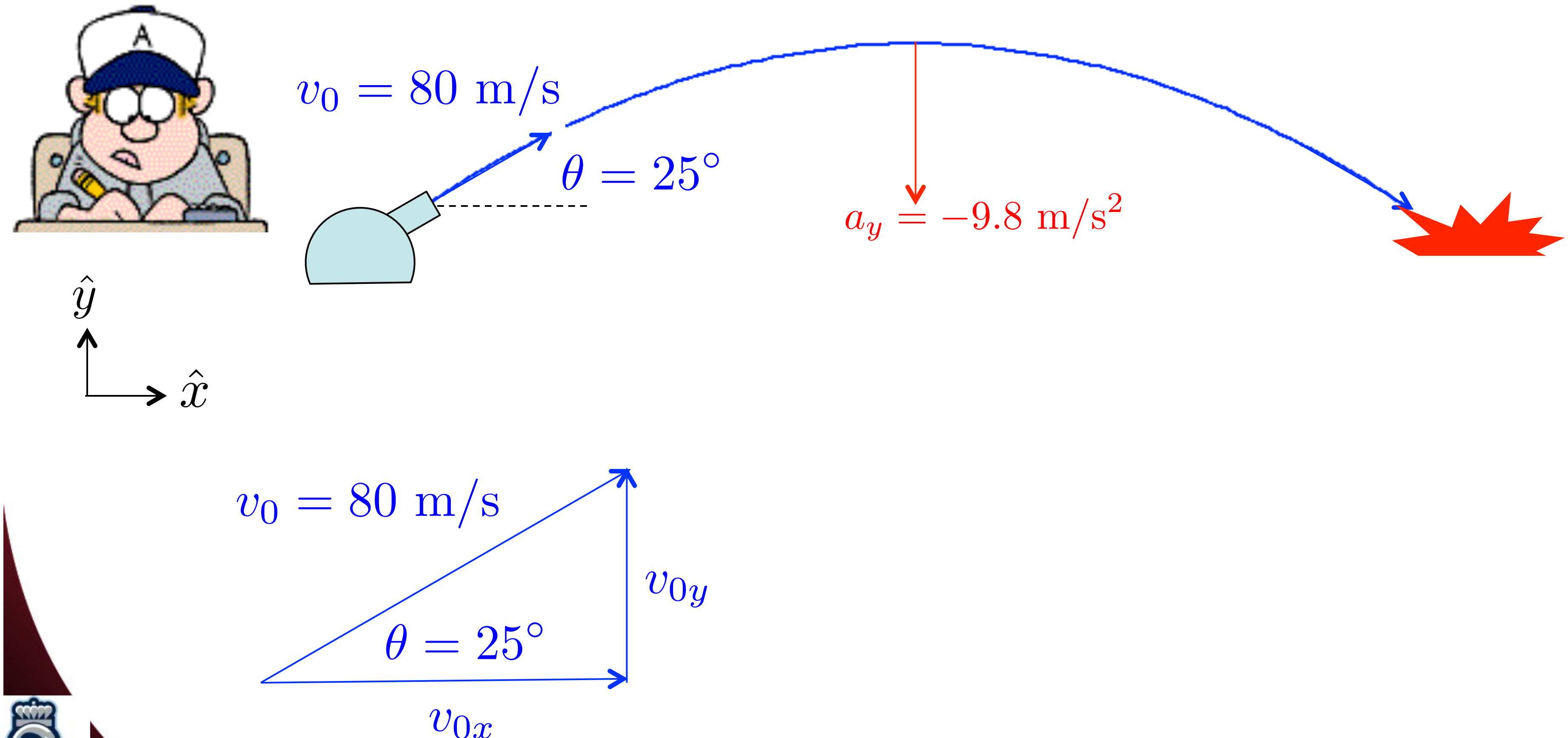

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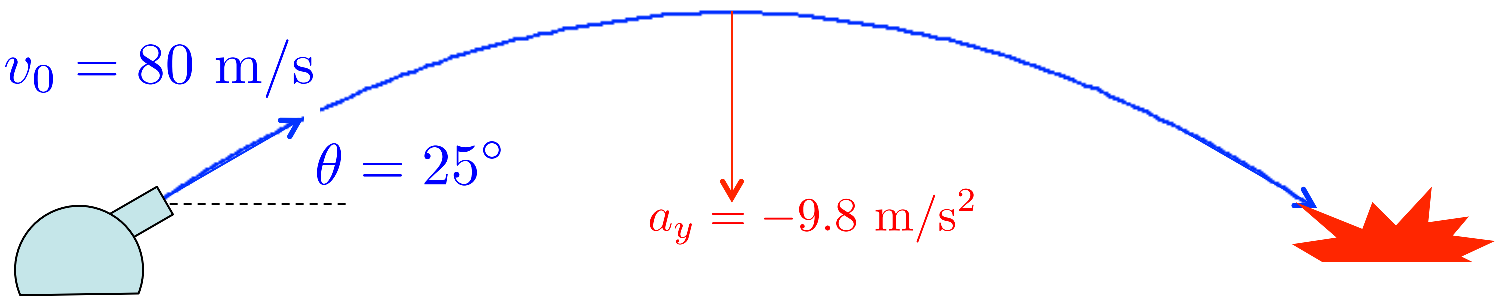
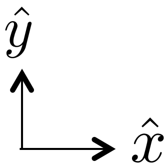
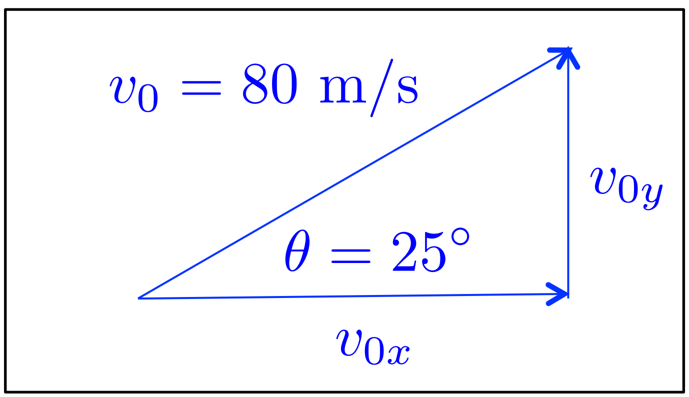




$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{v_{0y}}{v_0}$$



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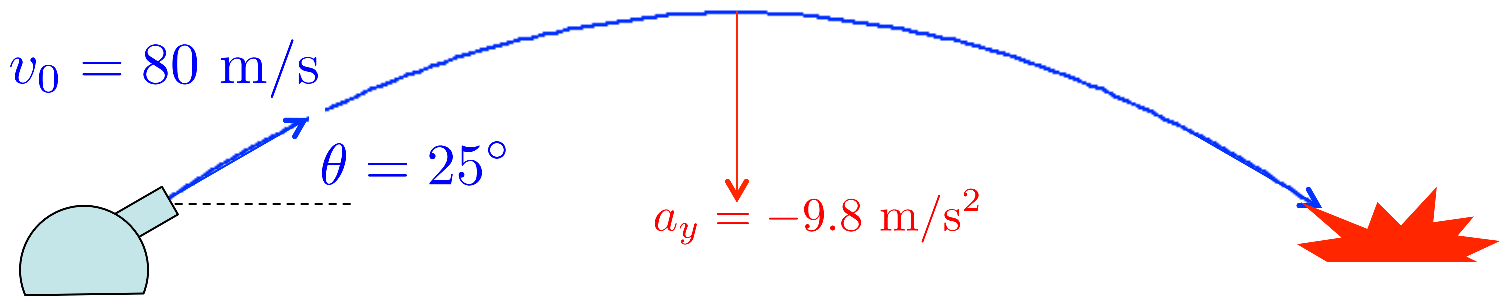

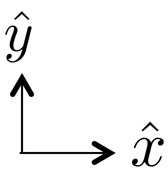

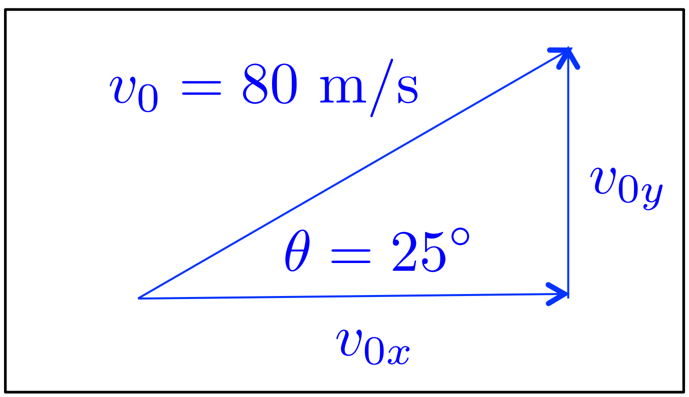

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{v_{0y}}{v_0}$$

$$v_{0y} = (\text{sign}) v_0 \sin\theta = (+34 \text{ m/s})$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypoteneuse}} = \frac{v_{0x}}{v_0}$$

$$v_{0x} = (\text{sign}) v_0 \cos\theta = (+73 \text{ m/s})$$

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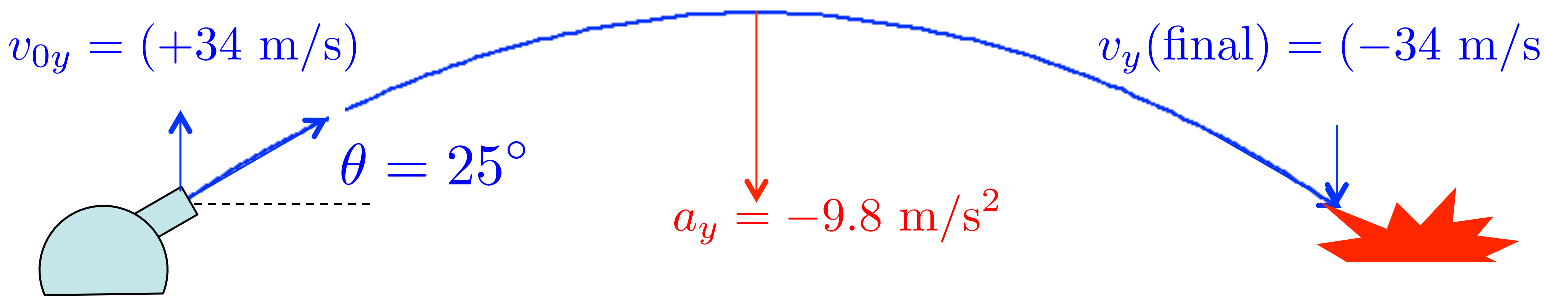

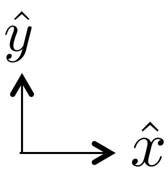

For vertical motion:

$$\Delta v(t) = at$$

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$

$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$

$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

$$v_y(\text{end}) - v_{0y} = a_y t$$

$$t = \frac{v_y(\text{final}) - v_{0y}}{a_y} = \frac{(-34 \text{ m/s}) - (+34 \text{ m/s})}{(-9.8 \text{ m/s}^2)}$$

$$\boxed{t = 6.8 \text{ s}}$$

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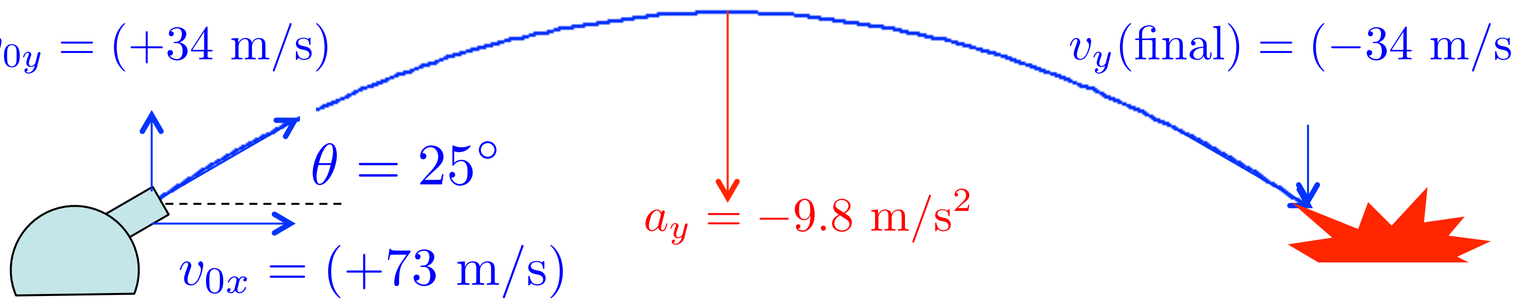
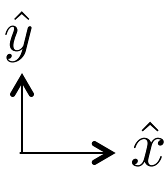




For horizontal motion: $a_x = 0\ \mathrm{m/s^2}$

$$\Delta v(t) = at$$

$$\Delta x(t) = \frac{1}{2}[v_0 + v(t)]t$$

$$\Delta x(t) = v_0 t + \frac{1}{2}at^2$$

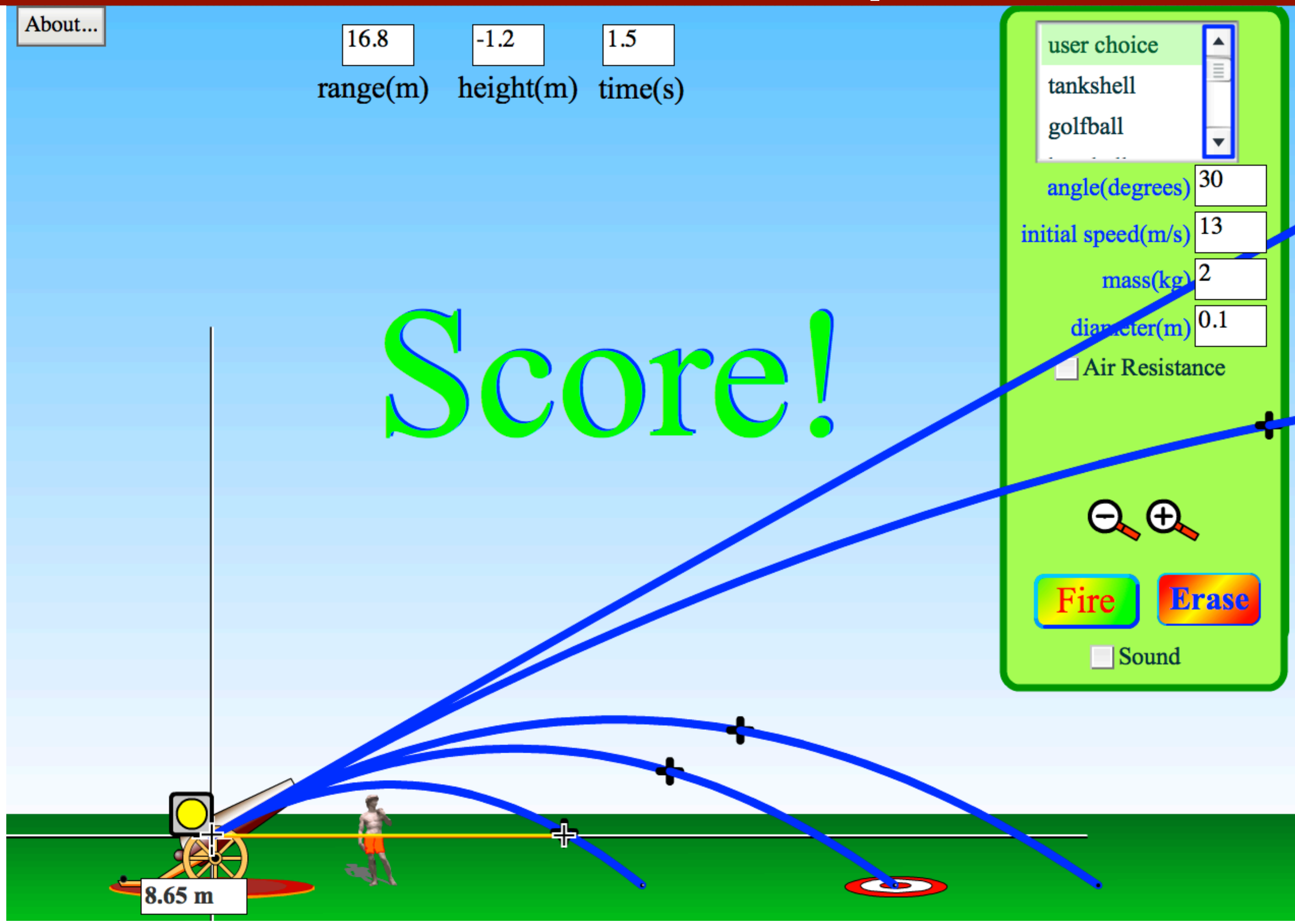
$$v^2(t) - v_0^2 = 2a\Delta x(t)$$

$$\Delta x = v_{0x}t = (+73\ \mathrm{m/s})(6.8\ \mathrm{s})$$

$$\boxed{\Delta x = 500\ \mathrm{m}}$$

(Two significant figures)

Interactive Example

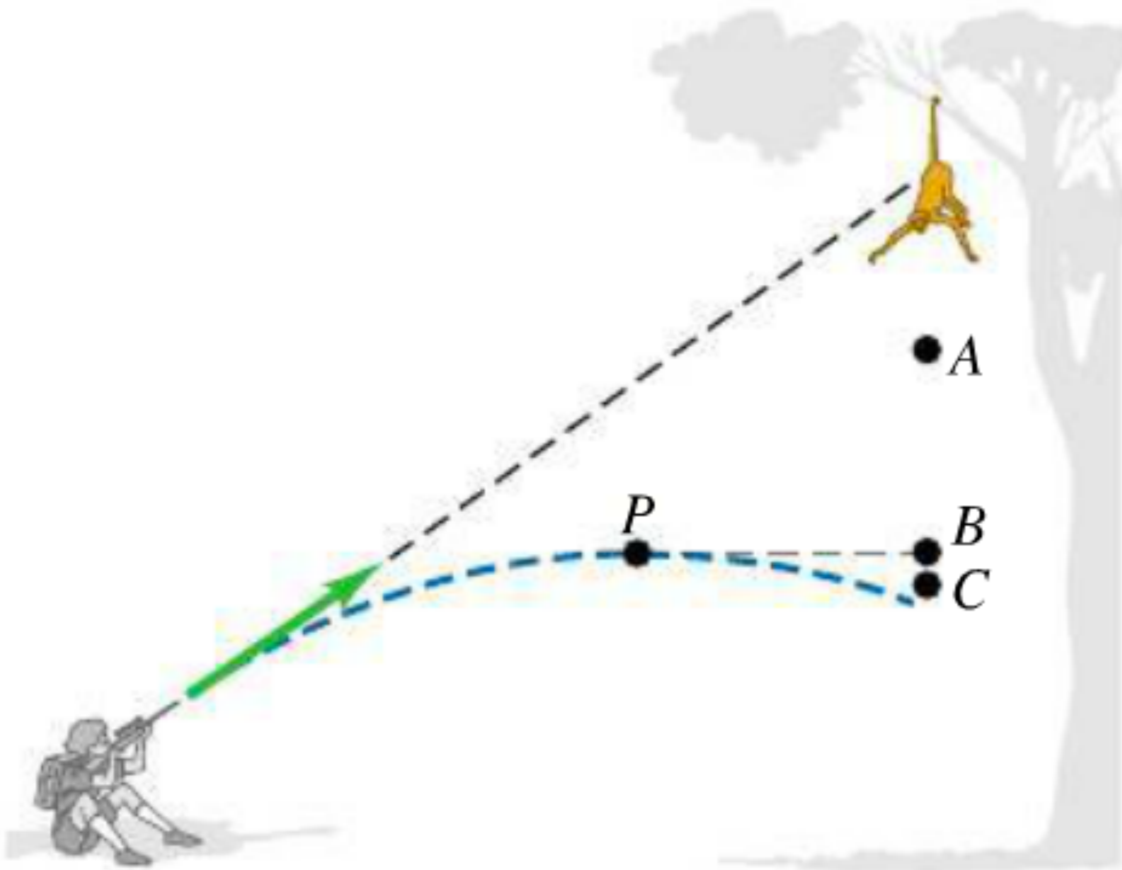


https://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html

Piano fling: https://www.youtube.com/watch?v=0O40Radaizk

A More Complicated Example (from text)

- A zookeeper fires a dart aiming at a monkey hanging in a tree. The monkey lets go the instant the dart leaves the gun. Show that the dart always hits the monkey.
  - Neglect air resistance and assume the dart hits the monkey before the monkey hits the ground and runs away.



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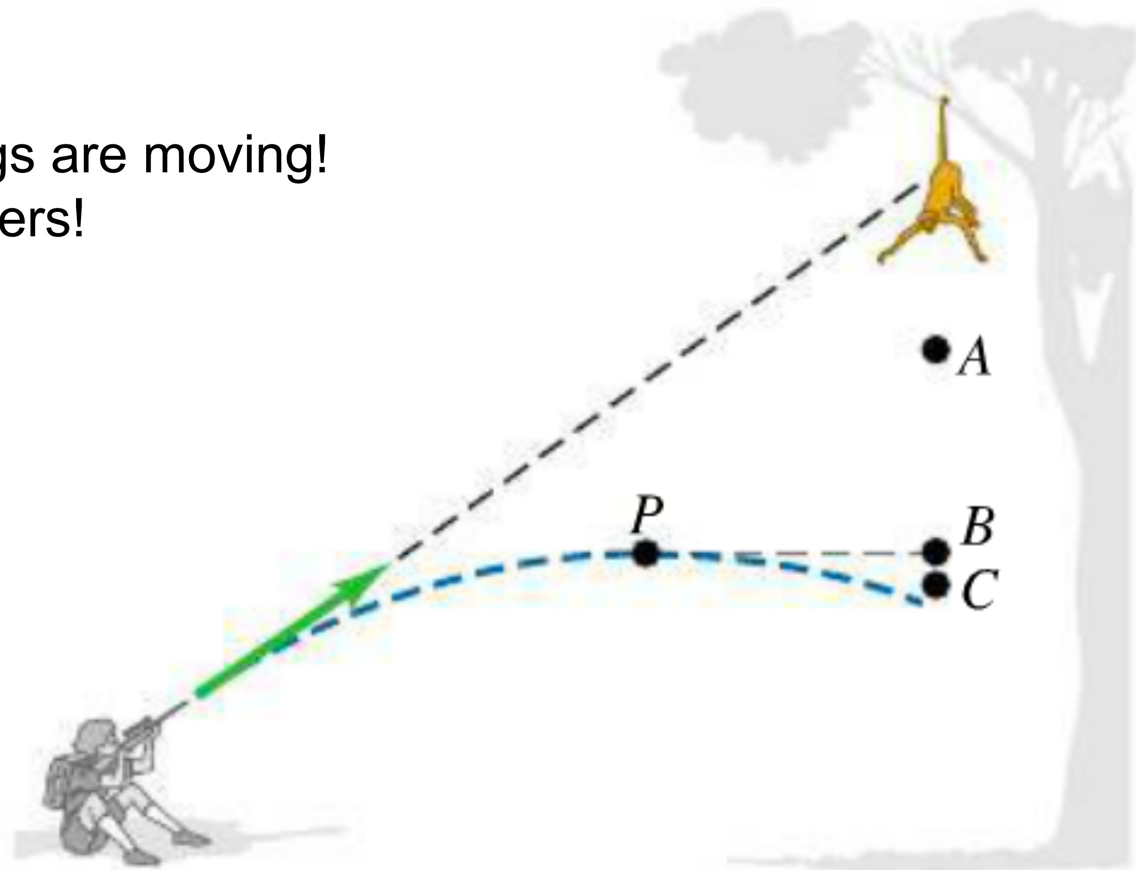
Two things are moving!
No numbers!



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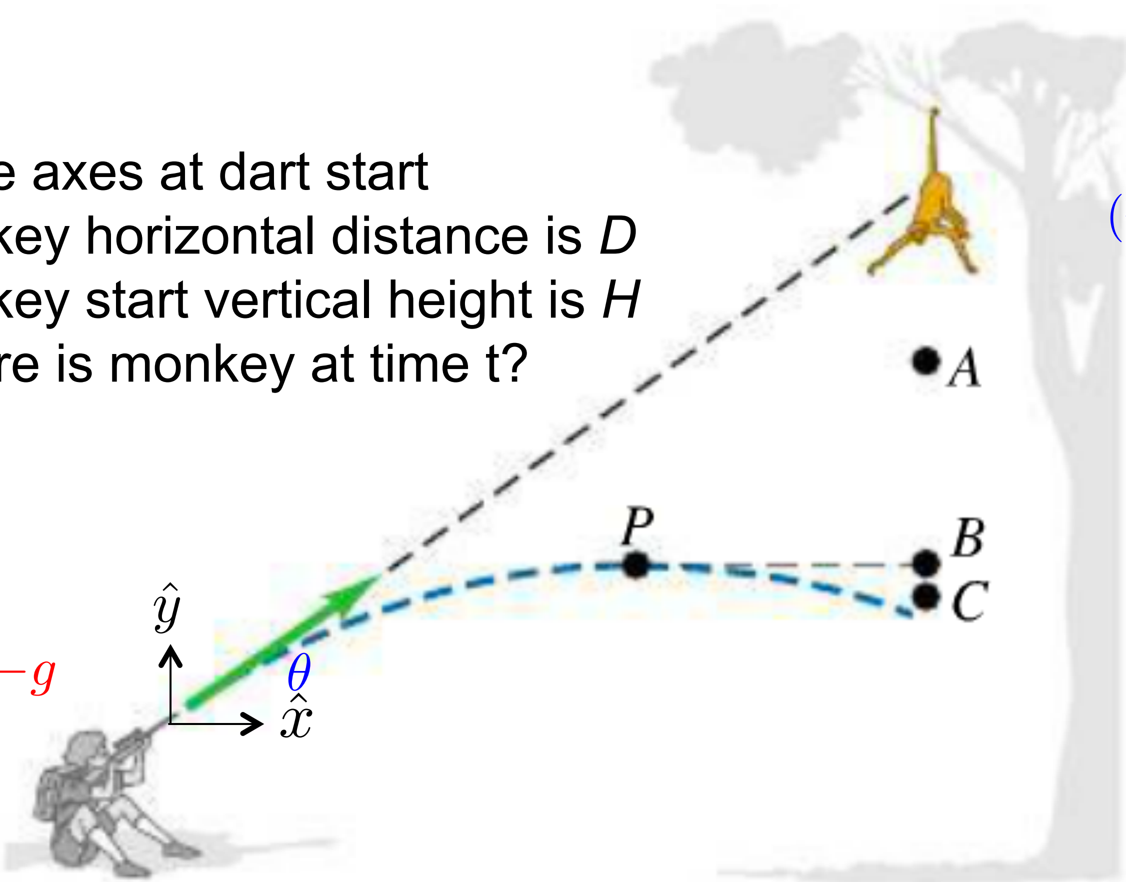
Place axes at dart start
Monkey horizontal distance is *D*
Monkey start vertical height is *H*
Where is monkey at time t?

$$(x_{0,\text{m}}, y_{0,\text{m}}) = (\text{D m}, \text{H m})$$
$$(v_{0x,\text{m}}, v_{0y,\text{m}}) = (0\ \text{m/s}, 0\ \text{m/s})$$

$$a_y = -g$$



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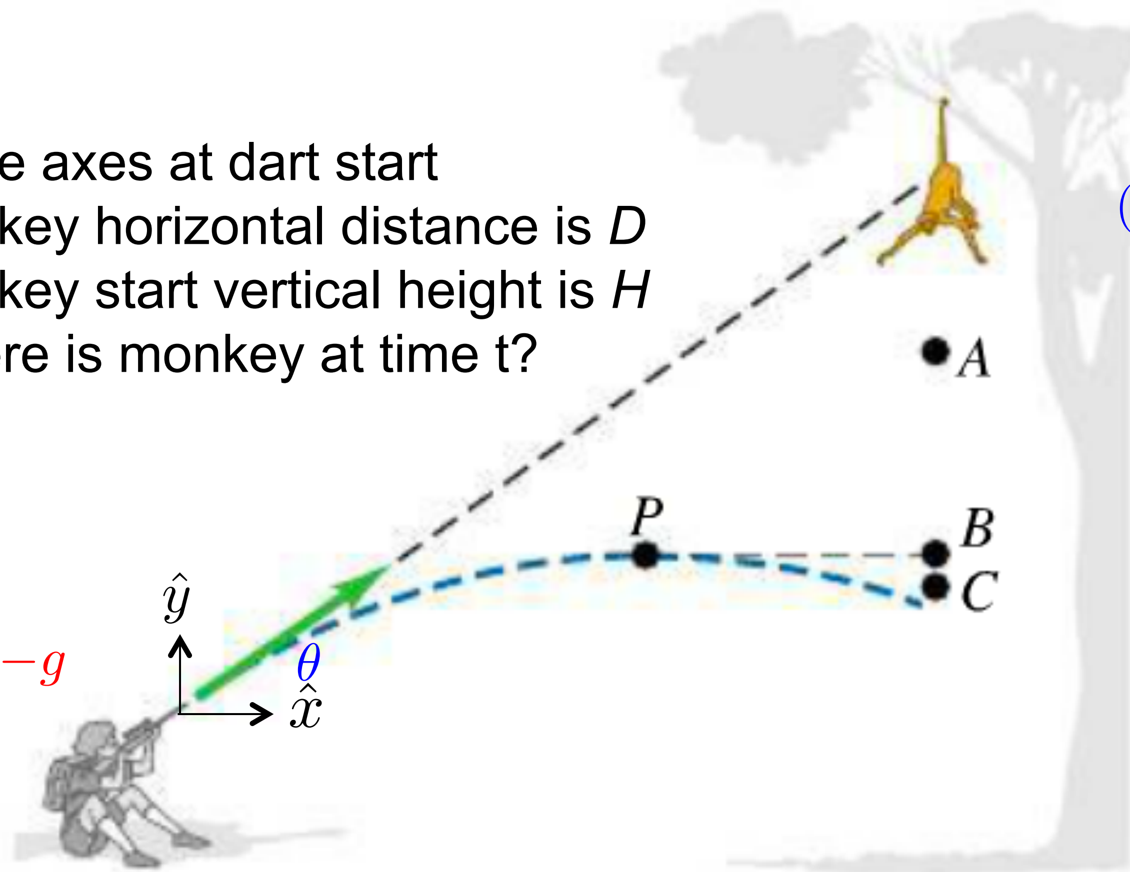

$$(x_{0,\mathrm{m}}, y_{0,\mathrm{m}}) = (\mathrm{D\ m}, \mathrm{H\ m})$$

$$(v_{0x,\mathrm{m}}, v_{0y,\mathrm{m}}) = (0\ \mathrm{m/s}, 0\ \mathrm{m/s})$$

$$\Delta y_\mathrm{m}(t) = v_{0y,\mathrm{m}} t + \frac{1}{2} a_y t^2$$

$$\Delta y_\mathrm{m}(t) = y_\mathrm{m}(t) - H = -\frac{1}{2} g t^2$$

$$y_\mathrm{m}(t) = H - \frac{1}{2} g t^2$$

$$x_\mathrm{m}(t) = D$$

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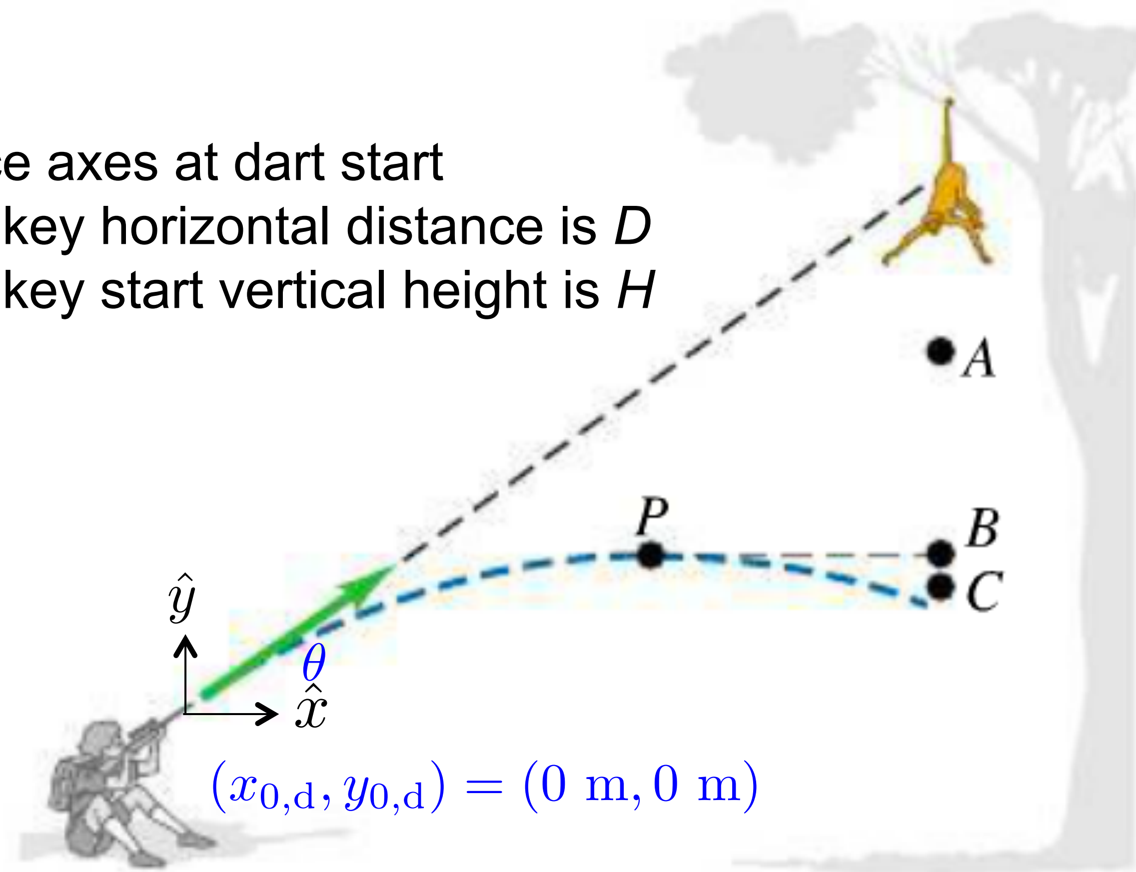

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$(x_{0,\mathrm{d}}, y_{0,\mathrm{d}}) = (0\ \mathrm{m}, 0\ \mathrm{m})$

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Dart initial velocity is $v_{0,d}$
Where is dart at time t?

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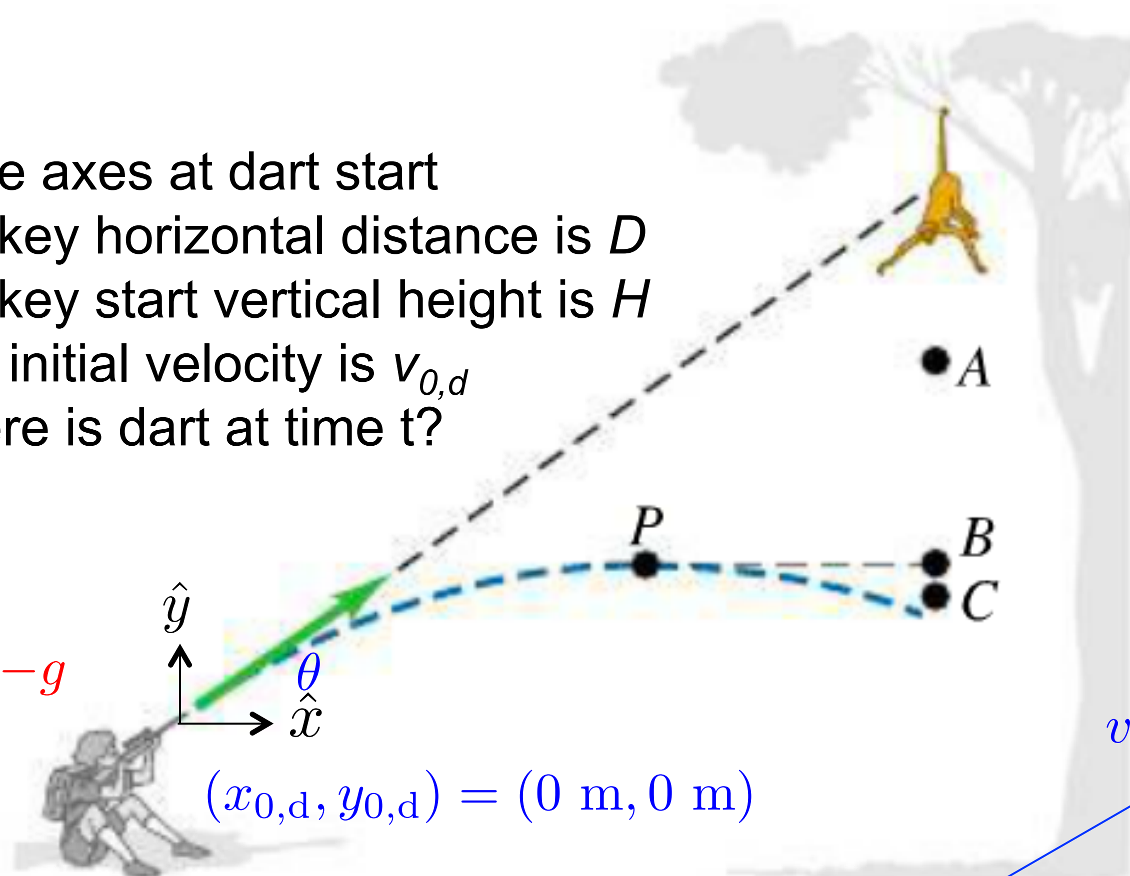

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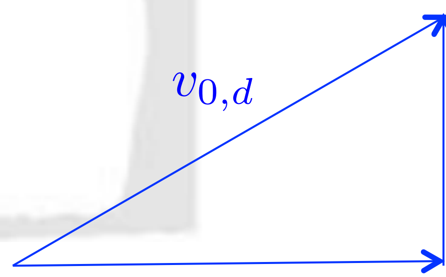

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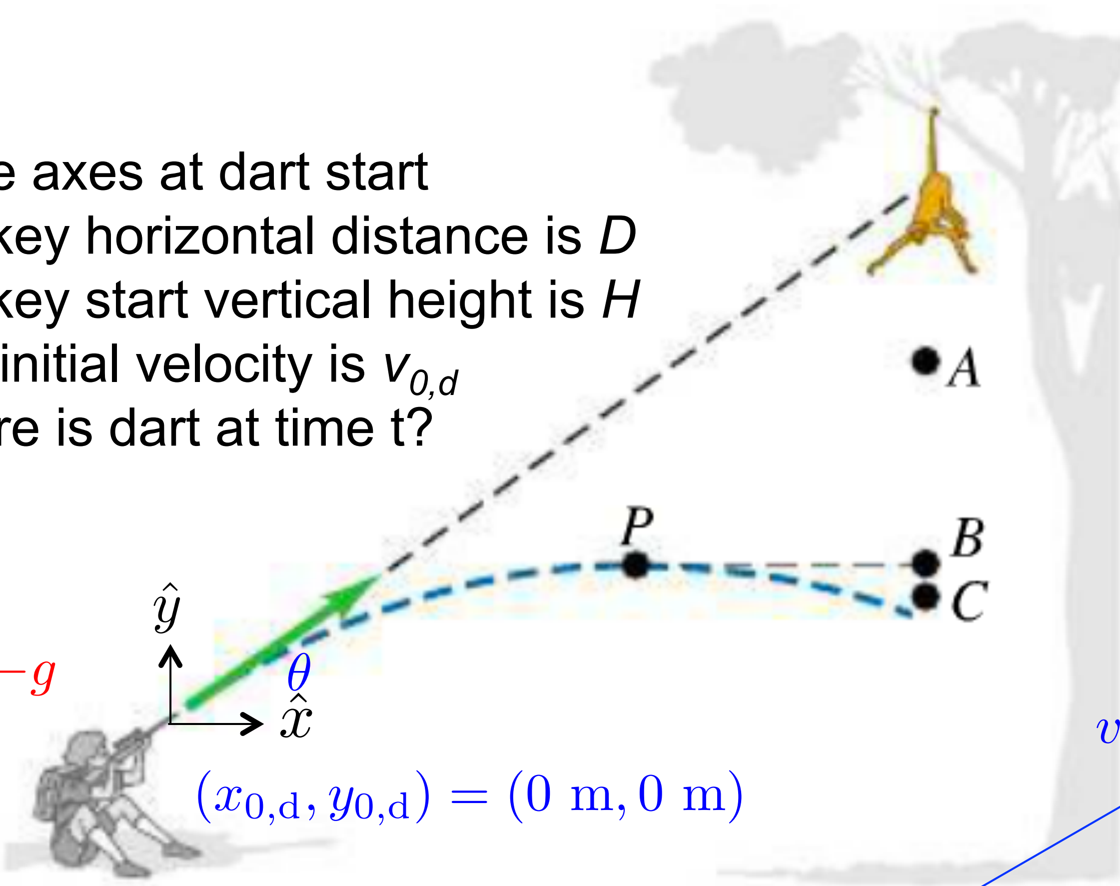


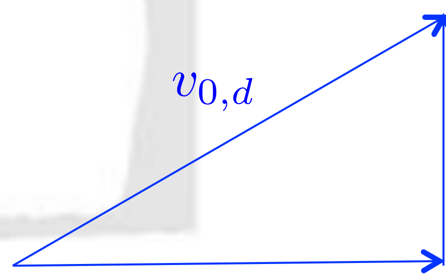
$$(x_{0,\mathrm{d}}, y_{0,\mathrm{d}}) = (0\ \mathrm{m}, 0\ \mathrm{m})$$

Horizontal

$$\Delta x(t) = v_{0x,\mathrm{d}} t = v_{0,\mathrm{d}}\, t\, \cos\theta$$

When x(t)=D (dart and monkey have same horizontal location)

$$t = \frac{\Delta x(t)}{v_{0,\mathrm{d}} \cos\theta} = \frac{\mathrm{D}}{v_{0,\mathrm{d}} \cos\theta}$$



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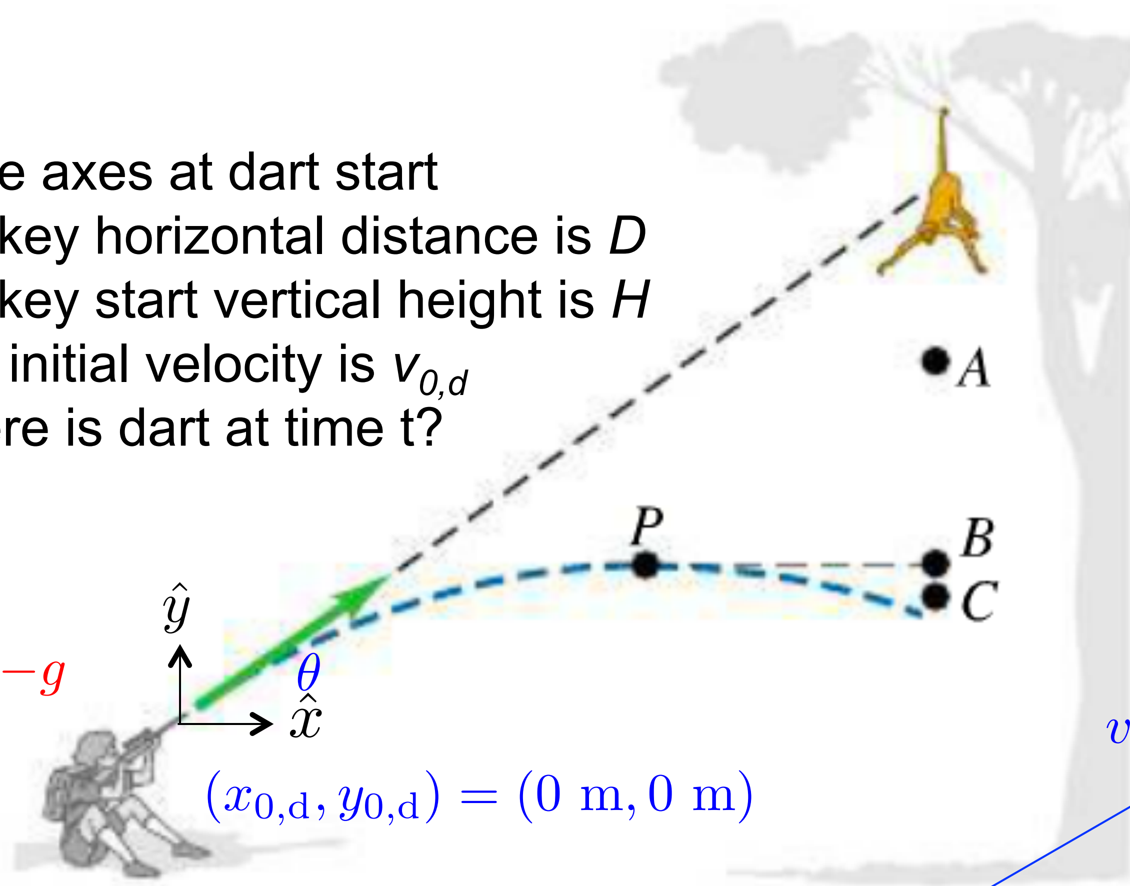


$(x_{0,\mathrm{d}}, y_{0,\mathrm{d}}) = (0\ \mathrm{m}, 0\ \mathrm{m})$

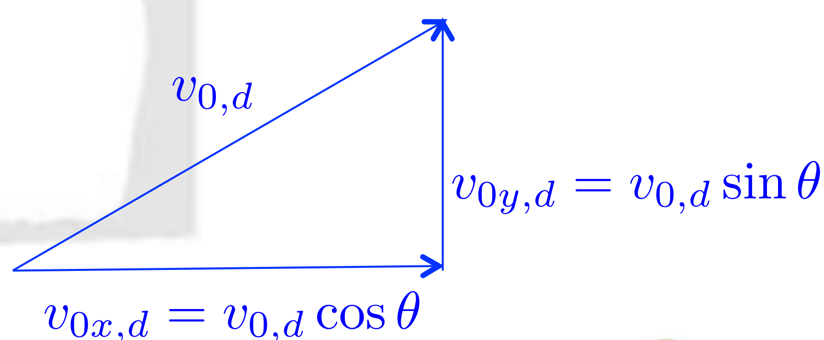
Vertical

$$\Delta y(t) = v_{0y,\mathrm{d}} t + \frac{1}{2} a_y t^2$$

$$y(t) = v_{0,\mathrm{d}}\, t \sin\theta - \frac{1}{2} g t^2$$

For monkey

$$y_{\mathrm{m}}(t) = H - \frac{1}{2} g t^2$$



Break Time

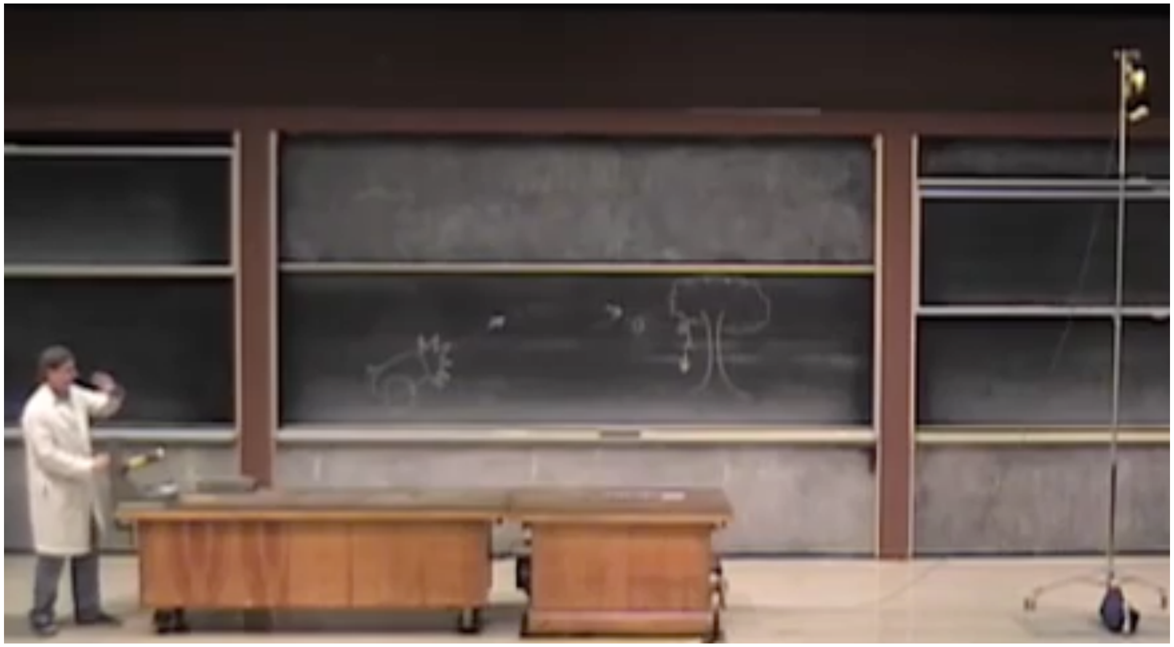
The Monkey and the Hunter

From Wikipedia, the free encyclopedia

"The Monkey and the Hunter" is a thought experiment often used to illustrate the effect of gravity on projectile motion.

The essentials of the problem are stated in many introductory guides to physics, such as Caltech's *The Mechanical Universe* television series and Gonick and Huffman's *Cartoon Guide to Physics*. In essence, the problem is as follows: A hunter with a blowgun goes out in the woods to hunt for monkeys and sees one hanging in a tree, at the same level as the hunter's head. The monkey, we suppose, releases its grip the instant the hunter fires his blowgun. Where should the hunter aim and when should he fire in order to hit the monkey?

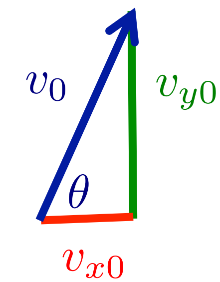
YouTube video: https://www.youtube.com/watch?v=cxvsHNRXLjw

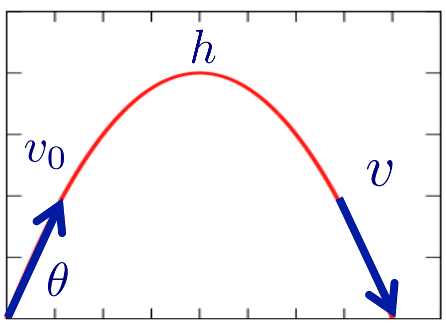


Example: Projectile Motion (Football Season)

- A quarterback throws a football at 45 mph. What angle does he need to throw it to hit a receiver 30 yards (90 feet) downfield? (Neglect air resistance and assume a level field. 45 mph is 66 feet/sec.)

- Call the angle θ - then we have $v_{x0} = v_0 \cos\theta$ and $v_{y0} = v_0 \sin\theta$
- We can employ a trick: from **symmetry**, $v_y = -v_{y0}$





$x - x_0 = 90$ feet

$$v_y = v_{y0} + at \quad \Rightarrow \quad t = -\frac{2v_{y0}}{a}$$

$$x - x_0 = v_{0x} t = \left(-\frac{2v_{y0}}{a}\right) v_{x0} = -\frac{2v_0^2}{a} \cos\theta \sin\theta$$

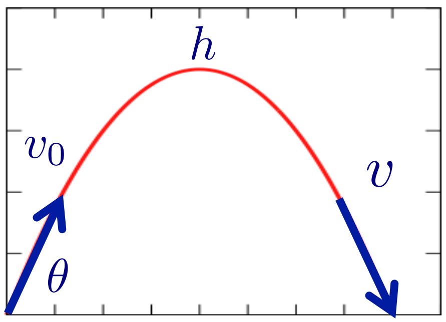
$$\text{Trigonometric identity}: \quad \sin\theta\cos\theta = \frac{1}{2}\sin(2\theta)$$

$$\sin(2\theta) = -\frac{a(x-x_0)}{v_0^2} = -\frac{(-32 \text{ feet/s}^2)(90 \text{ feet})}{(66 \text{ feet/s})^2} = 0.66$$

$$\boxed{\theta = 20.7^\circ}$$

Projectile Motion: Observations and Animation

- For **a projectile launched from ground level to ground level**, and using $v_y = -v_{y0}$, we can **derive** some interesting results



$$x - x_0 \text{ (range)} = -\frac{v_0^2}{a}\sin(2\theta)$$

Max at 45 degrees

$$h \text{ (max height)} = -\frac{v_0^2 \sin^2\theta}{2a}$$

Max at 90 degrees

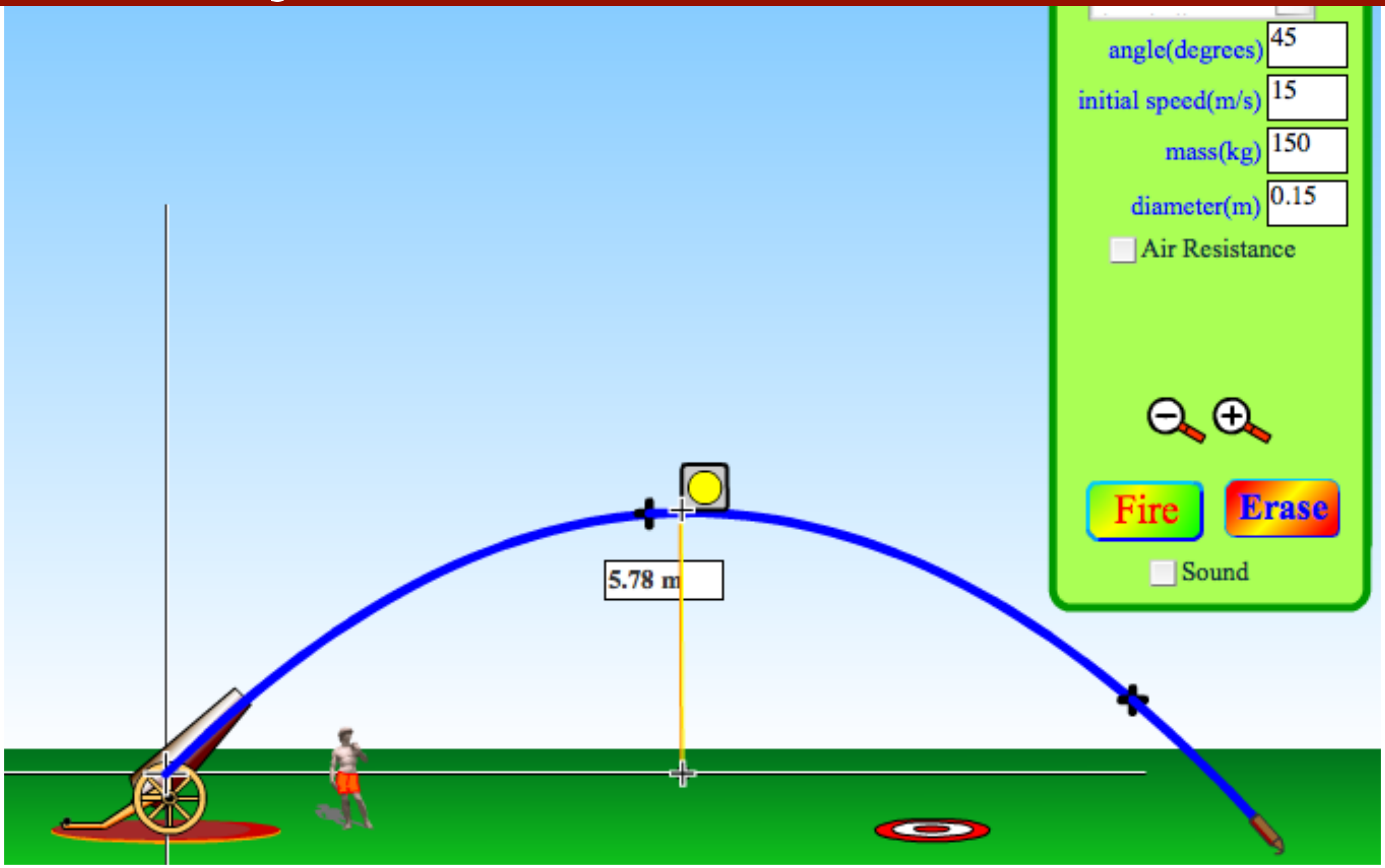
$$\frac{h}{x - x_0} = \frac{\sin^2\theta}{\sin(2\theta)} = \frac{1}{4}\tan\theta \quad \text{depends only on } \theta!$$

At a launch angle of 45 degrees to maximize the range of our projectile, the range $x - x_0 = -v_0^2/a$ is four times the achieved height.

We can experiment with this (and other projectile questions) with an applet located at

http://phet.colorado.edu/sims/projectile-motion/projectile-motion_en.html

Projectile Motion: Confirmation

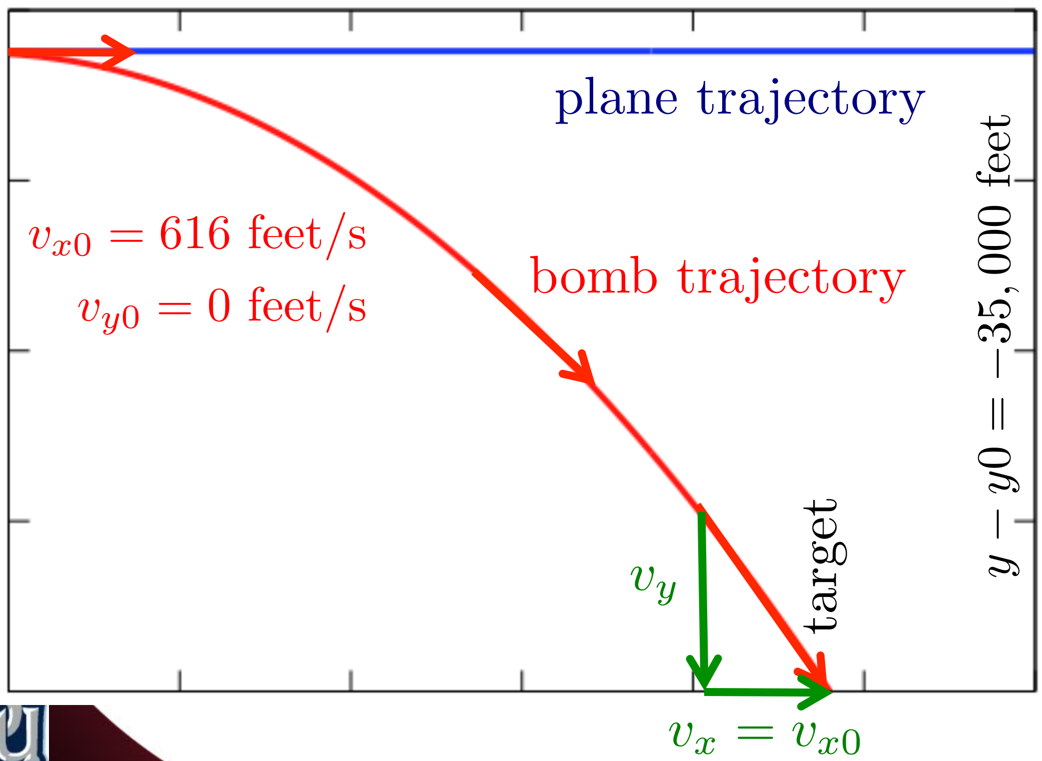


- Shoot projectile at 15 m/s at 45 degree angle
  - Height achieved is h=5.78 m, range achieved is $x\text{-}x_0$=23 m
  - Very close to $x - x_0 = 4h$ and

$$x - x_0 = -v_0^2/a = -(15\ \mathrm{m/s})^2/(9.8\ \mathrm{m/s^2}) = 22.96\ \mathrm{m}$$

Ponderable: Bomb Drop (10 minutes)

- A bomber is flying 420 mph (616 feet/s) at 35,000 feet altitude towards a ground-level target. $a=g=-32$ ft/s^2
  - How far before the target should the bomber drop the bomb?
  - Where is the bomber relative to the explosion when the bomb hits? (neglect air resistance ☺)



$$v_x = v_{x0}$$

$$x = x_0 + v_{x0} t$$

$$v_y = v_{y0} + at$$

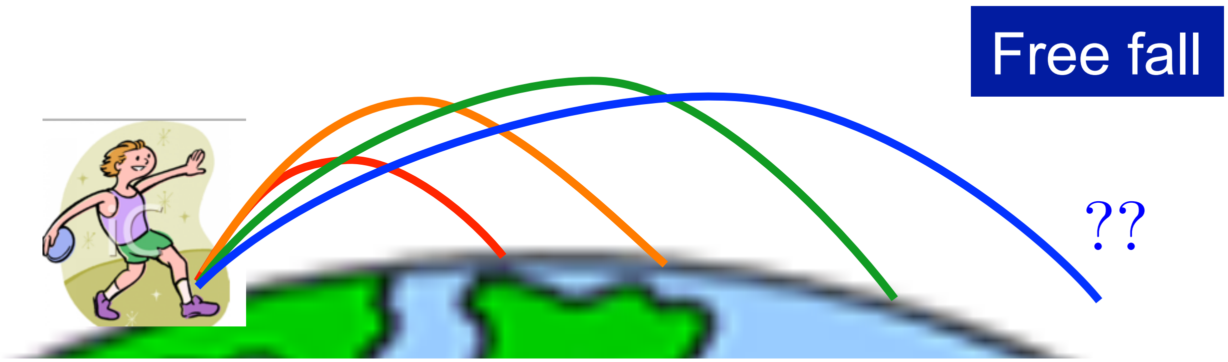
$$y = y_0 + v_{y0} t + \frac{1}{2} a t^2$$

We know $y-y_0$ and a, and $v_{y0}=0$, so we can find t…

Then we know v_{x0} and want to find $x-x_0$…

Ponderable: Falling Around The Earth

- We've been talking about projectile motion over level ground
  - The Earth is not flat: it is not "level ground" forever
  - Experience tells us that gravity always points towards the center of the Earth, wherever I am on the Earth
  - So it might be possible to shoot something really small and aerodynamic fast enough to "miss" the Earth even while the acceleration of gravity continuously makes it "fall"



Ponderable: Falling Around The Earth (10 minutes)

- Estimate (or guesstimate) how fast I have to shoot a projectile **horizontally** for it to circle around a perfectly spherical Earth and come back to me.

How do you even start to figure this out?

All you need are:

$$g = -9.8 \text{ m/s}^2$$
$$r_e \approx 6.37 \times 10^6 \text{ m}$$

The Earth's vertical drop in a horizontal distance d is

$$\Delta y \approx -\frac{d^2}{r_e}$$

Ponderable: Falling Around The Earth (Solution)

- Estimate (or guesstimate) how fast I have to shoot a projectile **horizontally** for it to circle around a perfectly spherical Earth and come back to me.

$$\Delta y \approx -\frac{d^2}{r_e}$$

Over a small segment of time Δt, vertical velocity is

$$v_y = \frac{\Delta y}{\Delta t} = -\frac{d^2}{r_e \Delta t}$$

and vertical acceleration is

$$a_y = g = \frac{\Delta v}{\Delta t} = -\frac{d^2}{r_e \Delta t^2}$$

Ponderable: Falling Around The Earth (Solution)

- Estimate (or guesstimate) how fast I have to shoot a projectile **horizontally** for it to circle around a perfectly spherical Earth and come back to me.

"Horizontally" we're not accelerating, so horizontal velocity is distance/time:

$$v_x = \frac{d}{\Delta t} \quad \Rightarrow \quad d = v_x \Delta t$$

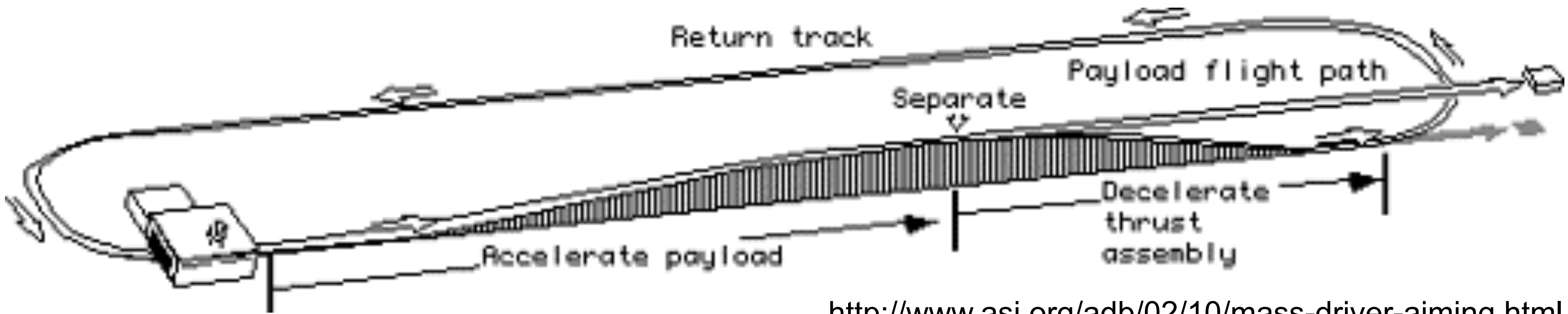
$$g = -\frac{v_x^2}{r_e}$$

$$v_x = \sqrt{-g r_e} = \boxed{7.9 \text{ km/s} = v_x}$$

Over 17000 miles/hour!

Orbit time of about 1.4 hours!

Aside: Space Launches and Mass Drivers



http://www.asi.org/adb/02/10/mass-driver-aiming.html

- An Earthbound launch vehicle like this has real challenges
  - High gravity ⇨ Very high launch velocity required
    - Requires extremely long track or absurdly high acceleration
  - Dense atmosphere ⇨ Very large air resistance
- But it's been considered for the moon for decades
  - g_{moon}=-1.6 m/s^2, r_{moon}=1737 km ⇨ v_{launch}=1.67 km/s=3730 mph
  - Perhaps okay for launching materials but not people
    - At 3g acceleration, the acceleration track is 50 km (30 miles) long!
    - (At 10g acceleration, it's still 16 km or 10 miles long…)

Circular Motion and Centripetal Acceleration

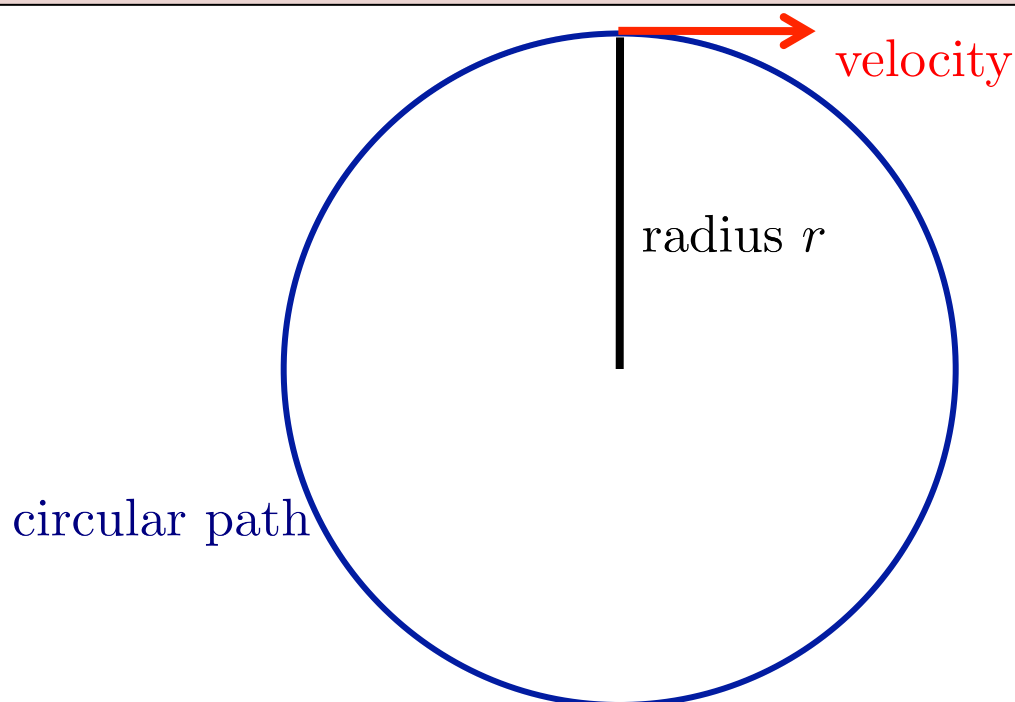
- Objects in circular gravitational orbits like this are examples of **circular motion**
  - Notice that nothing here depended on the height!
  - In fact, nothing depended on "gravity" except the constant acceleration towards the center of the earth
  - It turns out that is a pretty general result for circular motion
    - It's not trivial to derive, so it's kind of a fundamental equation
    - To keep an object moving with speed v in a circle of radius r, we need a **constant acceleration**

$$a_{\text{centrip}} = \frac{v^2}{r}$$

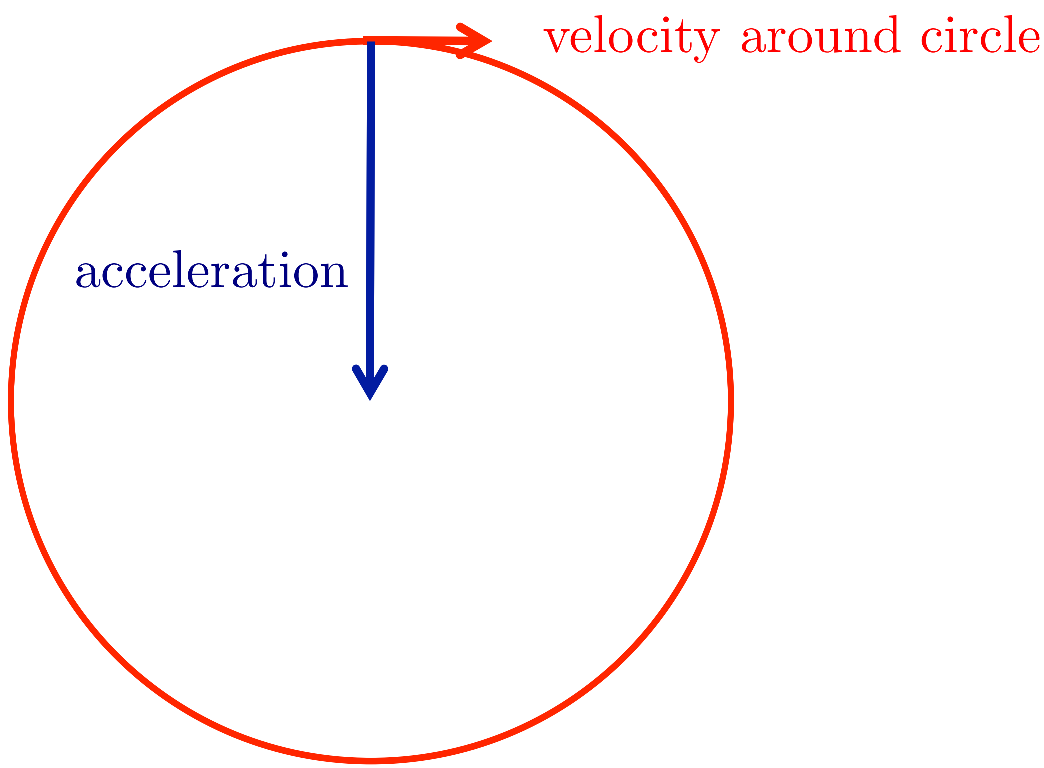
  - This is usually called **centripetal acceleration**

Ponderable (10 minutes)

- You're swinging something around on the end of a string. It moves in a circle of constant radius r with constant speed v.
  - In which direction does acceleration point when the object is at various points around the circle?
    - Remember, acceleration is how velocity changes over time…
  - Is the string tighter with a smaller radius, or a bigger radius?
  - If the string suddenly snaps, what direction does the object fly?

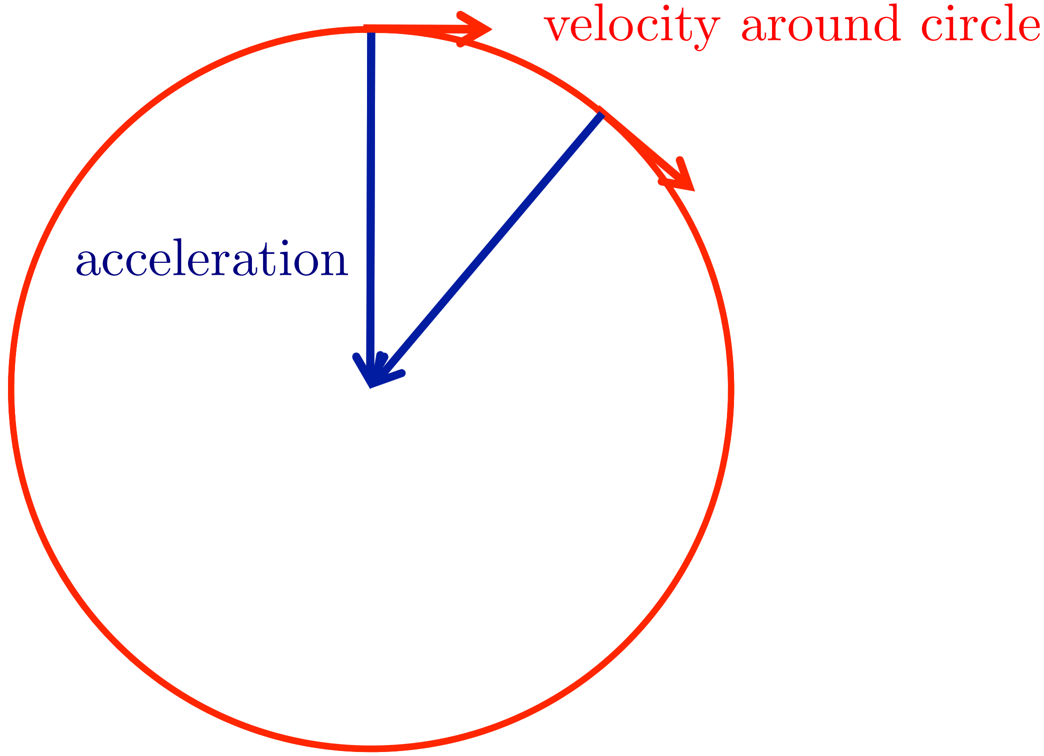


Centripetal Acceleration



- For an object moving with constant speed around a circle
  - The acceleration magnitude is constant but its direction is changing with time
  - Acceleration is always pointed towards the center of the circle
  - Remember, **acceleration is how velocity changes in time**

Centripetal Acceleration



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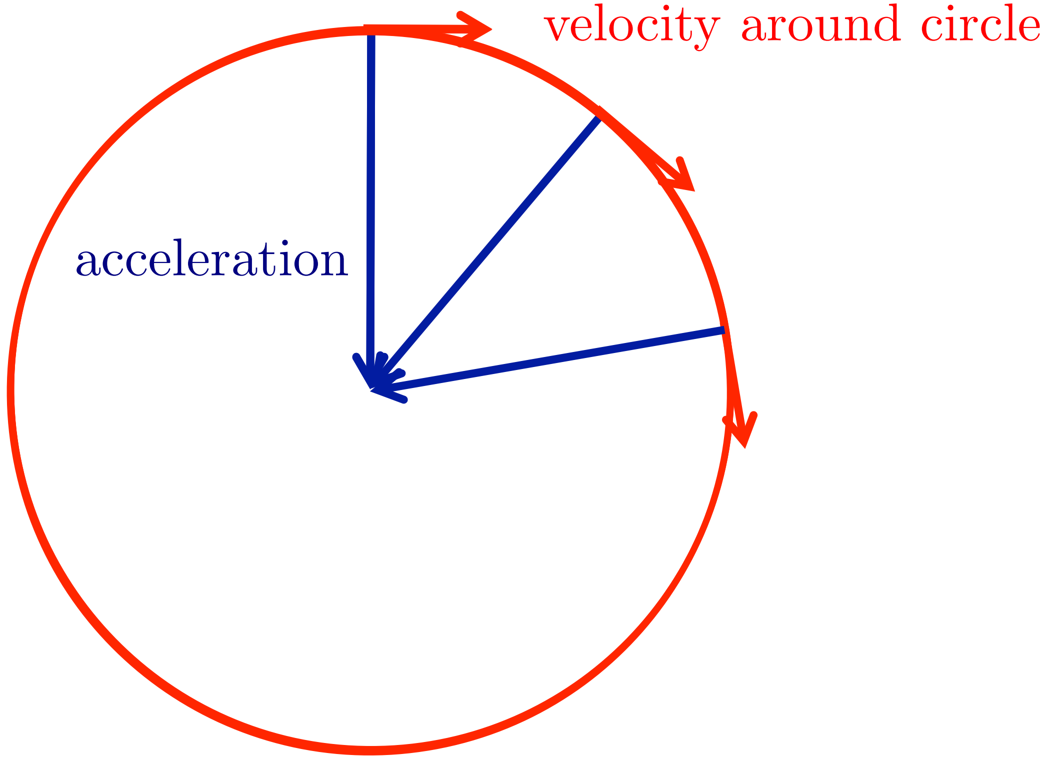
Centripetal Acceleration



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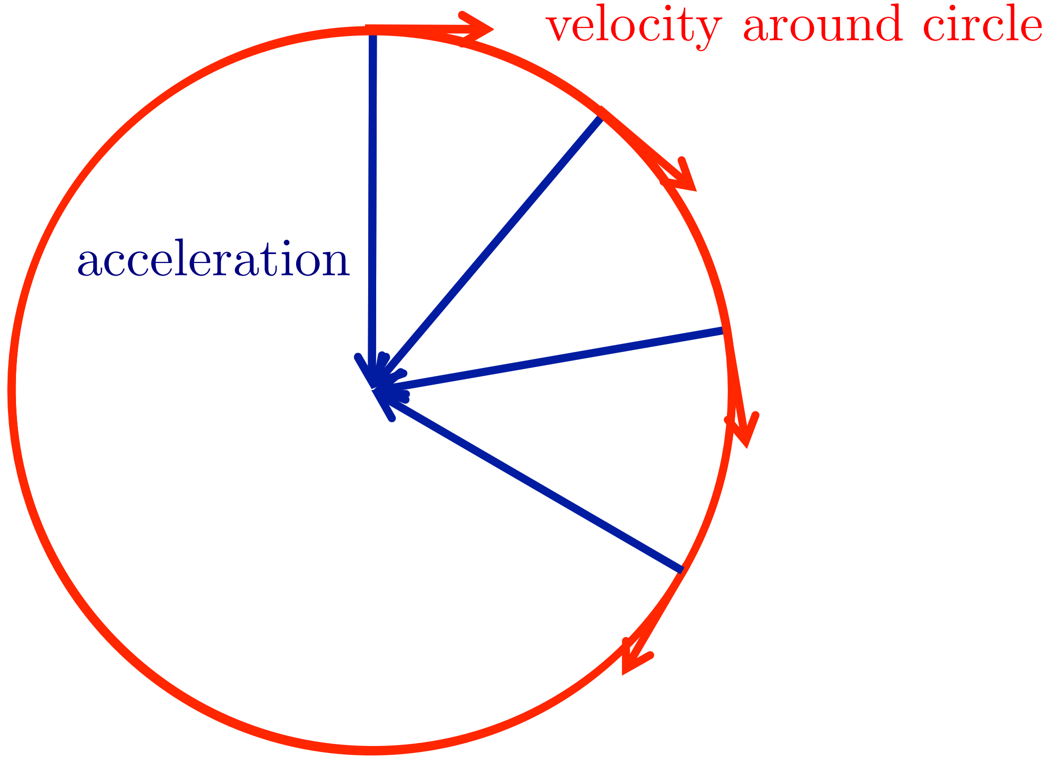
Centripetal Acceleration



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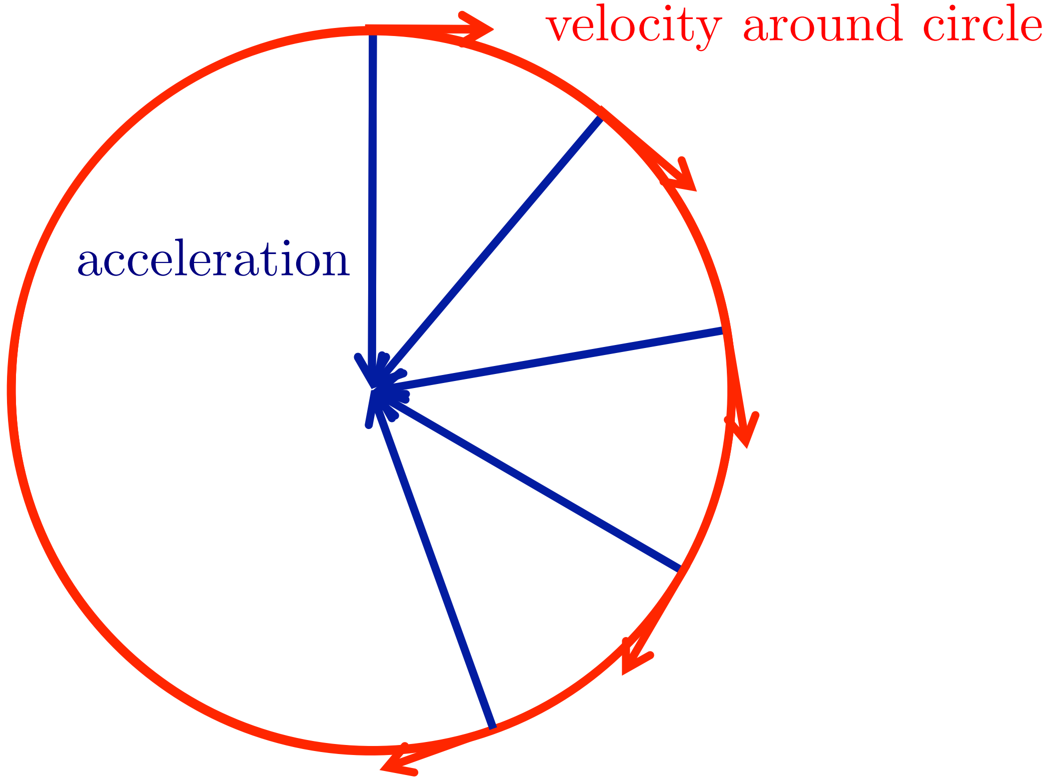
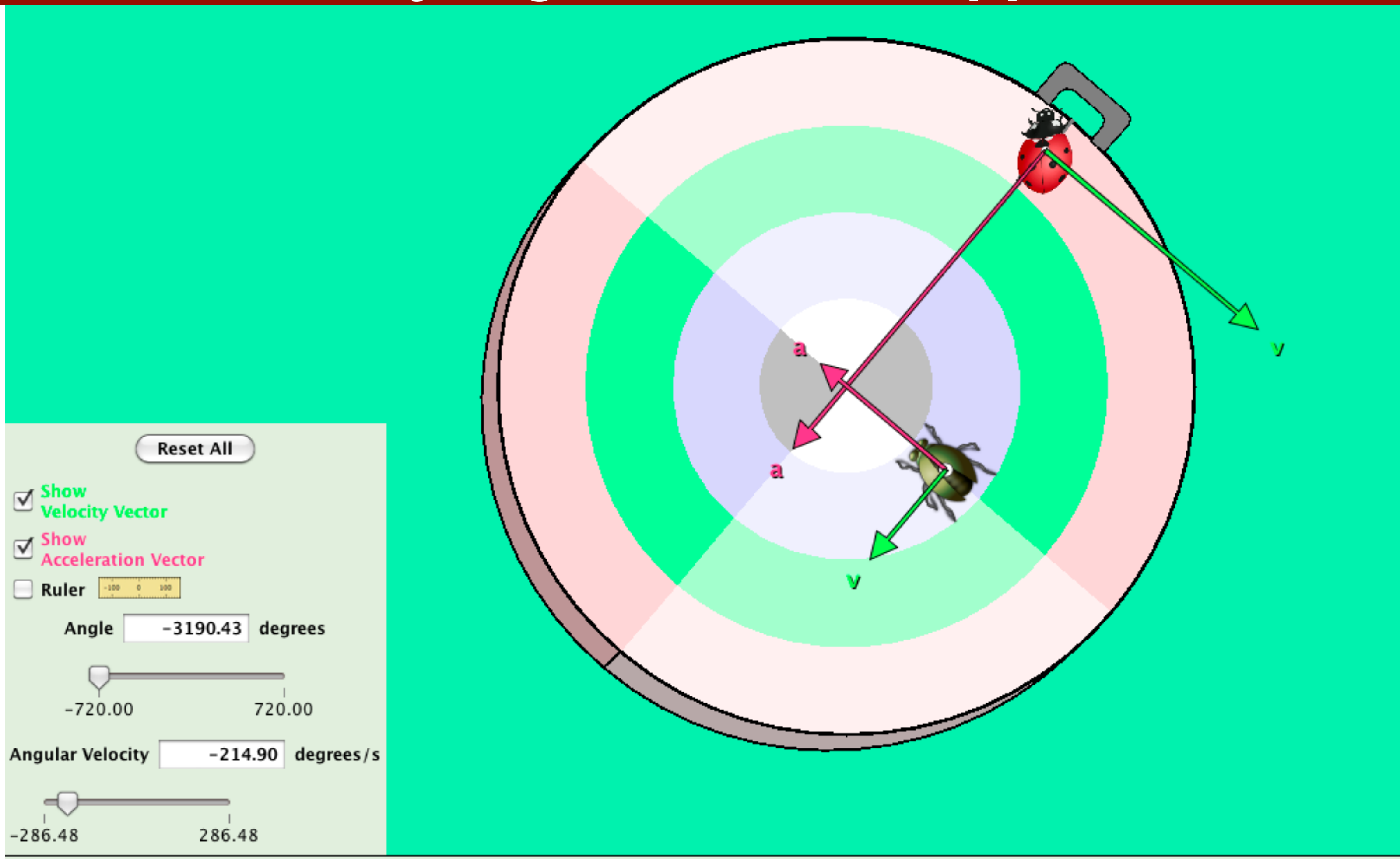
Centripetal Acceleration



- For an object moving with constant speed around a circle
  - The acceleration magnitude is constant but its direction is changing with time
  - Acceleration is always pointed towards the center of the circle
  - Applet time: http://phet.colorado.edu/en/simulation/rotation

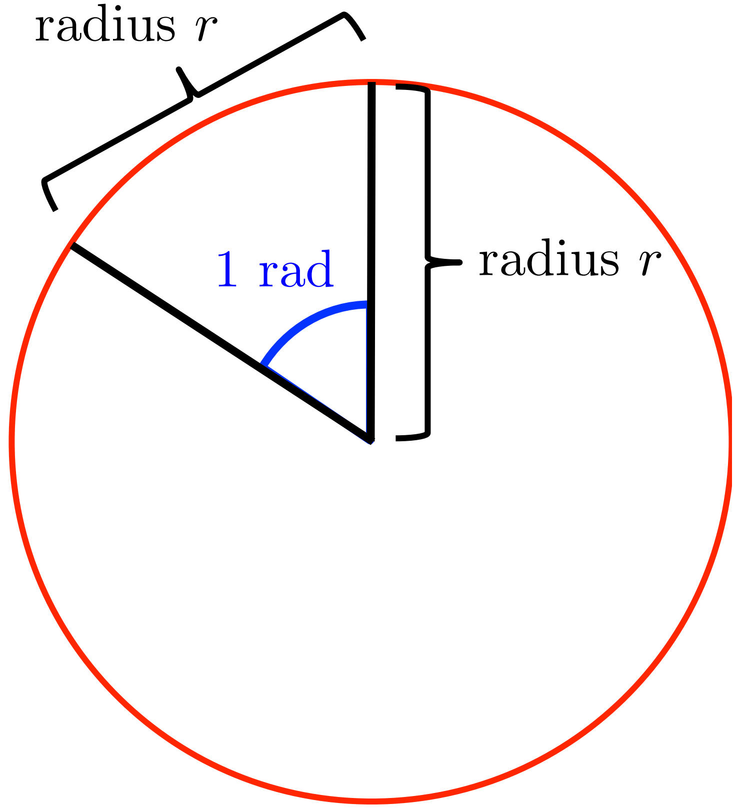
Ladybug Revolution Applet



- Place the bugs on the disk about here
  - Adjust the angular velocity to -215 degrees/sec to see this pic
  - Differences in magnitudes **and** directions of velocity **and** acceleration
  - Does this seem to follow $a=v^2/r$?

Circular Motion: Circle Math

- We need language and math to describe circular motion
  - The radius of the circle determines a "length scale"
  - The other direction is "around" the circle on the circle's arc
- We define a **new unit of angle**
  - An angle of one radian intercepts a semicircular arc of length r
  - So there are 2p radians in 360°



$$1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ$$

- More generally

$$\text{arc length} = r\,\theta(\text{in radians})$$

- So, e.g. one rpm angular velocity is a velocity of 2pr/minute.

Ponderable (10 minutes)

- Todd gets nostalgic and spins up his old **45 RPM** (revolutions per minute) record collection. Each record has a **7 inch total diameter**.
  - How fast is the outermost edge of the album moving in inches/sec?
  - How many "gees" of acceleration does a bug on the edge feel? (g=32 feet/s^2=384 inches/s^2)

Pendulum: Non-constant acceleration

- (see demo on the white board)

Pendulum: Non-constant acceleration

https://en.wikipedia.org/wiki/Acceleration#/media/File:Oscillating_pendulum.gif