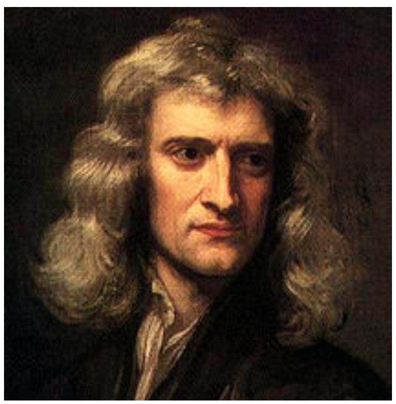
University Physics 226N/231N
Old Dominion University

More Circular Motion, then Newton's Laws



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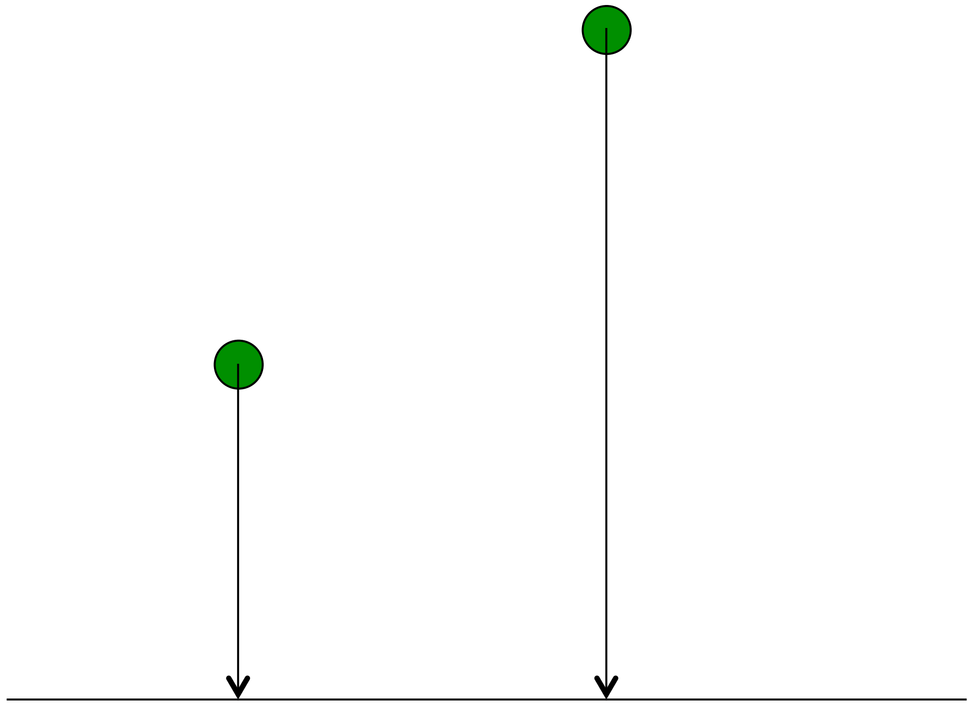
Happy Birthday to Amy Winehouse, Walter Koenig,
Michael Crabtree, and Lawrence Klein (1980 Nobel)!
Happy National Live Creative Day and National Cream Filled Donut Day!

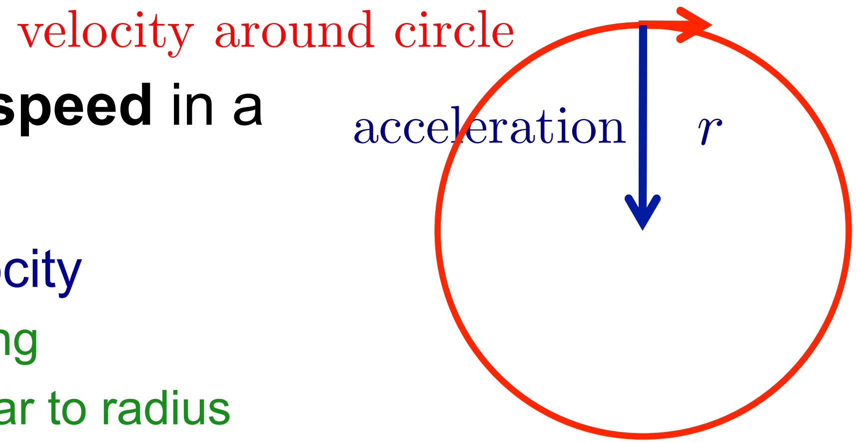
Please set your cell phones to "vibrate" or "silent" mode. Thanks!

Homework Quick Review

- Two identical stones are dropped from rest and feel no air resistance as they fall. Stone A is dropped from height *h*. Stone B is dropped from height *2h*. If stone A takes time *T* to reach the ground, stone B will take time…
  - *A) $T / 2$*
  - *B) $2T$*
  - *C) $4T$*
  - *D) $T / \sqrt{2}$*
  - *E) $T\sqrt{2}$*



Review: Circular Motion



- Objects moving at constant **speed** in a circular arc are:
  - Not moving at constant velocity
    - velocity *direction* is changing
    - velocity points perpendicular to radius
  - Moving in a state of constant *magnitude* of acceleration
    - Where acceleration is pointing towards the center of the circular arc
  - Magnitude of the radial "centripetal" acceleration is

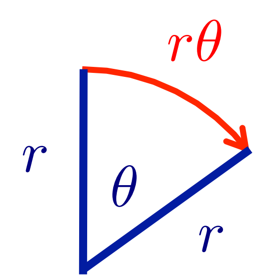
$$a = \frac{v^2}{r}$$

- Circular motion uses three angular units

$$1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ$$

- where

$$\text{arc length} = r\,\theta(\text{in radians})$$



Circular Motion: Circle Math

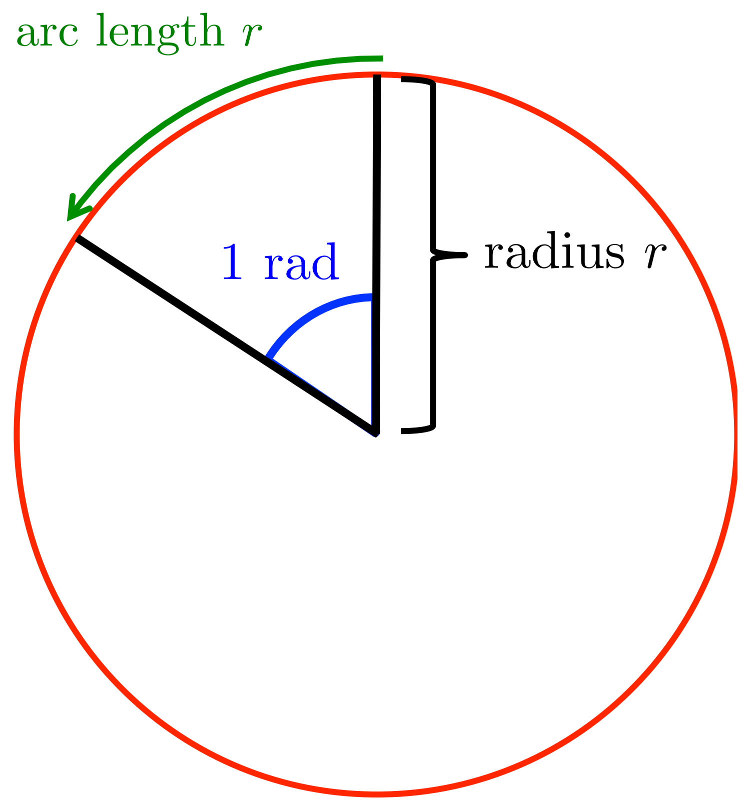
- We need language and math to describe circular motion
  - The radius of the circle determines a "length scale"
  - The other direction is "around" the circle on the circle's arc
- We define a **new unit of angle**
  - An angle of one radian intercepts a semicircular arc of length r
  - So there are 2π radians in 360°

$$1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ$$

- More generally

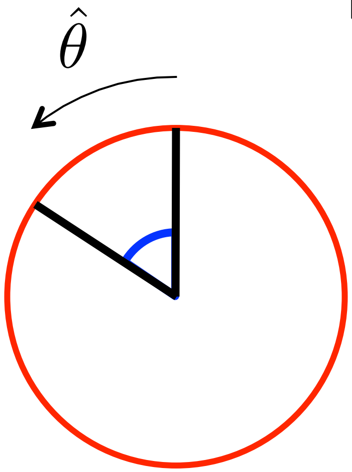
$$\text{arc length} = r\,\theta(\text{in radians})$$

- So, e.g. one rpm angular velocity is a velocity of 2πr/minute.



Circular Motion Angles: Examples

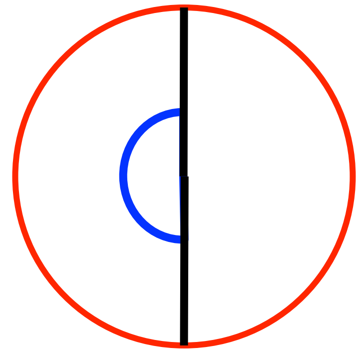
$$1 \text{ revolution} = 2\pi \text{ radians} = 360^\circ$$



$$1 \text{ rad}$$

$$= \frac{360^\circ}{2\pi} = 57.3^\circ$$

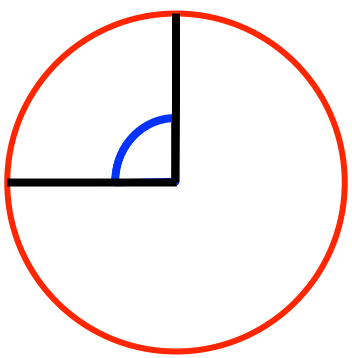
$$= \frac{1 \text{ rev}}{2\pi} \approx 0.159 \text{ rev}$$



$$180^\circ$$

$$= \pi \text{ rad}$$

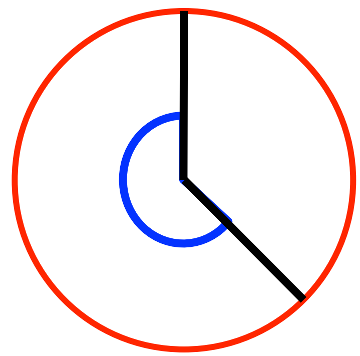
$$= \frac{1}{2} \text{ rev}$$



$$90^\circ$$

$$= \frac{\pi}{2} \text{ rad}$$

$$= \frac{1}{4} \text{ rev}$$

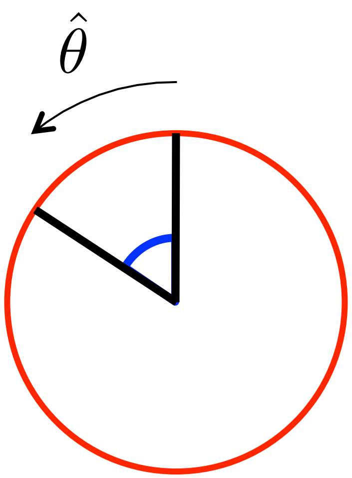


$$225^\circ$$

$$= \frac{5\pi}{4} \text{ rad}$$

$$= \frac{5}{8} \text{ rev}$$

Angle and Distance



$\text{arc length} = r\,\theta(\text{in radians})$

- Drawing $\hat{\theta}$ is treating it like a coordinate unit vector
  - Positive and negative vector directions

- We can now treat angle as a coordinate for rotational motion
  - "+33 revolutions" means $\Delta\theta = 33$ revolutions
  - Total distance Δs is related to circle radius *r*
    - (*s* is usually used to represent length of curve)

$$\Delta s = r\Delta\theta \quad (\Delta\theta \text{ in radians!})$$

- How angle changes with time is a new type of velocity – now we can do angular kinematics
  - We will get into that in Chapters 9-10

Example: Curved Road Acceleration

- The maximum lateral acceleration a certain car can handle without skidding is 0.80g. What is the maximum speed the car can go around an unbanked circular turn with radius 30m without skidding?

- Maximum acceleration is 0.80g = 0.80(9.8 m/s^2)=7.84 m/s^2
- For circular motion

$$a = \frac{v^2}{r} \qquad v = \sqrt{ar}$$

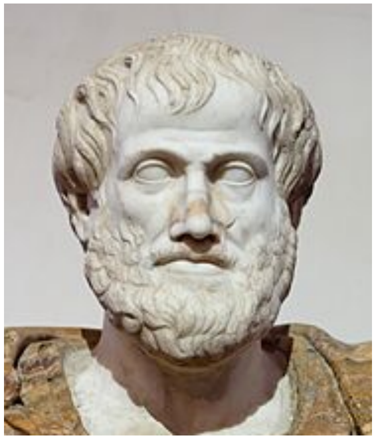
$$v = \sqrt{(7.84\ \mathrm{m/s^2})(30\ \mathrm{m})} = 15.3\ \mathrm{m/s} \quad \rightarrow \quad \boxed{15\ \mathrm{m/s} = v}$$

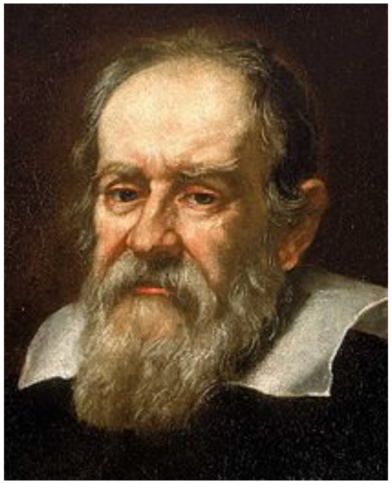
This is about 34 miles/hr. Be careful!

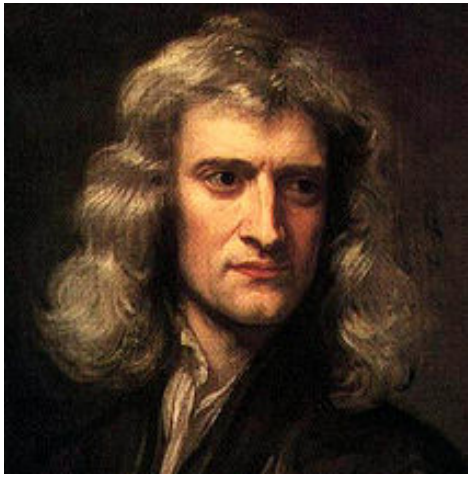
In the Footsteps of….

- We've been describing motion with three related attributes
  - **Position** in space: $\vec{x}$
  - **Velocity**, how position changes with time: $\vec{v} = \Delta\vec{x}/\Delta t = d\vec{x}/dt$
  - **Acceleration**, how velocity changes with time: $\vec{a} = \Delta\vec{v}/\Delta t = d\vec{v}/dt$
  - These are all **vectors** (magnitude and direction) and have **components**

- Now we'll revisit a ponderable and walk in the footsteps of giants…



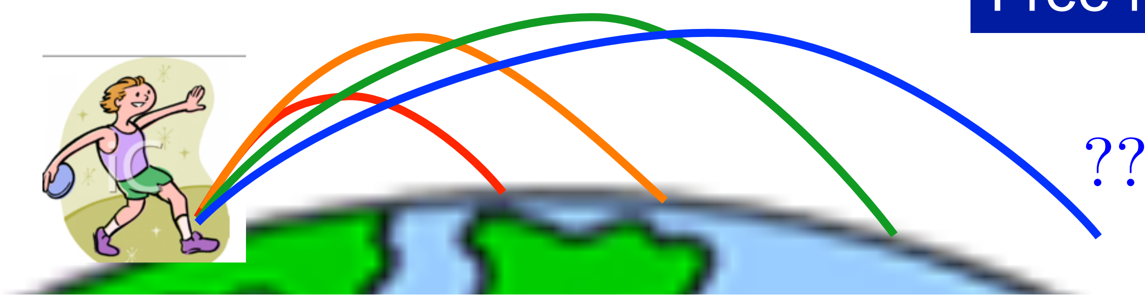




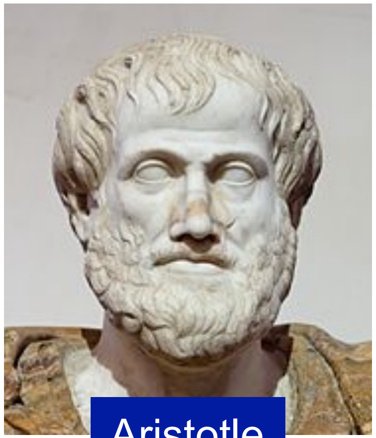
Prior Ponderable: Falling Around The Earth

- Taking projectile motion to the extreme…
  - The Earth is not flat: it is not "level ground" forever
  - Experience tells us that gravity always points towards the center of the Earth, wherever I am on the Earth
  - So it might be possible to shoot something really small and aerodynamic fast enough to "miss" the Earth even while the acceleration of gravity continuously makes it "fall"

Free fall



"Aristotle, Aristotle…"



Aristotle

- Aristotle's world view: my object will always eventually stop
  - Forces are continuously required for objects to move
  - That is, **forces are required to maintain velocity**
  - An object's "natural" state is at rest and it will always return to being at rest if forces don't keep it moving
- These are fairly reasonable statements
  - Our world is full of what we now know as frictional forces
  - Even people need to constantly exert themselves to keep going
- **But** there are some problems with this philosophy of physics

Ponderable: The Greek Aristotelian World (5 min)

- **All physics is based on observations of our universe**
  - Think of some observations that might have been accessible to the Greeks that create problems for Aristotle's world view

    Aristotle: "Force is required for motion" (motion=velocity)

  - Motion without forces? Different motion with same "force"?
  - Aristotle also believed **heavier** objects would fall **faster**
    - After all, they clearly have a larger attraction to the Earth!

Ponderable: The Greek Aristotelian World

- **All physics is based on observations of our universe**

  Aristotle: "Force is required for motion" (motion=velocity)

- Some challenges include
  - Same object behaving differently (e.g. rock on ice vs on rock)
    - Objects with different weights falling same distance in same time
  - Anything with force "at a distance": gravity and magnets
  - The perpetual motion of the sun, moon, planets, stars
    - If they move forever, what's "pushing" them?

“I call on the resting soul of Galileo…”

- A “thought experiment” (1589)
  Two different weight balls
  - Dropped separately
  - Dropped tied together

- **Are they fundamentally different?**
  (Don’t drop them on people’s heads!)

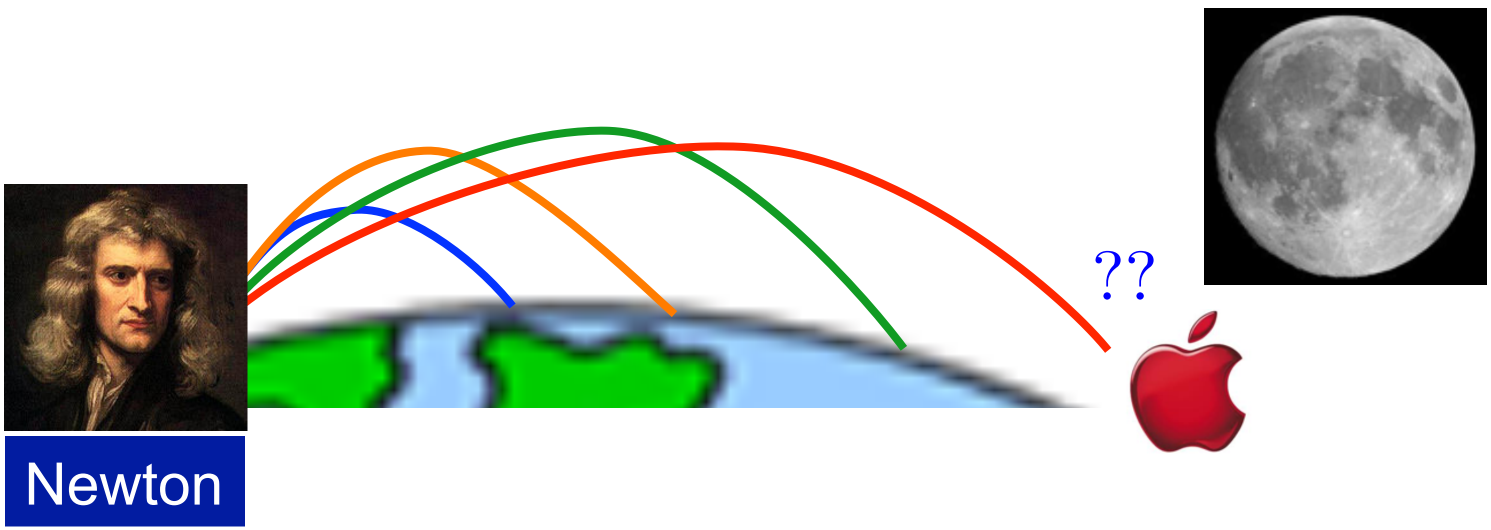
- The different forces of weight are balanced by different resistance to motion
  - All objects drop in same **time**
    - (Ignoring air resistance)
  - These led to early concepts of **Inertia** and **Mass**
    - Innate resistance to changes in motion

Galileo

Clock!

Prior Prior Ponderable: Falling Around The Earth

- Though the story about Newton's apple is apocryphal…
  - He did think about our ponderable about objects falling forever
  - He knew Galileo was right and Aristotle was wrong
  - He knew about the Copernican model: Earth goes around Sun
  - He also had a new "technology" to describe motion: **calculus**
- Newton's ponderable
  - **How is an apple thrown around the Earth like the moon?**



Pseudo-Ponderable and Break Time (10 min)

- We had figured out the speed we need to throw an object "around" the earth, using centripetal acceleration and gravity

$$v = \sqrt{-g\, r_e} \approx 7.9\ \mathrm{km/s}$$

- Assuming the moon is "falling" around the Earth, using just this and your knowledge that moon's orbital period is about 656 hours (27 days 7 hours)

Calculate the distance to the moon

The circumference of the orbit is the velocity times the orbital time…

Don't bother trying to look up this answer since this answer is wrong ☺

Back to Galileo and Newton

- We can resolve the inconsistencies!

- **Forces do not cause velocity**
- **Forces instead cause changes in velocity**
  - Hey, wait, this is just acceleration

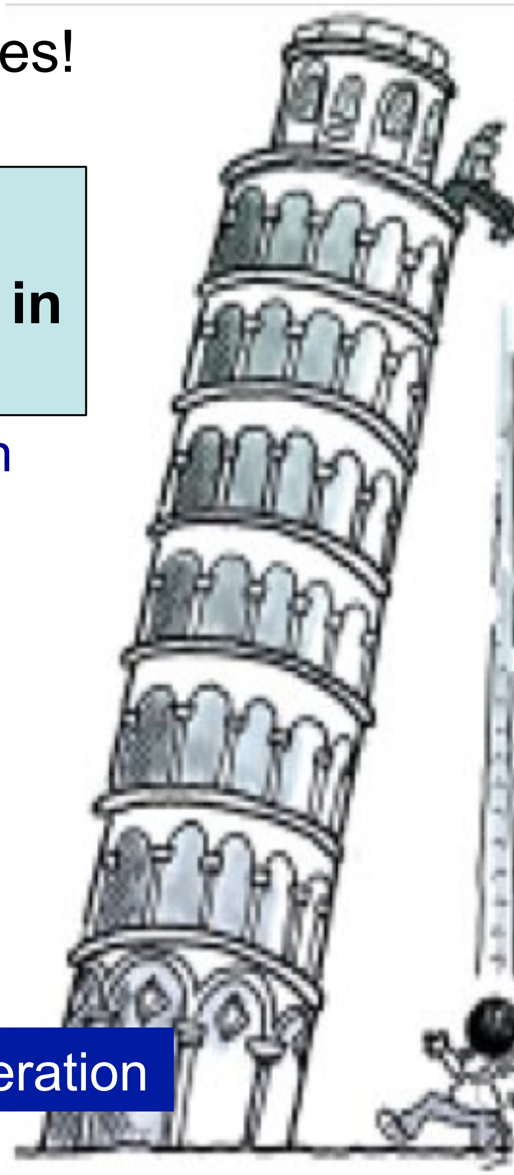
- Yes, **forces are vectors** that are directly related to **acceleration**

$$\vec{F}_{\text{net}} = m\vec{a}$$

Net Force

Mass or Inertia

Acceleration



Galileo

Clock!

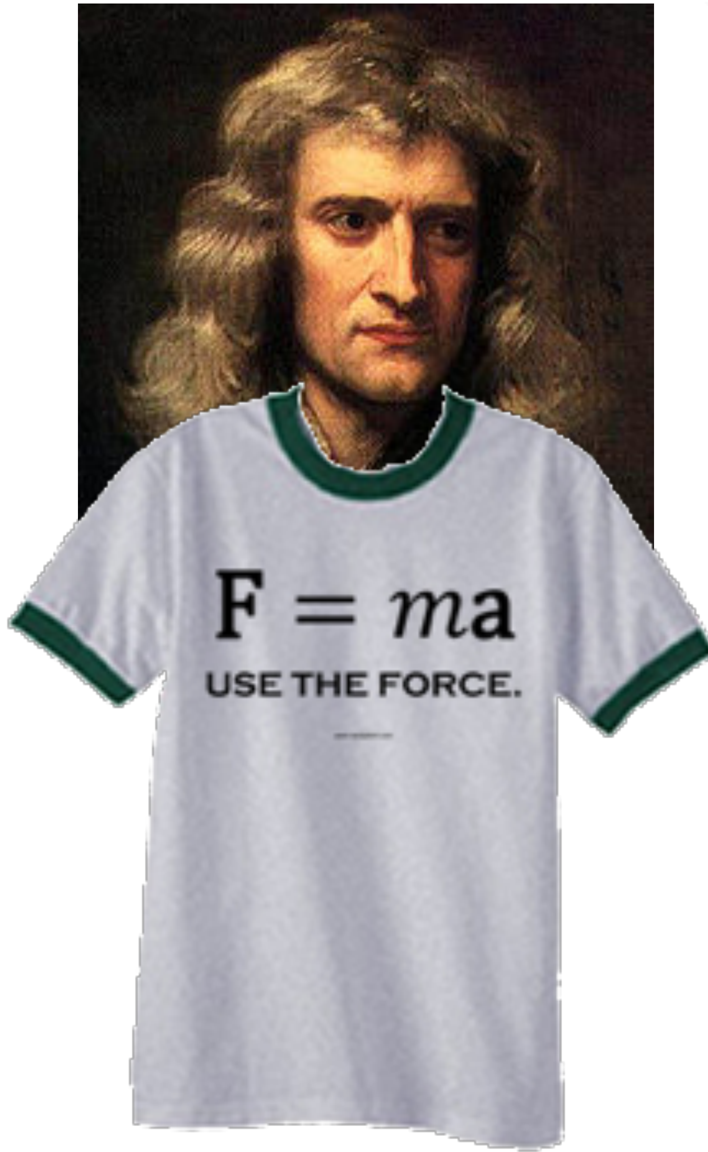
Use the Force, Newt!

- Newton's three "laws" of motion (1687)
- Newton's First Law

  A body in uniform motion remains in uniform motion, and a body at rest remains at rest, unless acted on by a nonzero net force.

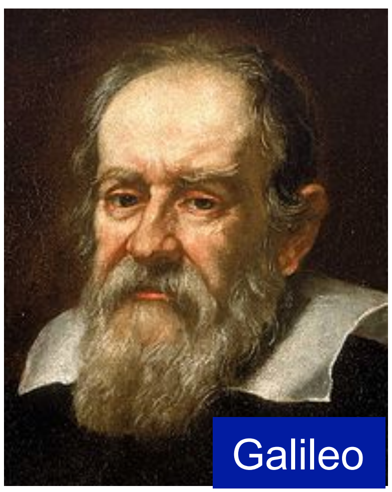
- Newton's Second Law
  - This was basically $\vec{F}_{\text{net}} = m\vec{a}$
- Newton's Third Law

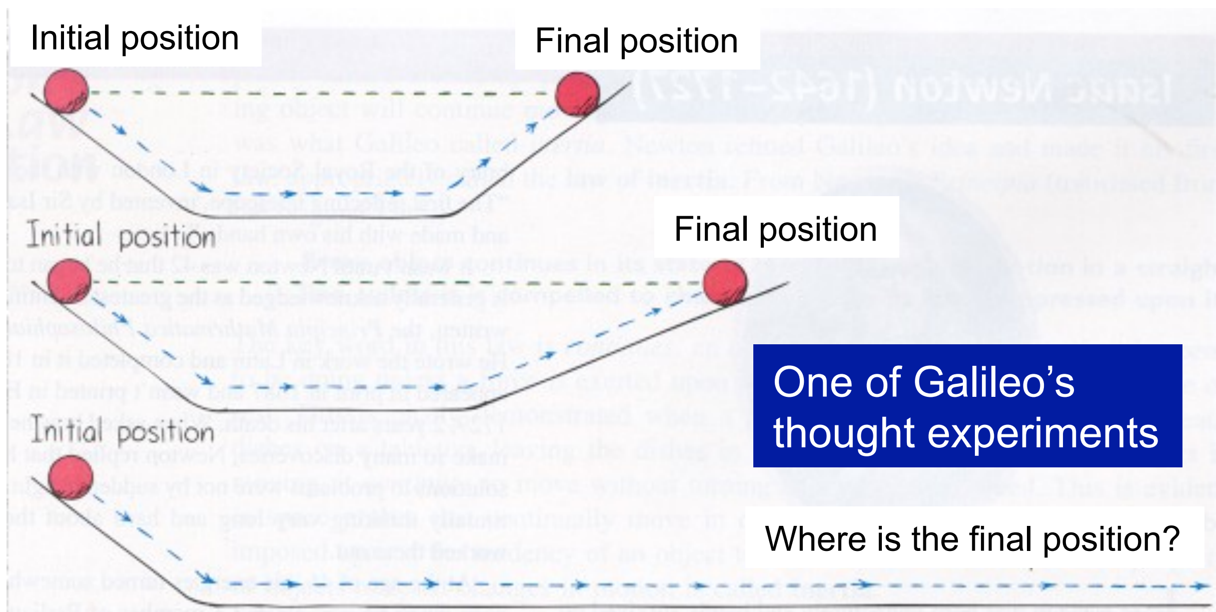
  If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.



Newton’s First Law

- Newton’s first law is a special case of the second law, when there’s no net force acting on an object, that is, when $\vec{F}_{\text{net}} = 0$
  - With no net forces, an object’s motion (velocity) doesn’t change.
  - If at rest it remains at rest. If in motion, it remains in uniform motion.
    - Uniform motion is motion at constant velocity in a straight line.
    - Thus the first law shows that uniform velocity is a natural state, requiring no explanation.


Galileo



Newton's Second Law

- The second law tells quantitatively how force causes changes in an object's "quantity of motion."
  - Newton defined "quantity of motion," now called **momentum**, as the product of an object's mass and velocity:

$$\vec{p} = m\vec{v}$$

Definition of momentum

  - Newton's second law equates the rate of change of momentum to the net force on an object:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{d\vec{p}}{dt}$$

Yes, that's a derivative

  - When mass is constant, Newton's second law becomes

$$\vec{F} = \frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

  - The force required to accelerate a 1-kg mass at the rate of 1 m/s^2 is defined to be 1 **Newton** (N).

Momentum

- You have everyday experience with momentum $\vec{p} = m\vec{v}$

If both are moving with same speed…

… the more massive object has higher momentum: which inflicts more damage in a crash?

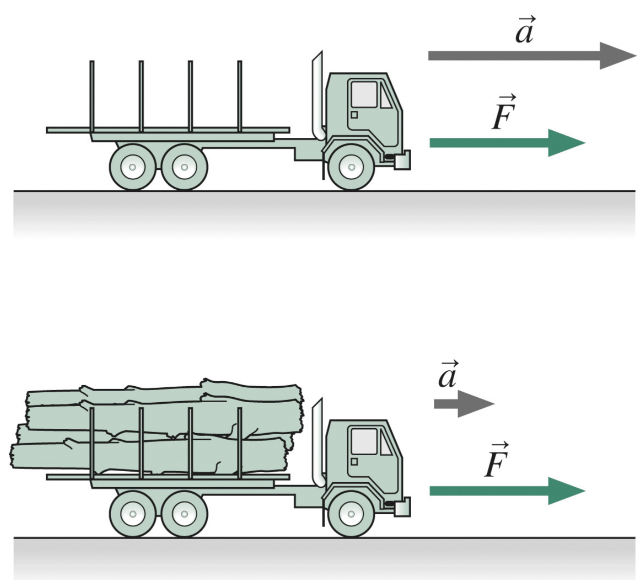
- Later we'll get into **conservation laws**
  - **Conservation of momentum** seems to be a **fundamental law**
  - **Conservation of energy** also seems to be a **fundamental law**
- Newton's second law lets us figure out motion even when mass changes
  - What happens when things fall apart or stick together when moving?

Mass, Inertia and Force

- If we solve the second law for the acceleration we find that

$$\vec{a} = \frac{\vec{F}}{m}$$

  showing that a given force is less effective in changing the motion of a more massive object.
  - The mass *m* that appears in Newton's laws is thus a measure of an object's **inertia** and determines the object's response to a given force.



- From Newton's second law for a force of magnitude *F*,

$$\vec{F} = m_{\text{known}}\vec{a}_{\text{known}}, \qquad \vec{F} = m_{\text{unknown}}\vec{a}_{\text{unknown}}$$

we get

$$\frac{m_{\text{unknown}}}{m_{\text{known}}} = \frac{a_{\text{known}}}{a_{\text{unknown}}}$$

Mass, Weight, and Gravity

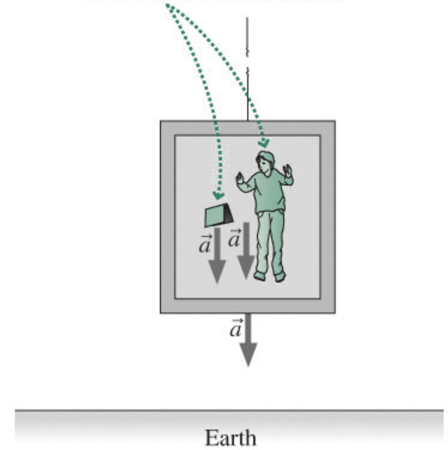
- **Weight** is the force of gravity on an object:

$$w = mg$$

  - Mass doesn't depend on the strength of gravity.
  - Weight depends on gravity, so varies with location:
    - Weight is different on different planets.
    - Near Earth's surface, $\vec{g}$ has magnitude 9.8 m/s^2 or 9.8 N/kg, and is directed downward.

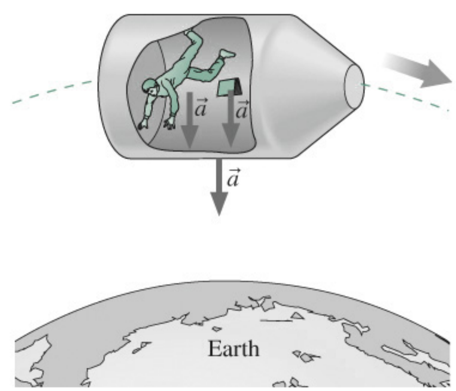
- All objects experience the same gravitational acceleration, regardless of mass.
  - Therefore objects in **free fall** with an observer (under the gravity alone) appear **weightless** (**not** massless) because they share a common accelerated motion.
  - This effect is noticeable in orbiting spacecraft
    - because the absence of air resistance means gravity is the only force acting.
    - because the apparent weightlessness continues indefinitely, as the orbit never intersects Earth.

In a freely falling elevator you and your book seem weightless because both fall with the same acceleration as the elevator.



(a)

Like the elevator in (a), an orbiting spacecraft is falling toward Earth, and because its occupants also fall with the same acceleration, they experience apparent weightlessness.



(b)

Newton's Second Law: Example

- A 740-kg elevator accelerates upward at 1.1 m/s^2, pulled by a cable of negligible mass. Find the tension force in the cable.
  - The object of interest is the elevator; the forces are gravity and the cable tension.
  - Newton's second law reads

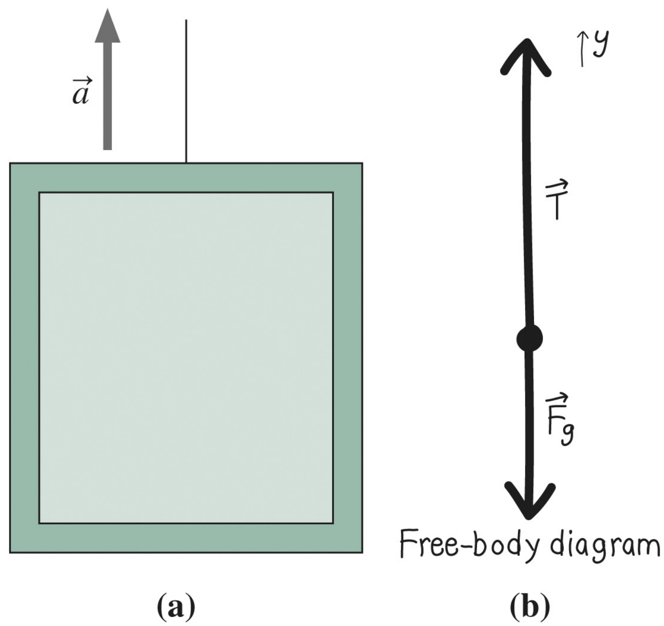
$$\vec{F}_{\text{net}} = \vec{T} + \vec{F}_g = m\vec{a}$$

  - In a coordinate system with *y*-axis upward, Newton's Second Law is

$$T_{\text{y}} + F_{\text{gy}} = ma_{\text{y}}$$

Solving gives

$$T_{\text{y}} = m(a_{\text{y}} + g) = 8.1\ \text{kN}$$



  - Does this make sense? Let's look at some cases:
    - When $a = 0$, $T = mg$ and the cable tension balances gravity.
    - When $T = 0$, $a = -g$, and the elevator falls freely.

Pseudo-Ponderable

- A non-zero net force acts on an object. Does that mean the object necessarily is moving in the same direction as the net force?

A. Yes

B. No

Pseudo-Ponderable

- A non-zero net force acts on an object. Does that mean the object necessarily is moving in the same direction as the net force?

A. Yes
B. No

- The English term for this is “deceleration”
- A net force (and thus an acceleration) can be in the opposite direction (or **any** direction) of the object’s motion (or velocity)
- Examples include
  - An elevator slowing down as it reaches a top or bottom floor
  - Braking in a moving vehicle
  - Circular motion (where the net force is towards the center of the circle, but velocity is around the circle)

End-of-Class Ponderable

- Someone is holding a Slinky by one end and letting the other end dangle down near the ground
- At this point the Slinky is in equilibrium – all forces balance, the net force is zero, and the Slinky does not accelerate
- So… **What happens when he lets the Slinky go? How does the overall slinky move?**

See a movie at Vertasium
Read about it at http://n.pr/QhyxTT