University Physics 226N/231N
Old Dominion University

Newton's Laws Examples and Statics

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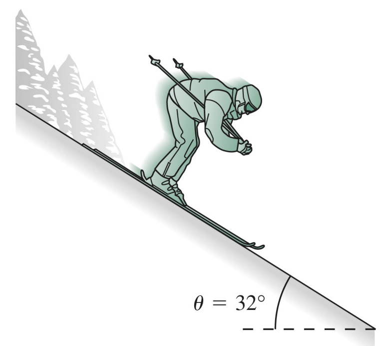


Monday, September 26, 2016

Reminder: The Second Midterm will be Weds Oct 19 2016

Happy Birthday to Olivia Newton-John, Martin Heidegger, Winsor McCay,
Johnny Appleseed, Serena Williams, and T.S. Eliot (1948 Nobel)!
Happy National Pancake Day and National Situational Awareness Day!

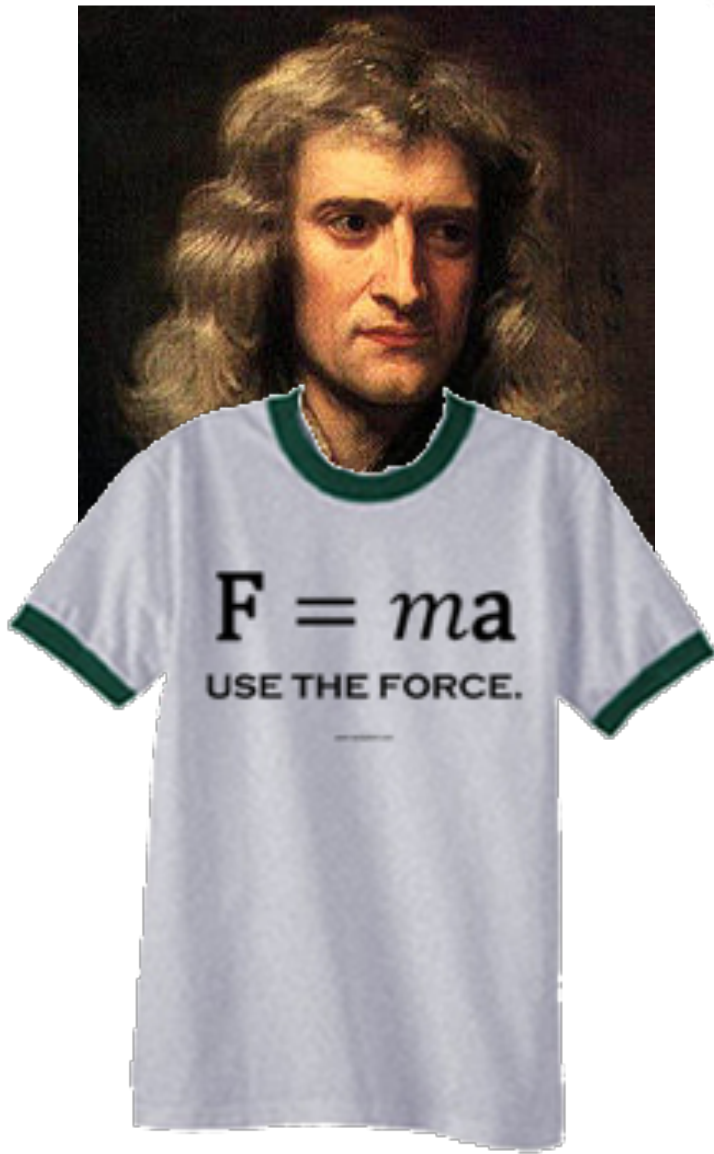
Please set your cell phones to "vibrate" or "silent" mode. Thanks!

Review: Use the Force, Newt!

- Newton's three "laws" of motion (1687)
- Newton's First Law

  A body in uniform motion remains in uniform motion, and a body at rest remains at rest, unless acted on by a nonzero net force.
- Newton's Second Law
  - This was basically $\vec{F}_{\mathrm{net}} = m\vec{a}$
- Newton's Third Law

  If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.



Review: Gravity is a Force.. Gravity is a Force…

- Kinematics: calculate with gravitational acceleration
  - Projectiles etc are free-falling objects
  - A simplified version of F=ma with one force, gravity

- From now on we will treat gravity as a **FORCE**
  - The force of gravity on an object is

$$\vec{F} = m\vec{g}$$

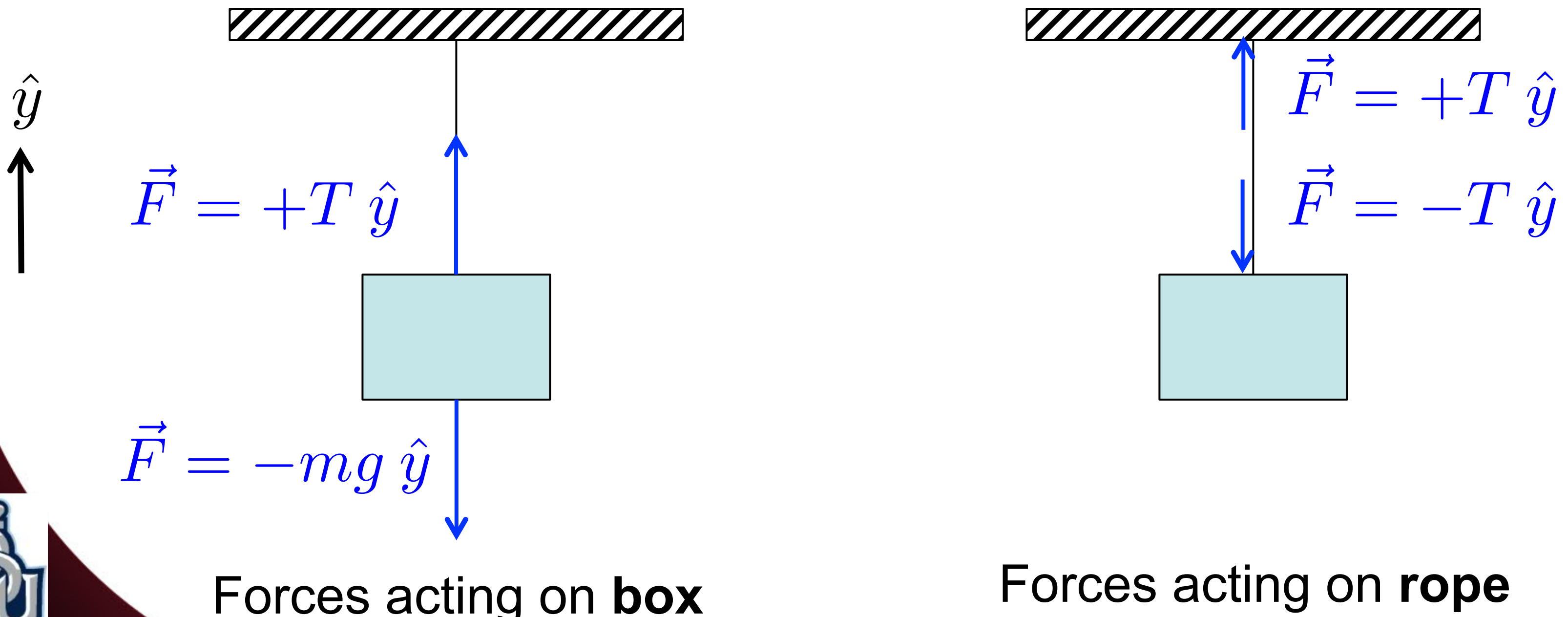
  - It is quite possibly one of **many** forces acting on an object
  - You should include it as a force when you perform calculations with Newton's 2nd Law

$$\vec{F}_{\rm net} = m\vec{a}$$

  - Do NOT include it on the right hand side with acceleration!
    - Trying to do so can be very confusing and quite possibly wrong

Review: Newton's Second Law and Force Diagrams

- Recall Newton's second law: $\vec{F}_{\rm net} = m\vec{a}$
- All these variables refer to the forces, mass, and resulting acceleration vector of a **single object**
- You must be consistent in calculating these for the same single object
- **Example:** box hanging from a rope at rest: $\vec{a}_{\rm box} = \vec{a}_{\rm rope} = 0\ {\rm m/s^2}$



Review: Newton's Second Law

- A 740-kg elevator accelerates upward at 1.1 m/s^2, pulled by a cable of negligible mass. Find the tension force in the cable.
  - The object of interest is the elevator; the forces are gravity and the cable tension.
  - Newton's second law reads

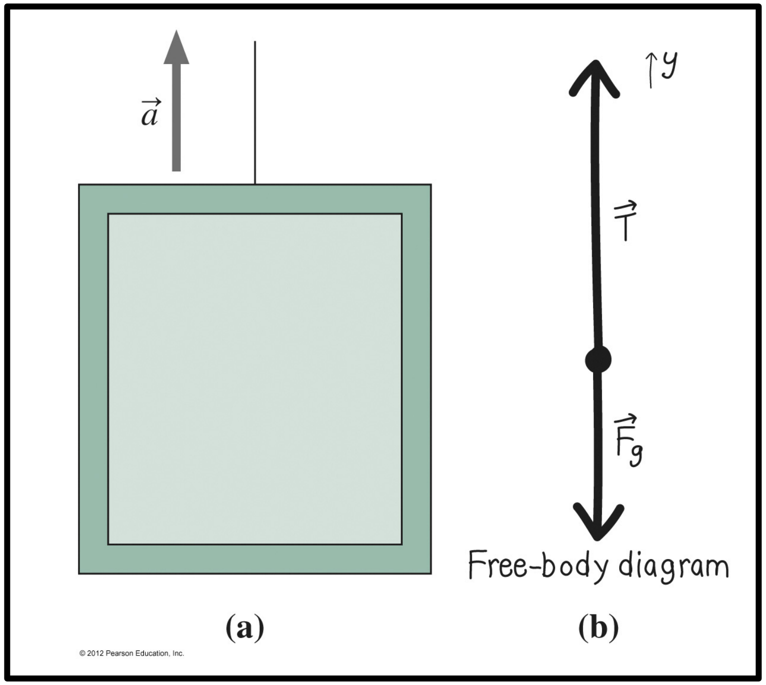
$$\vec{F}_{\rm net} = \vec{T} + \vec{F}_g = m\vec{a}$$

  - In a coordinate system with *y*-axis upward, Newton's Second Law is

$$T_y - F_{gy} = T_y - mg = ma_y$$

Solving gives

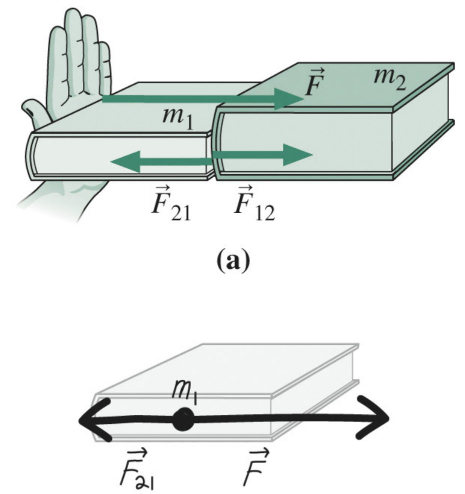
$$T_{\rm y} = m(a_{\rm y} + g) = 8.1 \text{ kN}$$



  - This tension is **larger** than the weight of the elevator
    - The cable tension is supporting the elevator **and** accelerating it

Review: Newton's Third Law

- Forces **always** come in pairs.
  - If object A exerts a force on object B, then object B exerts an oppositely directed force of equal magnitude on A.
  - Obsolete language: "For every action there is an equal but opposite reaction."
  - The two forces always act on *different* objects; they can't cancel each other.
- Example:
  - Push on book of mass m_1 with force $\vec{F}$
  - Note third-law pair $\vec{F}_{21}$ and $\vec{F}_{12}$
  - Third law is necessary for a consistent description of motion in Newtonian physics.

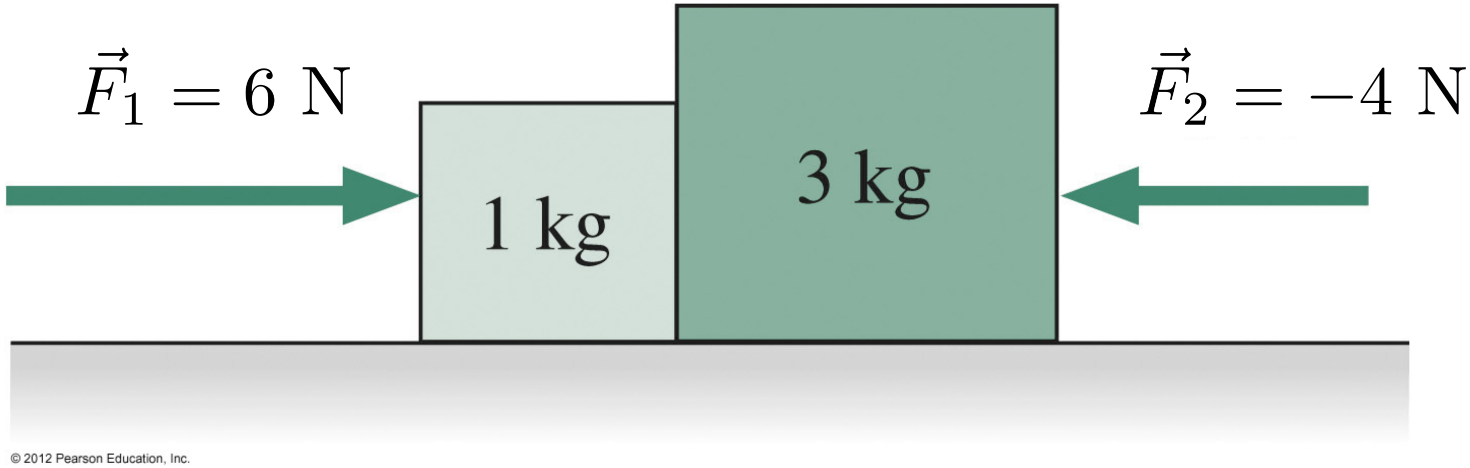


Two horizontal forces act **on the book** m_1

$\vec{F}_{21}$ from book m_2

$\vec{F}$ from hand

Newton's 2nd/3rd Law: Pseudo-Ponderable



This figure shows two blocks with two forces acting on the pair. The net force on the **larger** block is

A. Less than 2 N.
B. Equal to 2 N.
C. Greater than 2 N.

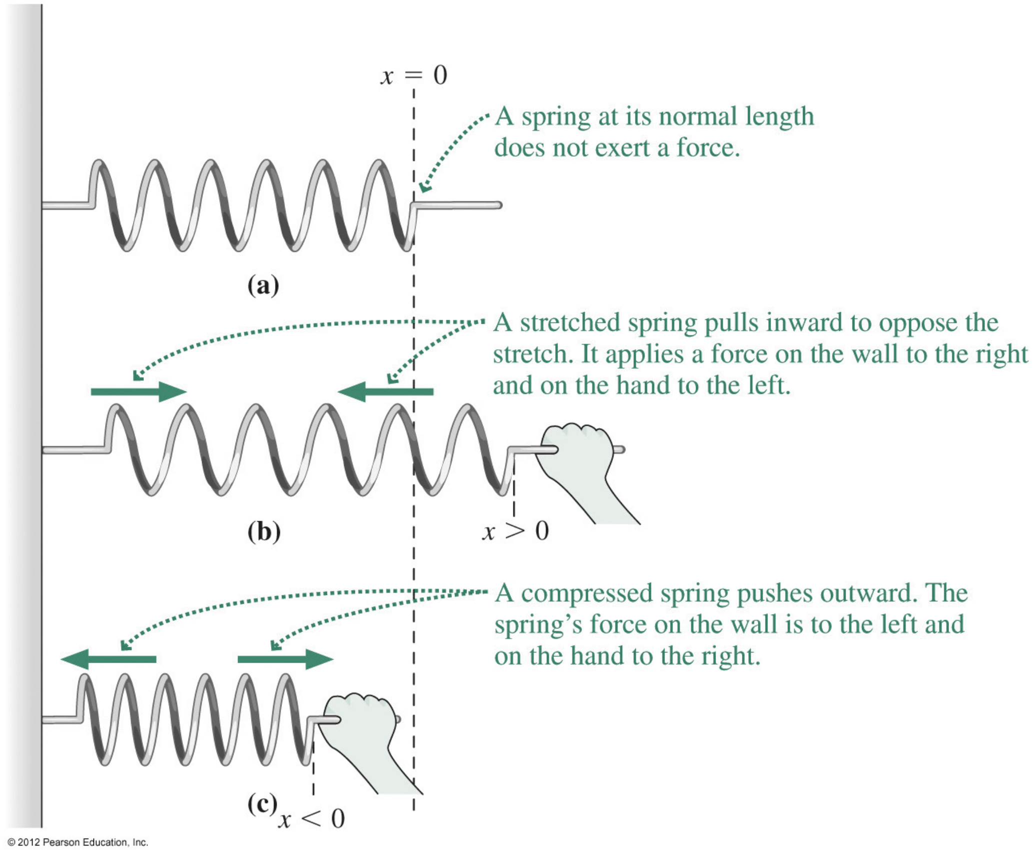
What is the acceleration $\vec{a}$ of each block?

Spring Forces

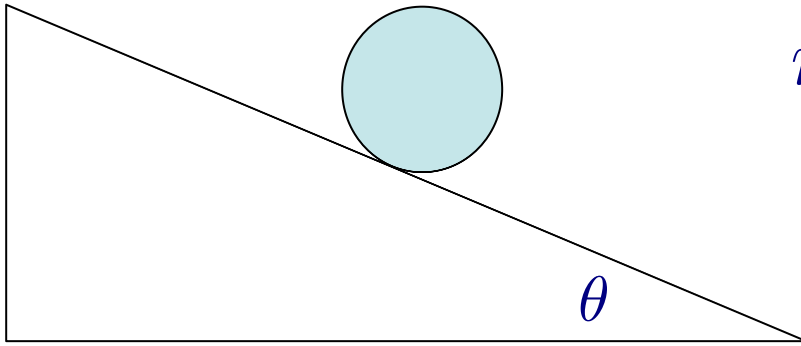
- A stretched or compressed spring produces a force proportional to the stretch or compression from its equilibrium configuration: $\vec{F}_{\text{spring}} = -k\vec{x}$

- The spring force is a **restoring force** because its direction is opposite that of the tension or compression.

- Springs provide convenient devices for measuring force.



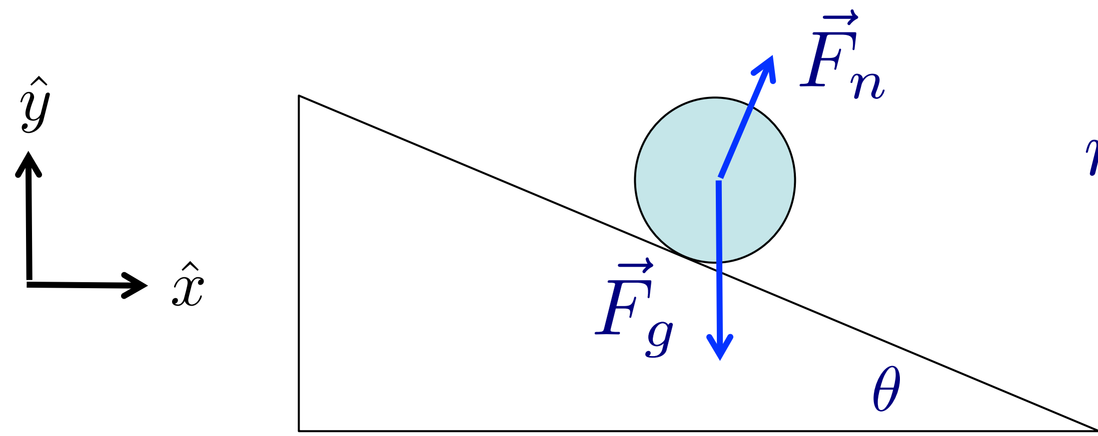
Newton's Second/Third Laws: Tougher Example



$$m = 5.0 \text{ kg}$$
$$\theta = 20^\circ$$

- A small ball of m=5.0 kg is on an inclined plane of 20 degrees
  - Find the ball's acceleration down the plane

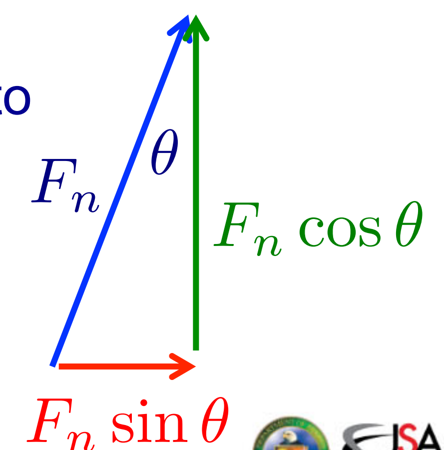
Newton's Second/Third Laws: Tougher Example



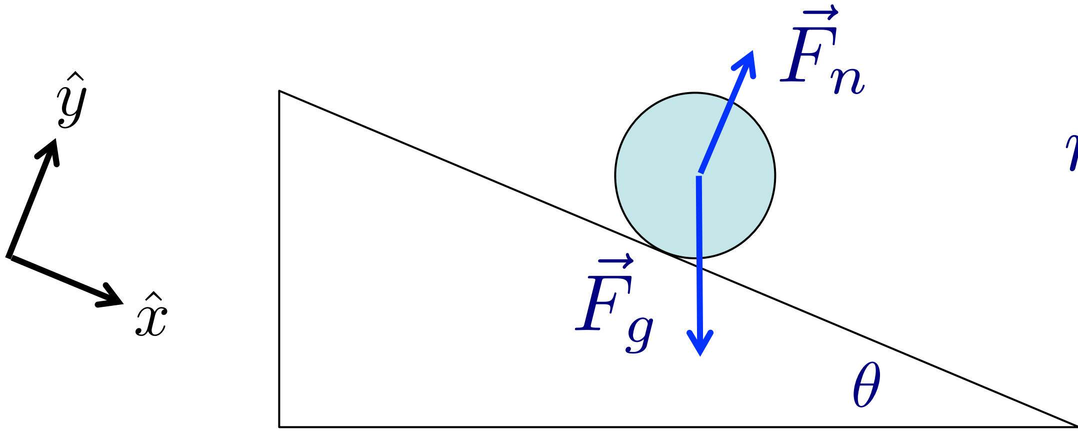
$$m = 5.0 \text{ kg}$$
$$\theta = 20^\circ$$
$$F_g = mg = 49 \text{ N}$$

- A small ball of m=5.0 kg is on an inclined plane of 20 degrees
  - What are all the forces acting on the ball? (no friction for now!)
    - Note that forces are vectors: they have direction and magnitude!
    - Two forces: gravity pointing down and push of plane pointing perpendicular to the surface of the plane.
  - **What are the components of the forces**? One way to look at it is with the x,y axes shown above

$$F_{g,x} = 0 \qquad F_{n,x} = F_n \sin\theta$$
$$F_{g,y} = -F_g \qquad F_{n,y} = F_n \cos\theta$$



Newton's Second/Third Laws: Tougher Example



$$m = 5.0 \text{ kg}$$

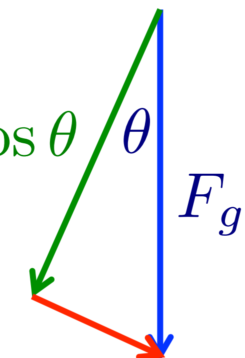
$$\theta = 20^\circ$$

$$F_g = mg = 49 \text{ N}$$

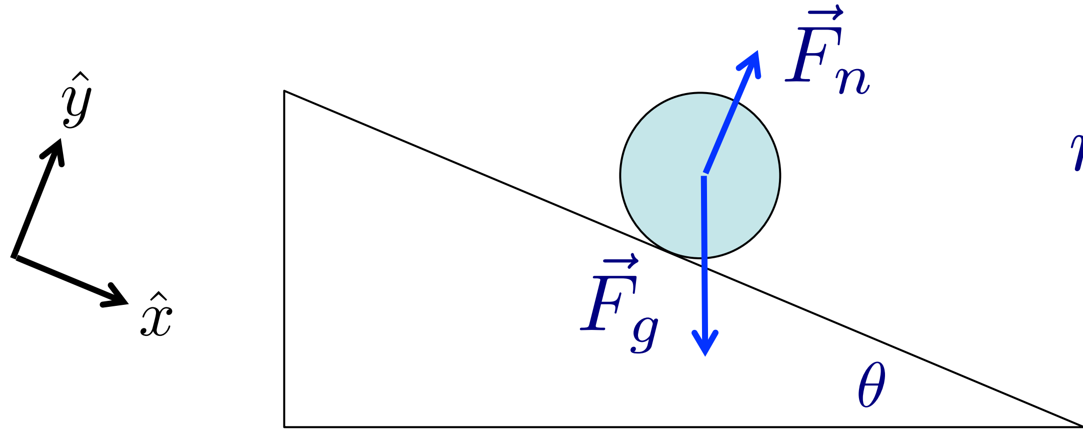
- A small ball of m=5.0 kg is on an inclined plane of 20 degrees
  - **What are the components of the forces**? Another way to look at it is with axes parallel to and perpendicular to the plane
    - This makes the final acceleration easier to calculate – we know that the net force and acceleration are "down the plane"

$$F_{n,x} = 0 \qquad F_{g,x} = F_g \sin\theta$$

$$F_{n,y} = F_n \qquad F_g, y = -F_g \cos\theta$$



Newton's Second/Third Laws: Tougher Example

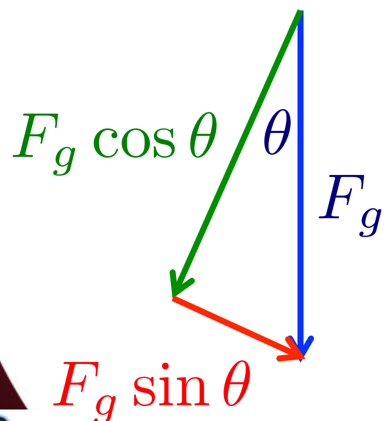


$$m = 5.0 \text{ kg}$$

$$\theta = 20^\circ$$

$$F_g = mg = 49 \text{ N}$$

- A small ball of m=5.0 kg is on an inclined plane of 20 degrees
  - **What is the normal force from the plane**, F_n?



$$F_{g,x} = F_g \sin\theta \qquad F_{n,x} = 0$$

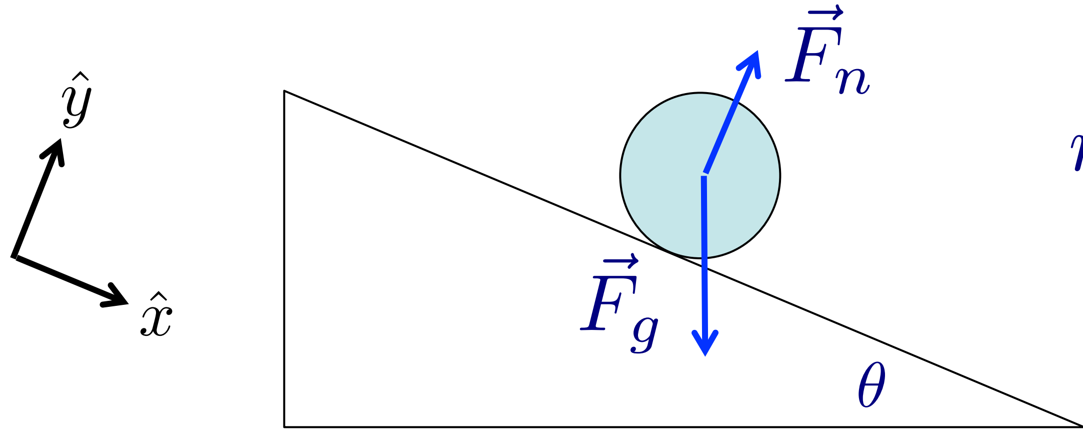
$$F_{g,y} = -F_g \cos\theta \qquad F_{n,y} = F_n$$

$$F_{\text{net,y}} = F_n - F_g \cos\theta = 0$$

No acceleration perpendicular to the plane

$$F_n = F_g \cos\theta = (49 \text{ N}) \cos(20^\circ) = \boxed{46 \text{ N} = F_n}$$

Newton's Second/Third Laws: Tougher Example

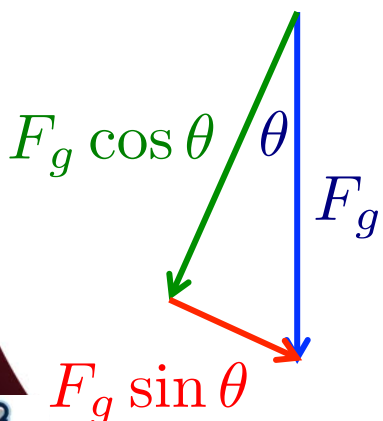


$$m = 5.0 \text{ kg}$$

$$\theta = 20^\circ$$

$$F_g = mg = 49 \text{ N}$$

- A small ball of m=5.0 kg is on an inclined plane of 20 degrees
  - **What is the acceleration of the ball down the plane?** a_x



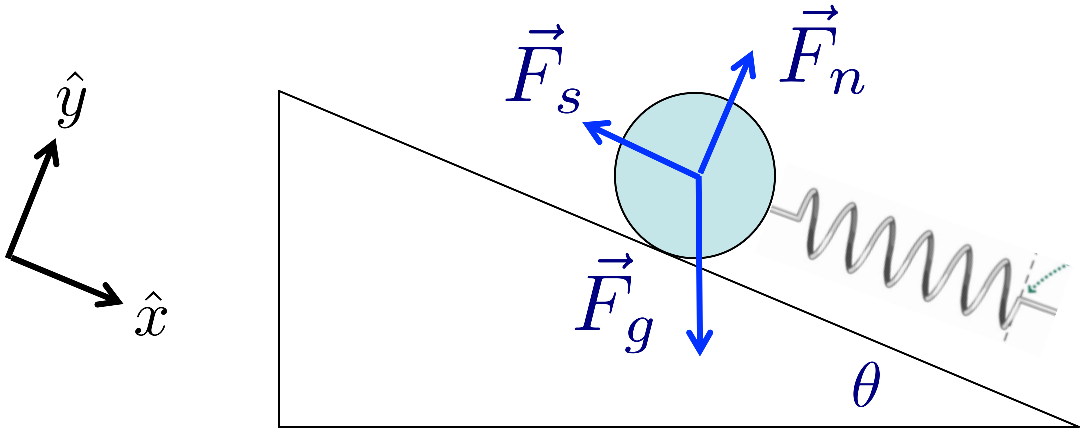
$$F_{g,x} = F_g \sin\theta \qquad F_{n,x} = 0$$

$$F_{g,y} = -F_g \cos\theta \qquad F_{n,y} = F_n$$

$$F_{\text{net,x}} = ma_x = F_{g,x} + F_{n,x} = F_g \sin\theta + 0 = mg\sin(20^\circ)$$

$$a_x = g\sin(20^\circ) = (9.8 \text{ m/s}^2)(0.342) = \boxed{3.35 \text{ m/s}^2 = a_x}$$

Ponderable

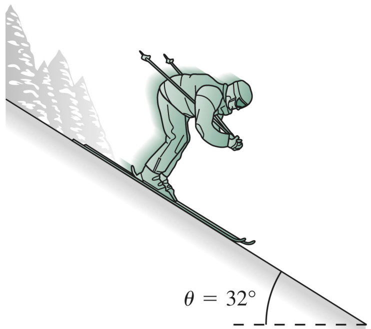


$$m = 5\ \text{kg}$$
$$\theta = 20^\circ$$
$$F_g = mg = 49\ \text{N}$$

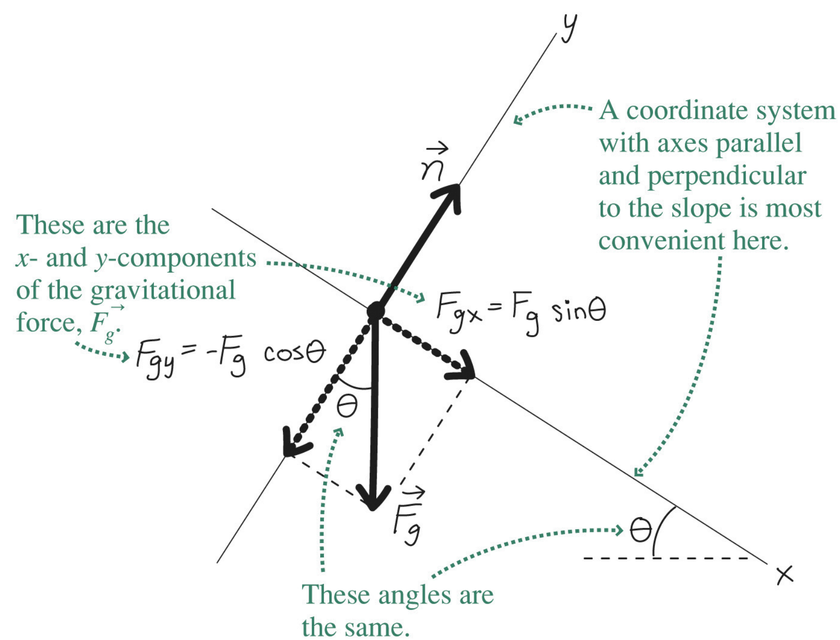
- A small ball of m=5 kg is on an inclined plane of 20 degrees
  - **Now we put a spring (k=2 N/m) beneath it to support it**
  - The spring compresses until the force it exerts counteracts gravity's force on the ball
    - **How far does the spring compress?**
    - **If we double the mass of the ball, how does the spring compression distance change?**

A Typical Problem: What's the skier's acceleration? What's the force the snow exerts on the skier?

- Physical diagram:



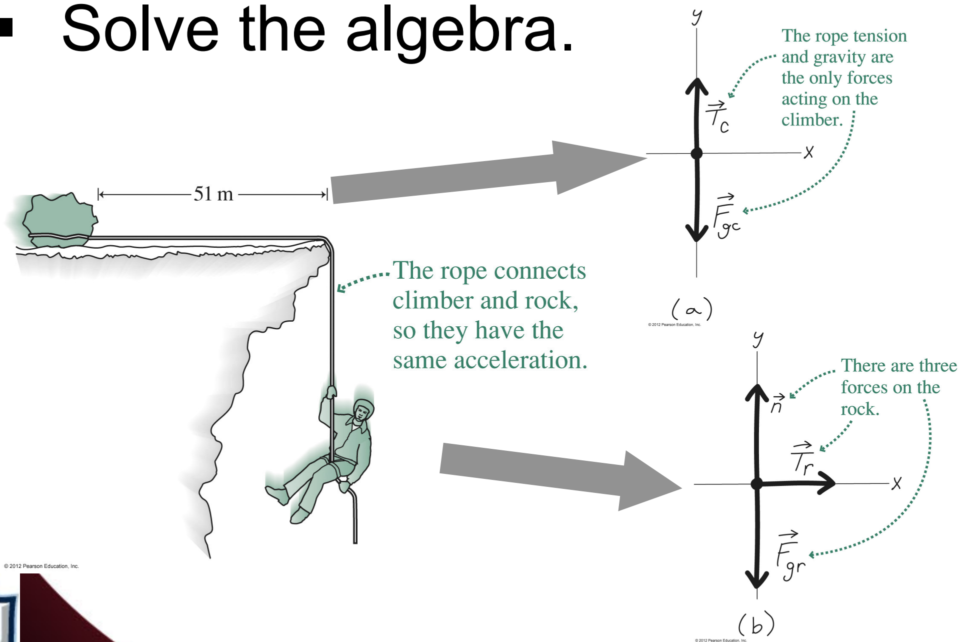
- "Free-body" diagram:



- Newton's law: $\vec{F}_{\text{net}} = \vec{n} + \vec{F}_g = m\vec{a}$
- In components:
  - x-component: $mg \sin\theta = ma$
  - y-component: $n - mg \cos\theta = 0$
- Solve (with m = 65 kg and θ = 32°) to get the answers:
  - $a = g \sin\theta = (9.8\ \text{m/s}^2)\sin 32° = 5.2\ \text{m/s}^2$
  - $n = mg \cos\theta = (65\ \text{kg})(9.8\ \text{m/s}^2)\cos 32° = 540\ \text{N}$

Multiple Objects

- Solve problems involving multiple objects by first identifying each object and all the forces on it.
- Draw a free-body diagram for each.
- Write Newton's law for each.
- Identify connections between the objects, which give common terms in the Newton's law equations.
- Solve the algebra.



- Newton's law:

$$\text{climber: } \vec{T}_c + \vec{F}_{gc} = m_c \vec{a}_c$$

$$\text{rock: } \vec{T}_r + \vec{F}_{gr} + \vec{n} = m_r \vec{a}_r$$

- In components:

$$\text{climber, } y\text{: } T - m_c g = -m_c a$$

$$\text{rock, } x\text{: } T = m_r a$$

$$\text{rock, } y\text{: } n - m_r g = 0$$

- Solution: $$a = \frac{m_c g}{m_c + m_r}$$

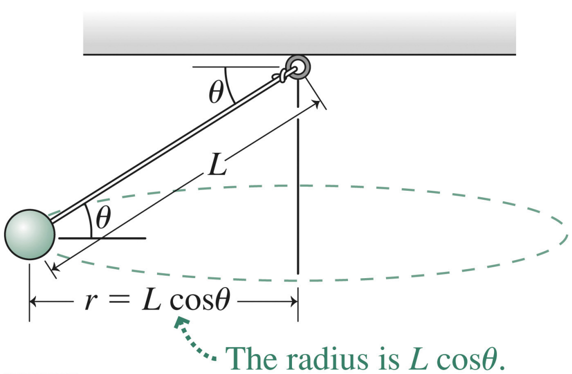
Circular Motion

- Problems involving circular motion are no different from other Newton's law problems – the geometry is just more complicated.
- Identify the forces, draw a free-body diagram, write Newton's law.
- The magnitude of the **centripetal force** on an object of mass *m* in circular motion with radius *r* is

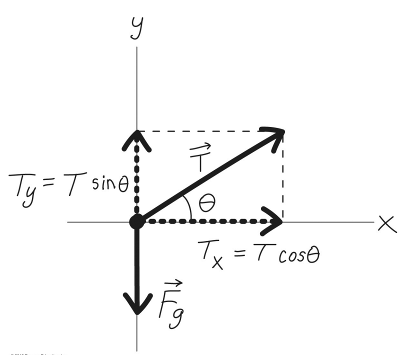
$$F = ma = \frac{mv^2}{r}$$

  - the acceleration has magnitude v^2/r and points toward the center of the circle

A ball whirling on a string:



Free-body diagram:



- Newton's law: $\vec{T} + \vec{F}_g = m\vec{a}$
- In components:

$$x: \quad T\cos\theta = \frac{mv^2}{L\cos\theta}$$

$$y: \quad T\sin\theta - mg = 0$$

- Solve for the ball's speed:

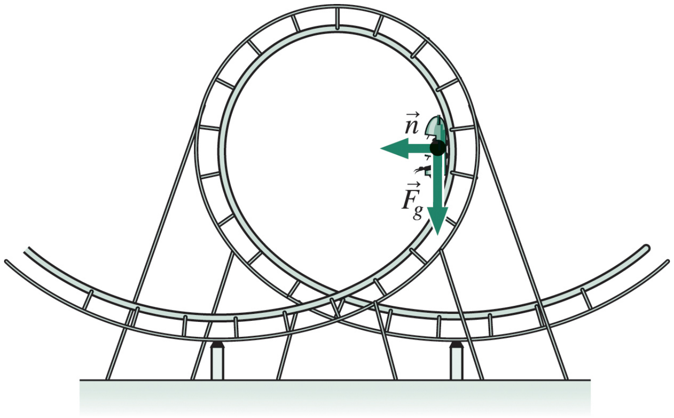
$$v = \sqrt{\frac{TL\cos^2\theta}{m}} = \sqrt{\frac{(mg/\sin\theta)L\cos^2\theta}{m}} = \sqrt{\frac{gL\cos^2\theta}{\sin\theta}}$$

Loop-the-Loop!

- The two forces acting on the roller-coaster car are:
  - gravity
  - normal force
- Gravity is always downward, and the normal force is perpendicular to the track.
- At the position shown, the two forces are at right angles:
  - The normal force acts perpendicular to the car's path, keeping its direction of motion changing.
  - Gravity acts opposite the car's velocity, slowing the car.
  - The net force is *not* toward the center

Newton's law:

$$\vec{n} + \vec{F}_g = m\vec{a}$$

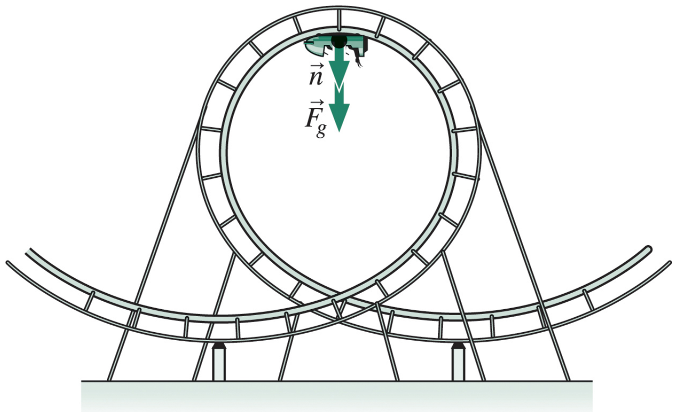


Loop-the-Loop!

- At the top of the loop, both forces are downward:

$$n_y = n,\ F_{gy} = mg \quad \Rightarrow \quad n + mg = \frac{mv^2}{r}$$

  - Solving for *v*, we obtain $v = \sqrt{nr/m + gr}$
  - For the car to stay in contact with the track, the normal force must be greater than zero.
  - So the minimum speed is the speed that let the normal force get arbitrarily close to zero at the top of the loop.
  - Then gravity alone provides the force that keeps the car in circular motion.

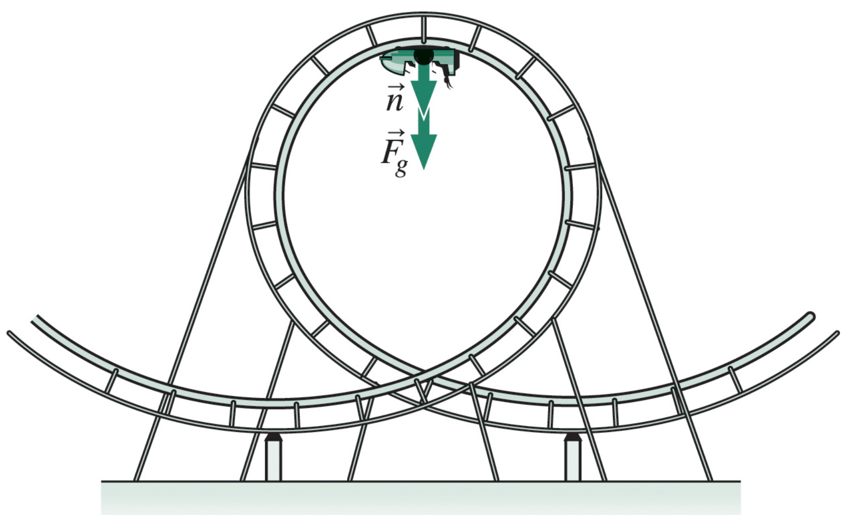


Loop-the-Loop!

- Therefore Newton's law has a single component, with the gravitational force *mg* providing the acceleration v^2/r that holds the car in its circular path:

$$\vec{F} = m\vec{a} \quad \rightarrow \quad mg = \frac{mv^2}{r}$$

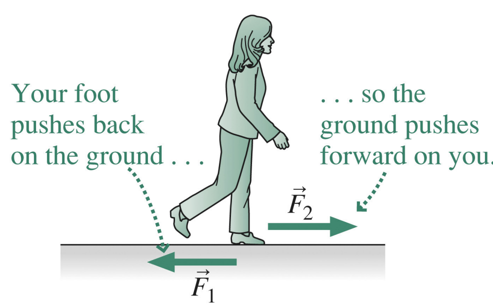
  - Solving for the minimum speed at the loop top gives $v = \sqrt{gr}$.
  - Slower than this at the top, and the car will leave the track!
  - Since this result is independent of mass, car and passengers will all remain on the track as long as $v \geq \sqrt{gr}$.

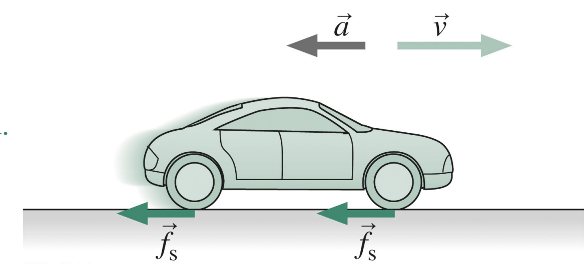


Friction

- **Friction** is a force that opposes the relative motion of two contacting surfaces.
- **Static friction** occurs when the surfaces aren't in motion; its magnitude is $f_s \geq \mu_s n$, where n is the normal force between the surfaces and μ_s is the **coefficient of static friction.**
- **Kinetic friction** occurs between surfaces in motion; its magnitude is $f_k = \mu_k n$.

Friction is important in walking, driving and a host of other applications:

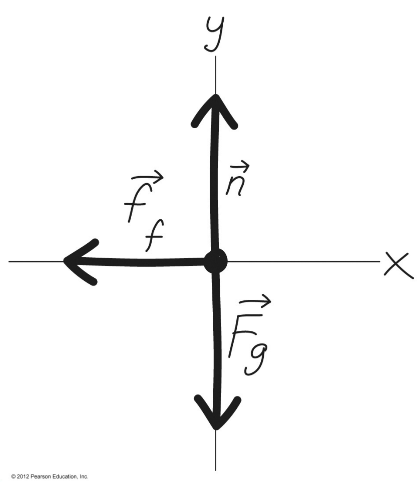




Solving Problems with Friction

- Problems with friction are like all other Newton's law problems.
- Identify the forces, draw a free-body diagram, write Newton's law.
- You'll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

- Example: A braking car: What's the acceleration?



- Newton's law: $\vec{F}_g + \vec{n} + \vec{f}_f = m\vec{a}$
- In components: $x: \quad -\mu n = m a_x$

  $y: \quad -mg + n = 0$
- Solve for *a*:

  y equation gives $n = mg$,

  so x equation gives $a_x = -\frac{\mu n}{m} = -\mu g$