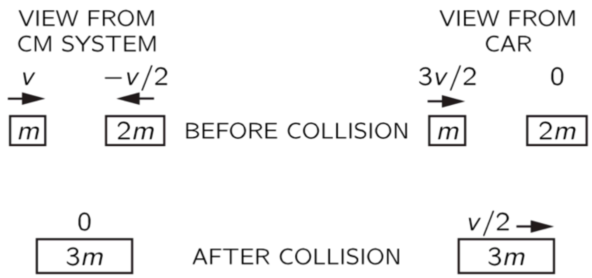


University Physics 226N/231N
Old Dominion University

Momentum, Impulse, Elastic Collisions

Dr. Todd Satogata (ODU/Jefferson Lab)

satogata@jlab.org

http://www.toddsatogata.net/2016-ODU

Wednesday, October 12, 2016

Reminder: The Second Midterm will be Weds Oct 19 2016

Happy Birthday to Hugh Jackman, Josh Hutcherson, Luciano Pavarotti,
John Moffat, and Arthur Harden (1929 Nobel)!
Happy National Gumbo Day and Yom Kippur (G'mar Tov)!

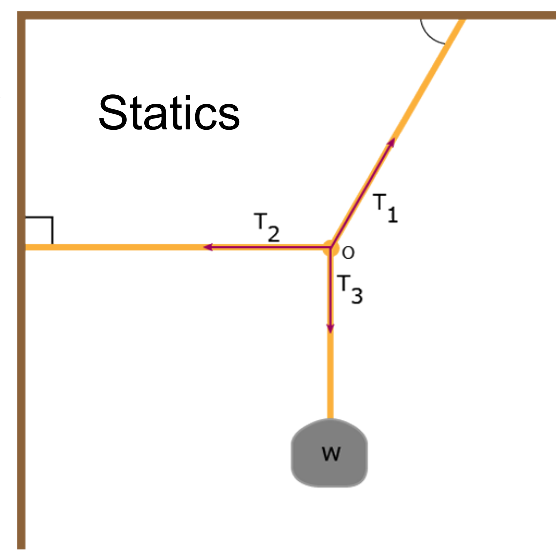
Please set your cell phones to "vibrate" or "silent" mode. Thanks!

What We Have Covered So Far

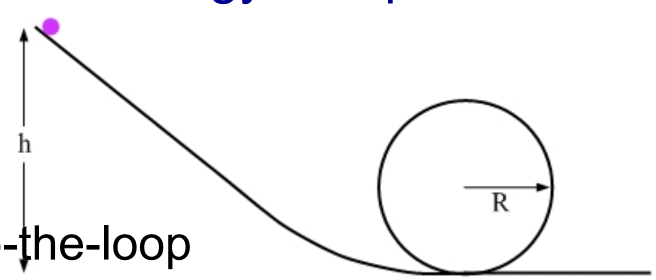
- Physical quantities and vectors
- Kinematics in one and two dimensions
- Centripetal acceleration
- Newton's laws
  - First law
  - Second Law
  - Third Law
  - Application of Newton's Laws
- Work and Energy
  - Friction:
  - Energy conservation
  - Potential energy and potentials

What We Have Covered So Far

- Physical quantities and vectors
- Kinematics in one and two dimensions
- Centripetal acceleration
- Newton's laws
  - First law → No force ➔ no acceleration
  - Second Law → $\vec{F}_{\rm net} = m\vec{a}$
  - Third Law → Forces occur in equal and opposite pairs
  - Application of Newton's Laws →



- Work and Energy
  - Friction: $F_{\rm f} = \mu_{\rm k} n \text{ or } \mu_{\rm s} n$
  - Energy conservation
  - Potential energy and potentials



Loop-the-loop

Forces and Newton's Second Law

- **Work** and **Energy** were defined in terms of forces being exerted over distances
  - For a constant force F in the same direction as distance Δx

$$\Delta W = F\,\Delta x \qquad\qquad F = \Delta W / \Delta x$$

  - For a general vector force that depends on $\vec{x}$

$$W = \int \vec{F}(x) \cdot d\vec{x}$$

(Note vector dot product)

- Newton's second law says $\vec{F}_{\rm net} = m\vec{a} = m\,\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$

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- We call $d(m\vec{v}) = \vec{F}_{\rm net}\,dt$ **impulse**
  - For the same change in momentum, you can have
    - A smaller force for a longer time
    - A larger force for a smaller time
  - Model rocket engines are classified by total impulse

Forces and Newton's Second Law

- **Work** and **Energy** were defined in terms of forces being exerted over distances
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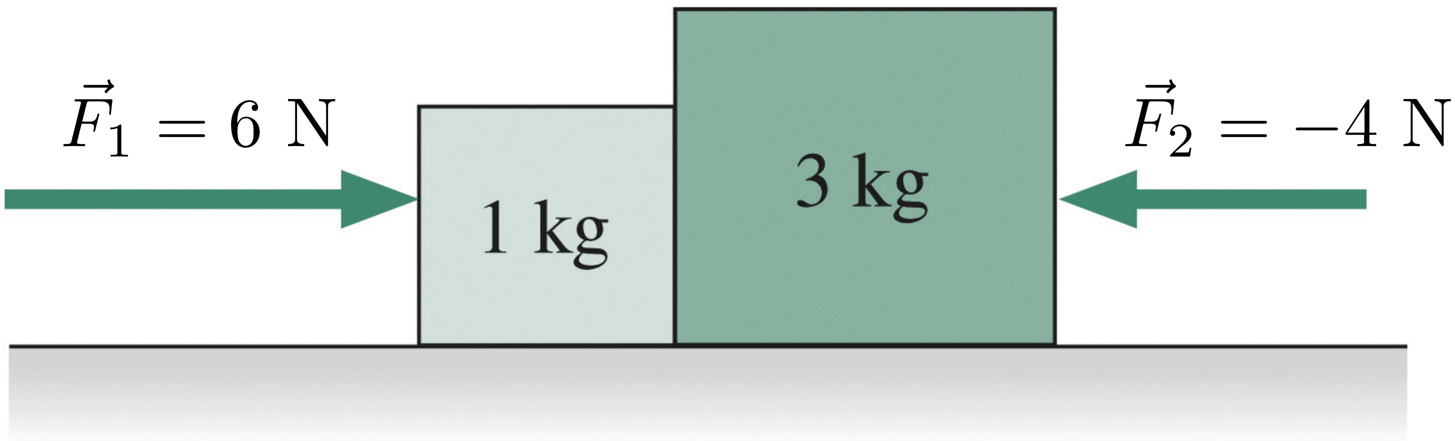
- Newton's second law says $\vec{F}_{\rm net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$
- We call $\vec{p} = m\vec{v}$ **momentum**
  - Momentum is a vector
  - Momentum, like velocity, depends on your **reference frame**
    - It depends on relative motion between you and whatever objects you are describing in your physics calculation

Newton's Third Law

- Recall Newton's Third Law:
  - When one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body. (Wikipedia)



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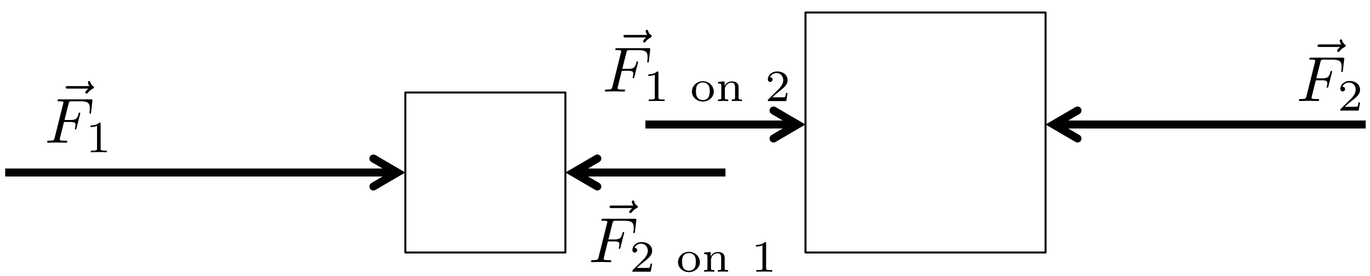
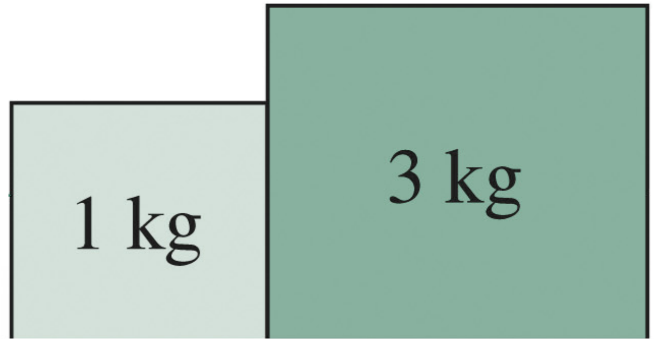
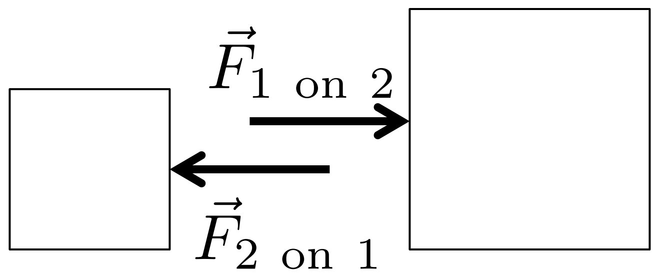
Equal and opposite forces between boxes



Newton's Third Law: No net external forces

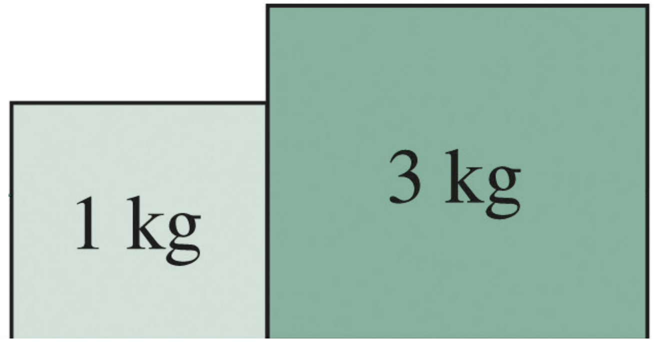


Equal and opposite forces between boxes

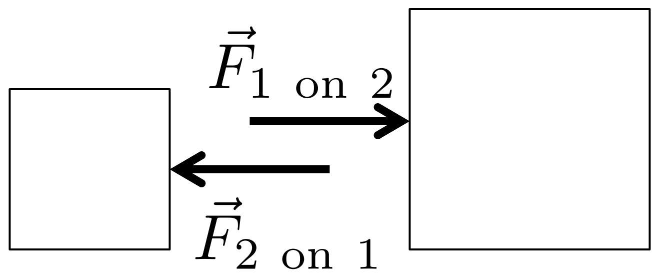


$$\vec{F}_{\text{net}} = 0$$

Newton's Third Law: No net external forces



Equal and opposite forces between boxes



$$\vec{F}_{\text{net}} = 0 = \frac{d\vec{p}}{dt} = \frac{d(\vec{p}_1 + \vec{p}_2)}{dt} \quad \Rightarrow \quad \boxed{\vec{p}_1 + \vec{p}_2 = \text{constant}}$$

Momentum is conserved (doesn't depend on time)

Conservation of Momentum Example 1

- Two objects of masses m_1=1.0 kg and m_2=3.0 kg are sitting at rest on a frictionless table with a small spring compressed between them. The spring is released, and afterwards the 1.0 kg mass moves to the left at 2.0 m/s.
  - What is the velocity of the 3.0 kg mass after the spring is released?

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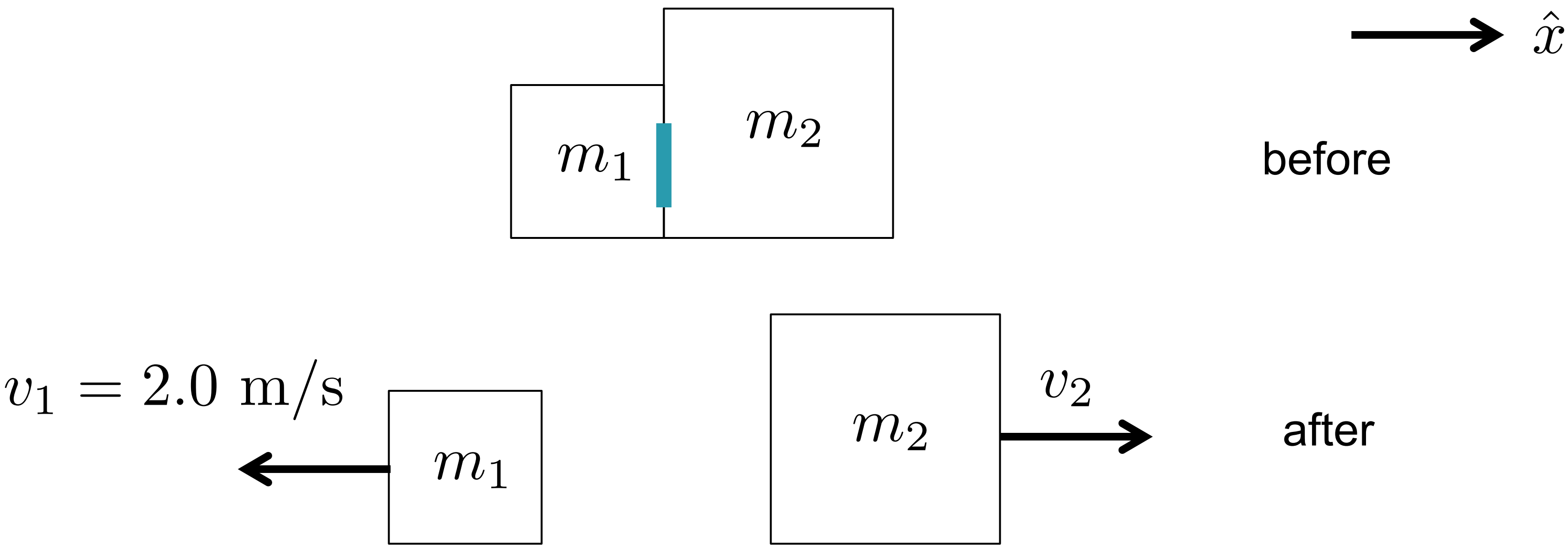

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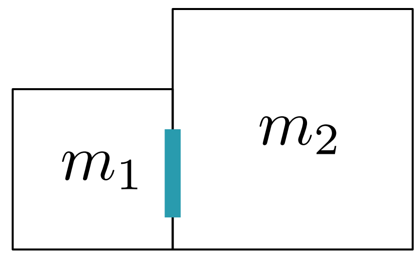
  - What is the velocity of the 3.0 kg mass after the spring is released?

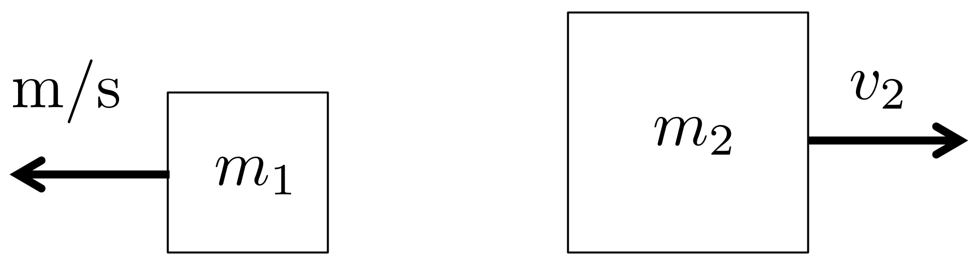
$$\vec{p}_{\rm initial} = 0.0 \text{ kg m/s}$$



$\longrightarrow \hat{x}$

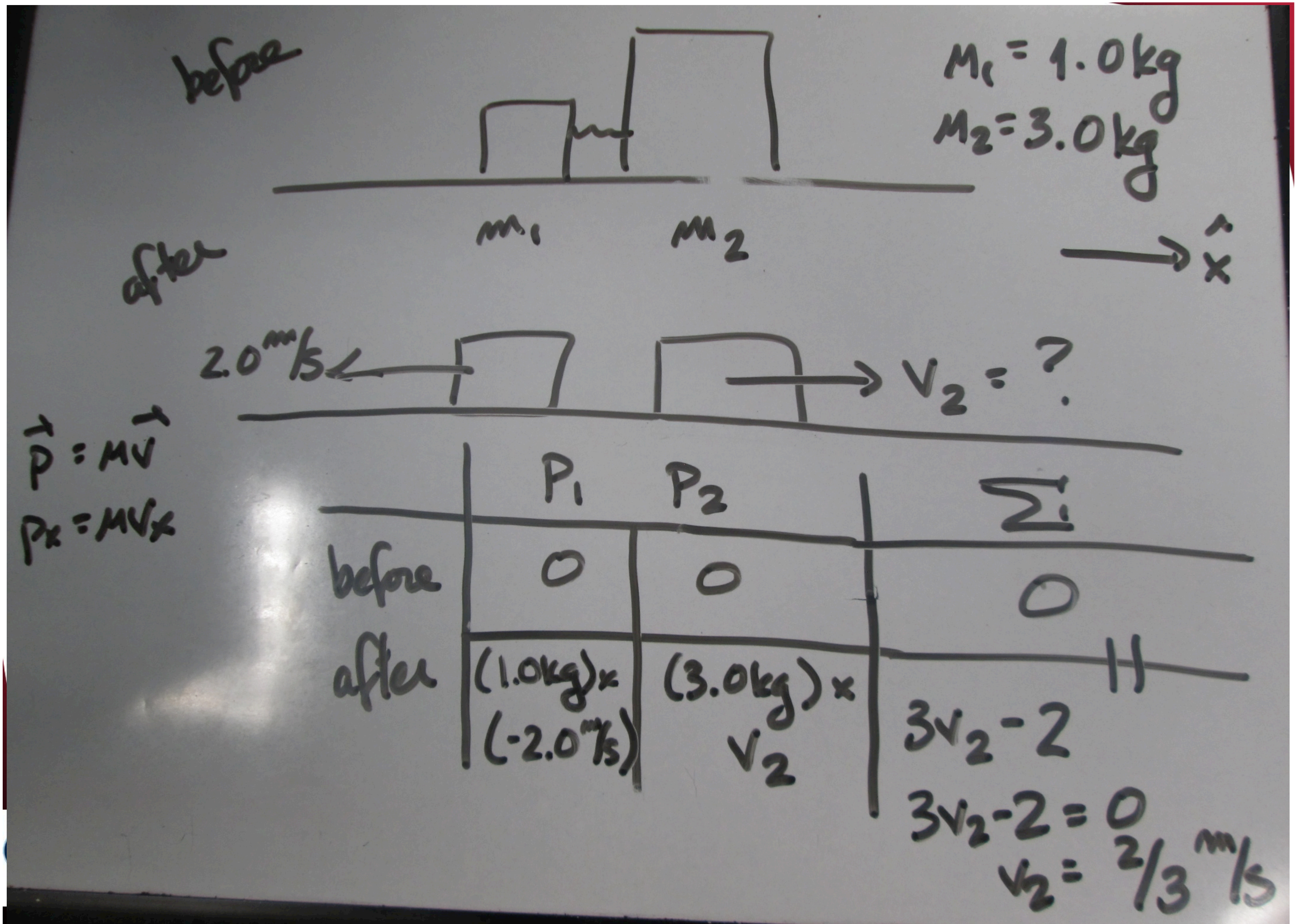
before

$$v_1 = 2.0 \text{ m/s}$$



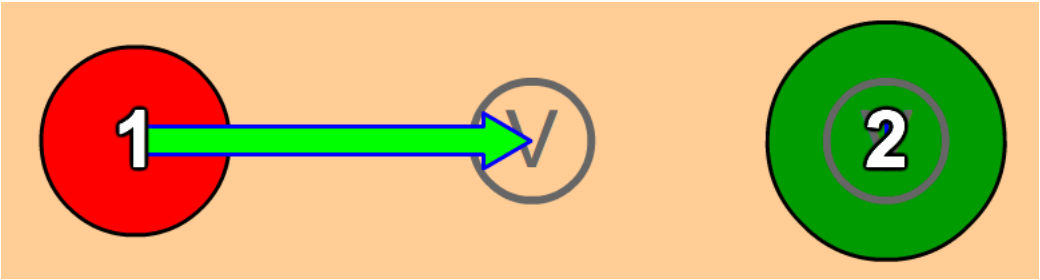
after

$$\vec{p}_{\rm final} = m_2 v_2 + m_1 v_1 = (3.0 \text{ kg}) v_2 + (1.0 \text{ kg})(-2.0 \text{ m/s})$$


before
after
$m_1 = 1.0\,kg$
$m_2 = 3.0\,kg$
m_1
m_2
$\hat{x}$
2.0 m/s
$v_2 = ?$
$\vec{p} = m\vec{v}$
$p_x = mv_x$
p_1
p_2
Σ
before
0
0
0
after
(1.0kg) x (-2.0 m/s)
(3.0kg) x v_2
$3v_2 - 2$
$3v_2 - 2 = 0$
$v_2 = 2/3$ m/s

Conservation of Momentum Example 2

- A 0.5 kg mass is moving in a line at 1.0 m/s towards a second stationary mass of 1.0 kg on a frictionless surface. After they bounce elastically off each other, what are their final velocities?



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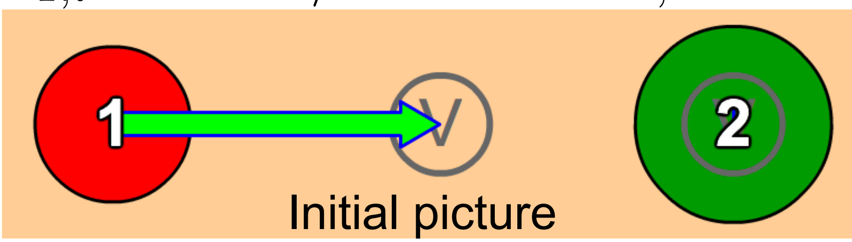
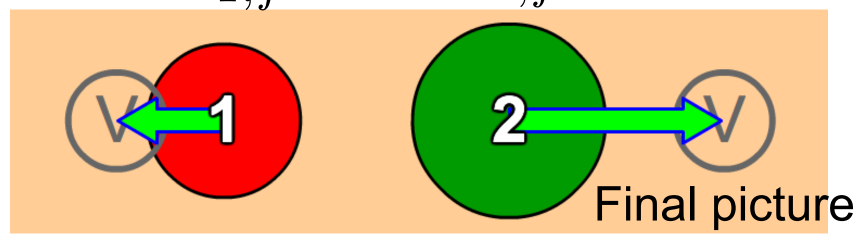
$\longrightarrow \hat{x}$

$v_{1,i} = 1.0\ \text{m/s}$ $\quad v_{2,i} = 0$



$v_{1,f}$ $\quad v_{2,f}$



$$p_{\text{initial}} = m_1 v_{1,i} + m_2 v_{2,i} = (0.5\ \text{kg})(1.0\ \text{m/s}) = 0.5\ \text{kg m/s}$$

$$p_{\text{final}} = m_1 v_{1,f} + m_2 v_{2,f} = (0.5\ \text{kg}) v_{1,f} + (1.0\ \text{kg}) v_{2,f}$$

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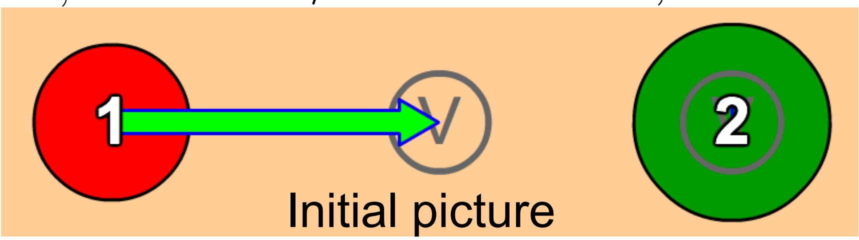
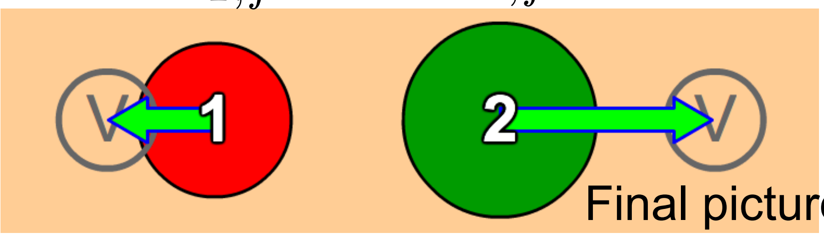


$v_{1,f}$ $\qquad v_{2,f}$



$$p_{\text{initial}} = m_1 v_{1,i} + m_2 v_{2,i} = (0.5 \text{ kg})(1.0 \text{ m/s}) = 0.5 \text{ kg m/s}$$

$$p_{\text{final}} = m_1 v_{1,f} + m_2 v_{2,f} = (0.5 \text{ kg}) v_{1,f} + (1.0 \text{ kg}) v_{2,f}$$

Uh oh… We have only one equation (conservation of linear momentum) but two unknowns (the two final velocities).

We also have to use **conservation of energy** to solve this problem
But we can only use it because the collision is **elastic**.

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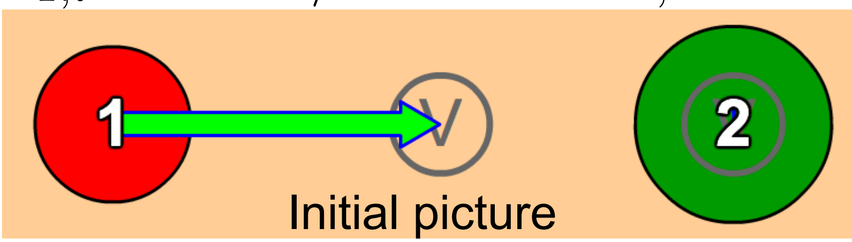
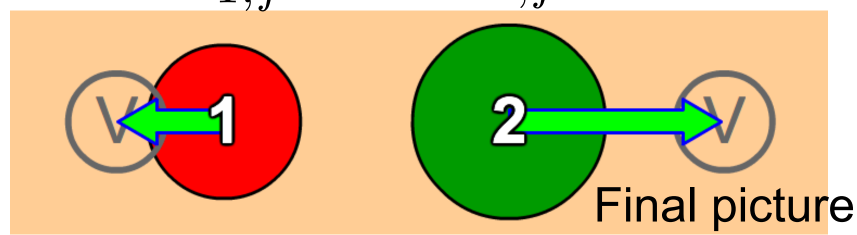


$v_{1,f}$ $\quad v_{2,f}$



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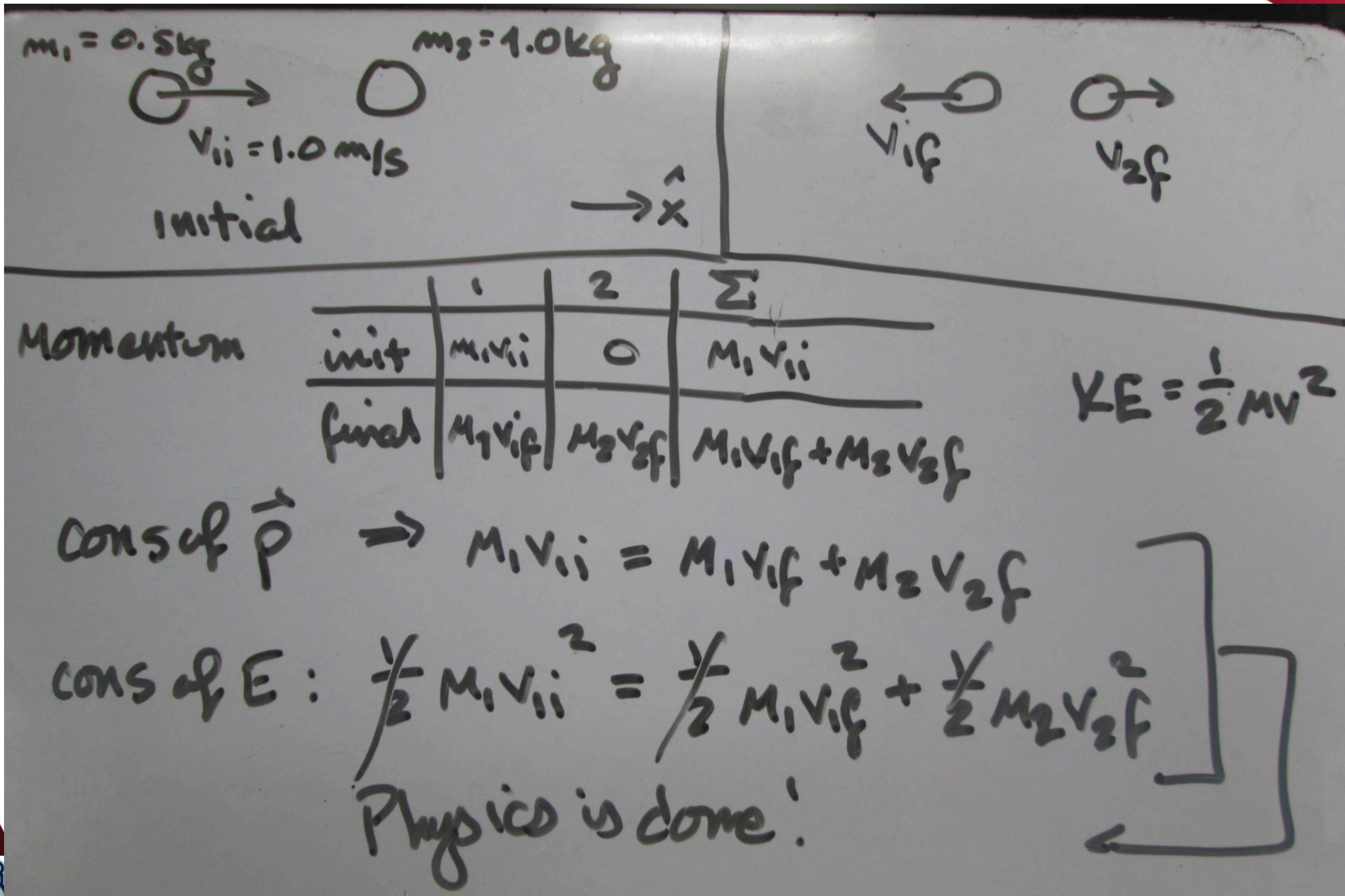
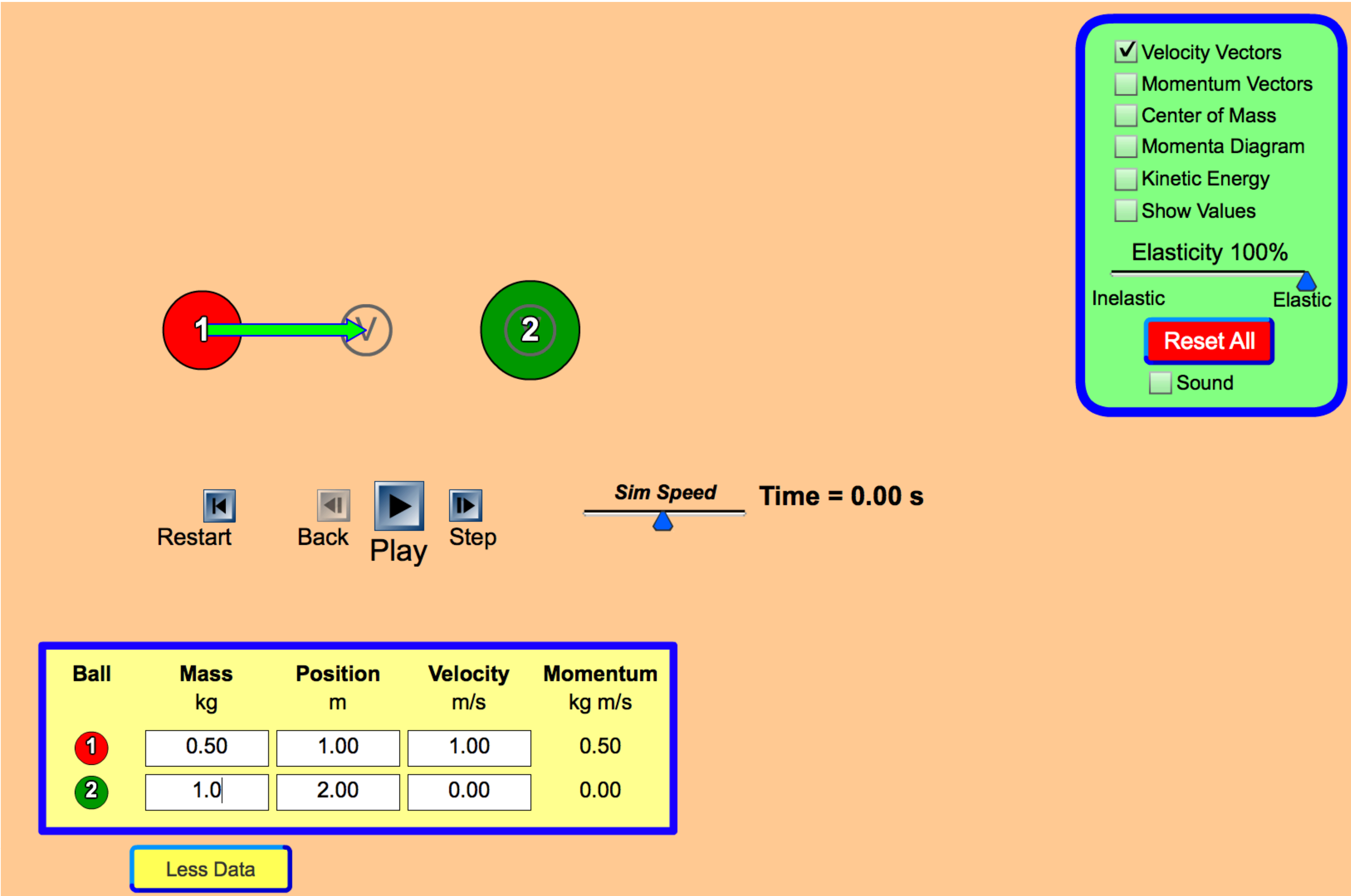
$$p_{\text{final}} = m_1 v_{1,f} + m_2 v_{2,f} = (0.5 \text{ kg}) v_{1,f} + (1.0 \text{ kg}) v_{2,f}$$

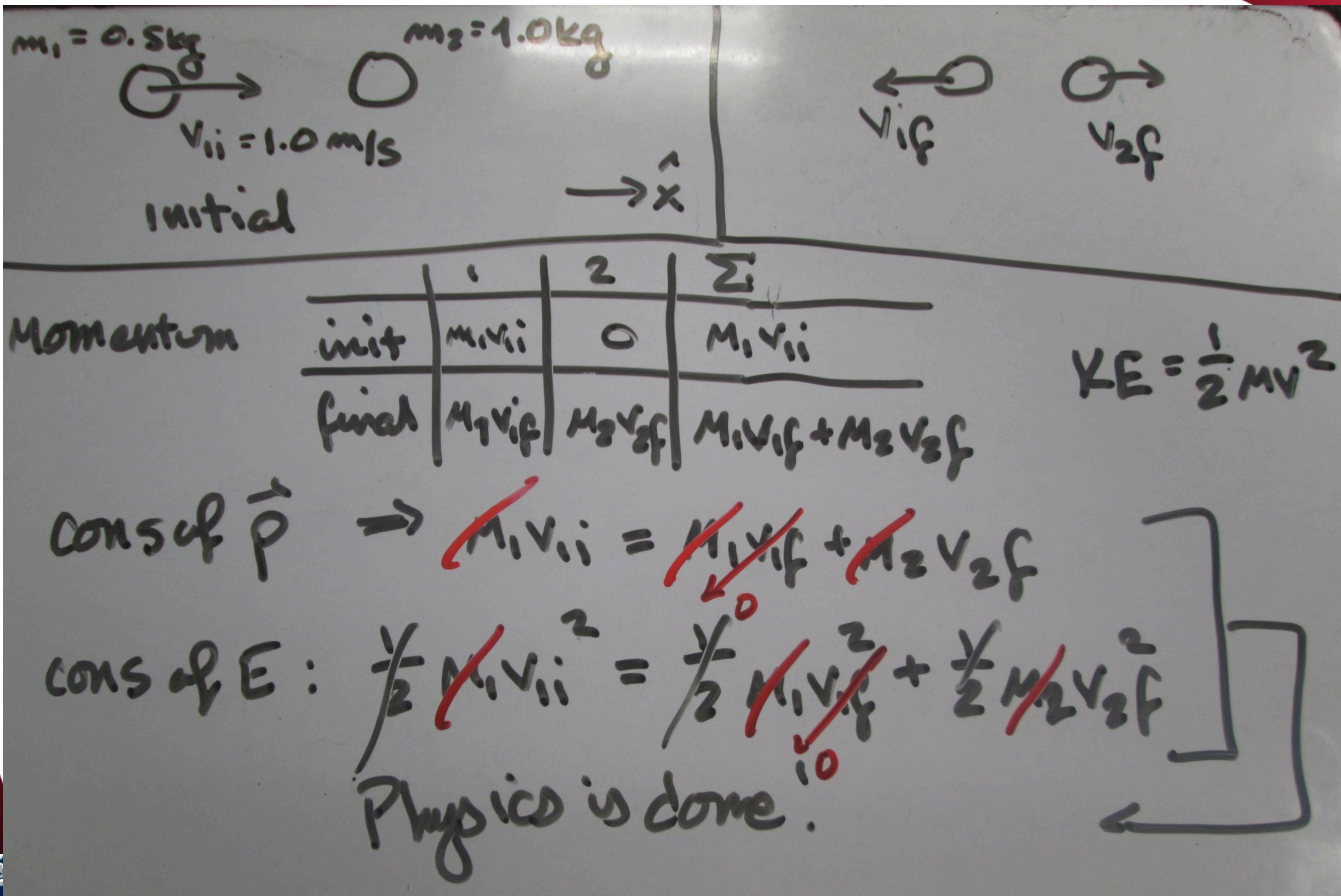
(Dr. Todd works the remainder of this one out on a white board)


m1 = 0.5kg
m2 = 1.0kg
v1i = 1.0 m/s
initial
x̂
v1f
v2f
Momentum
1
2
Σ
init
m1v1i
0
M1v1i
final
M1v1f
M2v2f
M1v1f + M2v2f
KE = 1/2 mv²
cons of p⃗ ⇒ M1v1i = M1v1f + M2v2f
cons of E : 1/2 M1v1i² = 1/2 M1v1f² + 1/2 M2v2f²
Physics is done!

PHET Collision Lab



https://phet.colorado.edu/sims/collision-lab/collision-lab_en.html


$m_1 = 0.5$ kg
$m_2 = 1.0$ kg
$v_{1i} = 1.0$ m/s
initial
$\hat{x}$
v_{1f}
v_{2f}
1 2 Σ
Momentum
init $m_1 v_{1i}$ 0 $m_1 v_{1i}$
final $m_1 v_{1f}$ $m_2 v_{2f}$ $m_1 v_{1f} + m_2 v_{2f}$
$KE = \frac{1}{2} m v^2$
cons of $\vec{p}$ → $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$
cons of E: $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$
Physics is done!

I SEE (Collisions, Energy/Momentum Conservation)

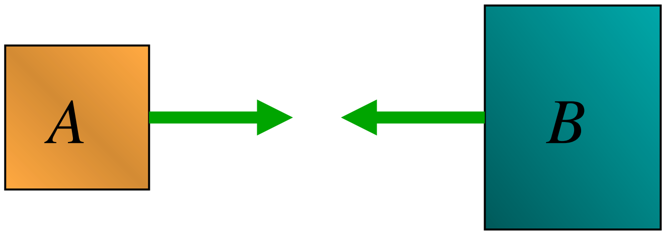
- **I: Identify** the relevant concepts
  - What are the physical quantities are known, and unknown?
  - These problems involve **objects before and after interactions**
    - Usually those objects do not have external forces acting on them
    - Advanced reading: https://www.lhup.edu/~dsimanek/ideas/bounce.htm
    - Determine whether the collisions are elastic (energy conserving)
- **S: Set Up** the problem
  - **Tell the story**: Draw a picture and choose equations to solve.
  - Draw your coordinate system. Decompose all vectors into components.
  - Write out conservation of momentum in each dimension
  - Write out conservation of energy if the collision is elastic
- **E: Execute** the solution
  - “Do the math” or “crunch the numbers”.
  - Will likely involve solving “multiple equations in multiple unknowns”
- **E: Evaluate** your answer
  - Does your answer make sense? Check the units.

Inelastic Collisions

- In **inelastic collisions**, energy is not conserved
  - Some energy is “lost” from the energy of the initial objects
  - Examples of places this energy goes include
    - Inelastic deformation (e.g. car crashes, beanbags)
    - Mechanical storage (e.g. spring compressing)
    - Friction and heat
- You will be told when collisions are inelastic
  - Objects sticking to each other after collision
  - Objects obviously deforming (e.g. bullet sticking in block)
- Some other types of problems are also clearly inelastic
  - Objects being “blown apart” or “thrown apart”

Q8.5

Two objects with different masses collide with and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



A. the same amount of total momentum and the same total kinetic energy.

B. the same amount of total momentum but less total kinetic energy.

C. less total momentum but the same amount of total kinetic energy.

D. less total momentum and less total kinetic energy.

E. Not enough information is given to decide.

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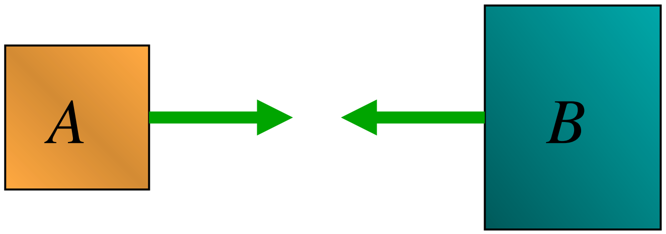


A. the same amount of total momentum and the same total kinetic energy.

✔ B. the same amount of total momentum but less total kinetic energy.

C. less total momentum but the same amount of total kinetic energy.

D. less total momentum and less total kinetic energy.

E. Not enough information is given to decide.

Q8.8

Block *A* on the left has mass 1.00 kg. Block *B* on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does p_A (the magnitude of momentum of block *A*) compare to p_B (the magnitude of momentum of block *B*)?

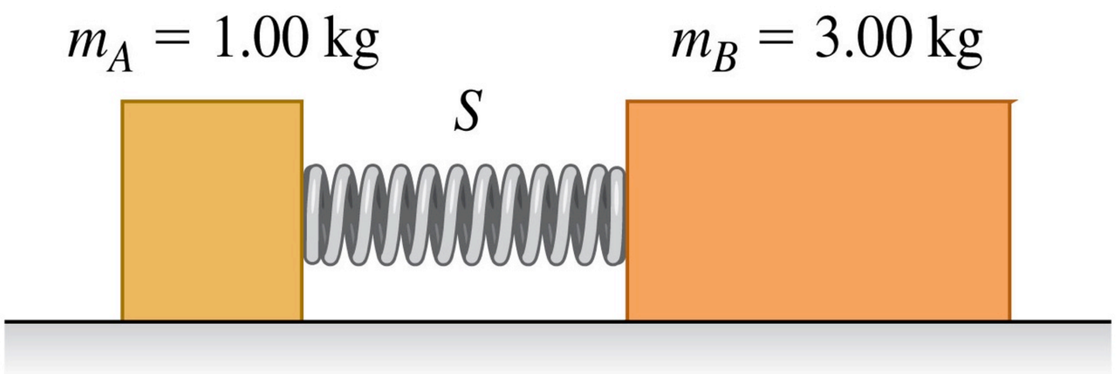
A. $p_A = p_B/9$

B. $p_A = p_B/3$

C. $p_A = p_B$

D. $p_A = 3p_B$

E. $p_A = 9p_B$



A8.8

Block A on the left has mass 1.00 kg. Block B on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does p_A (the magnitude of momentum of block A) compare to p_B (the magnitude of momentum of block B)?

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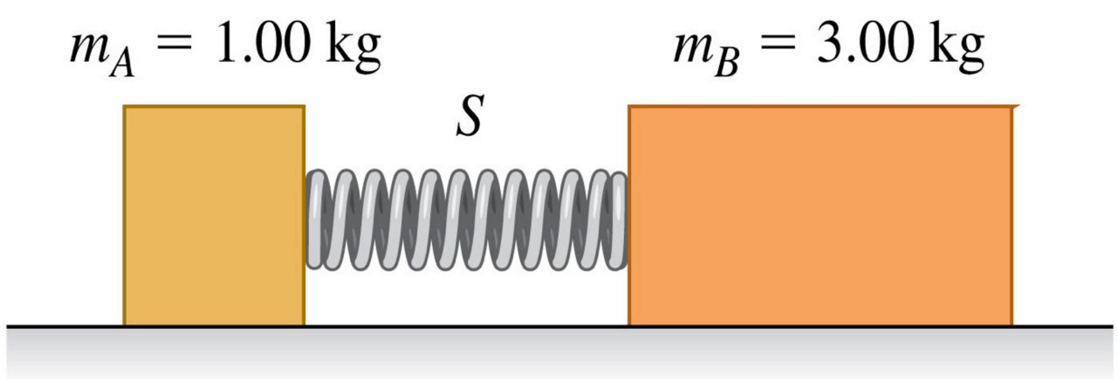


Q8.9

Block A on the left has mass 1.00 kg. Block B on the right has mass 3.00 kg. The blocks are forced together, compressing the spring. Then the system is released from rest on a level, frictionless surface. After the blocks are released, how does K_A (the kinetic energy of block A) compare to K_B (the kinetic energy of block B)?

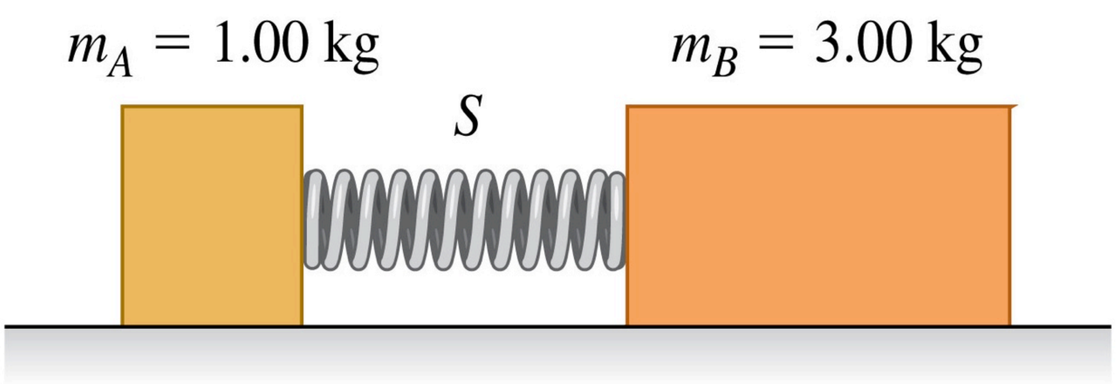
A. $K_A = K_B/9$

B. $K_A = K_B/3$

C. $K_A = K_B$

D. $K_A = 3K_B$

E. $K_A = 9K_B$



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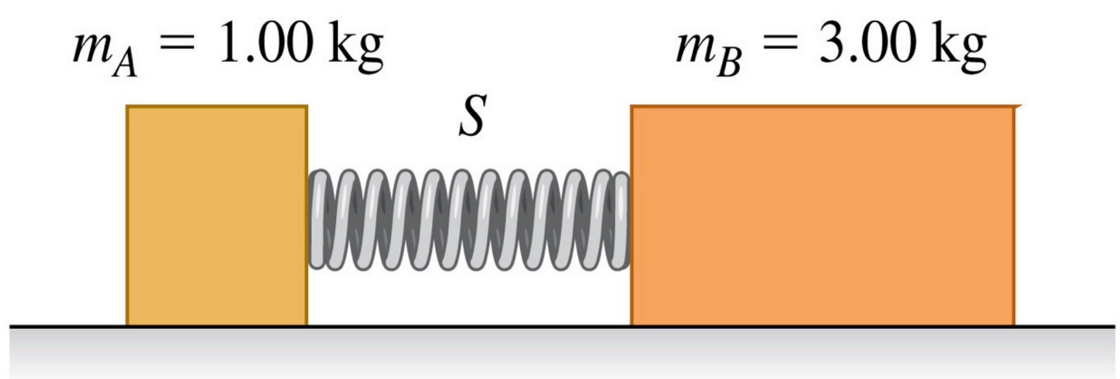
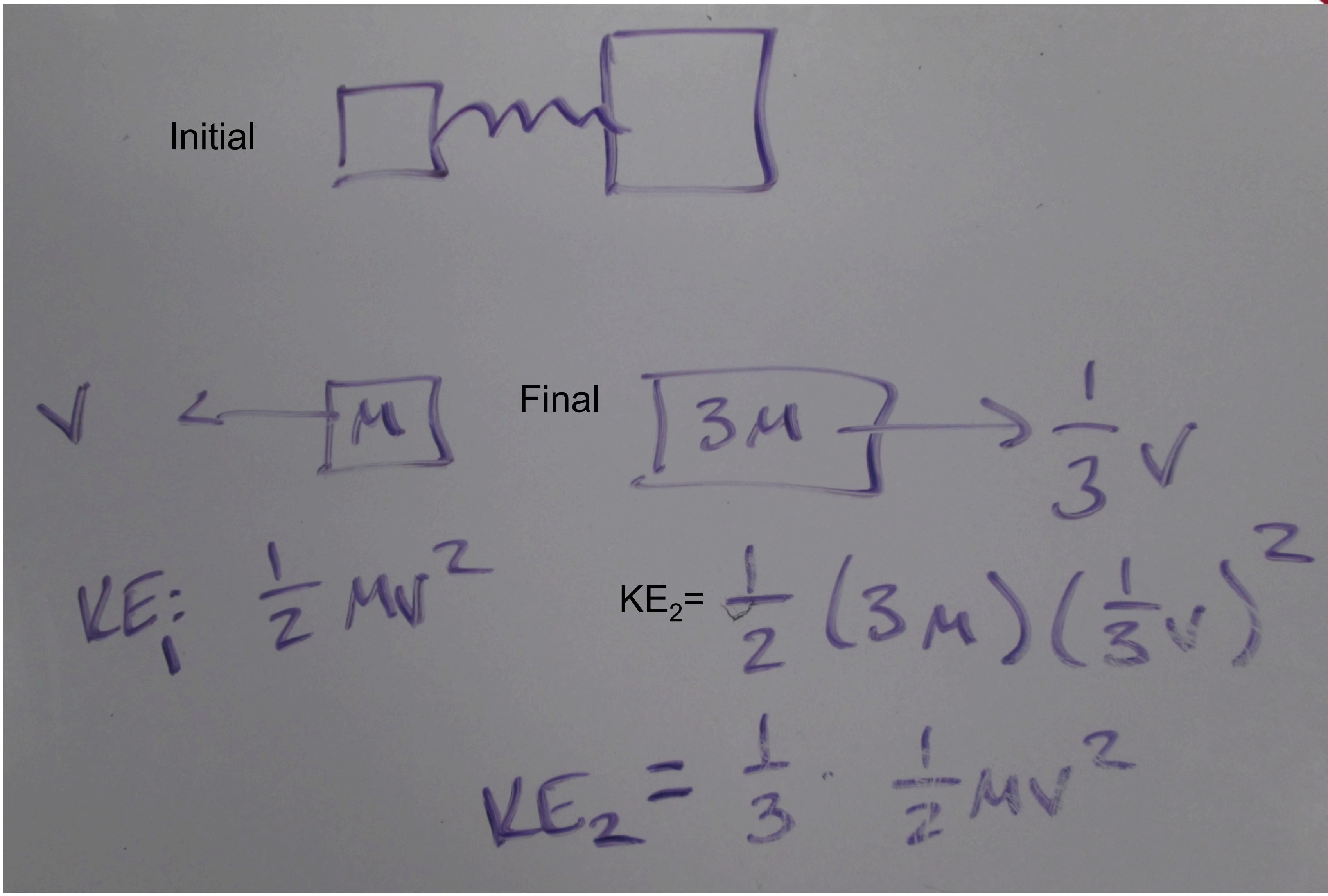
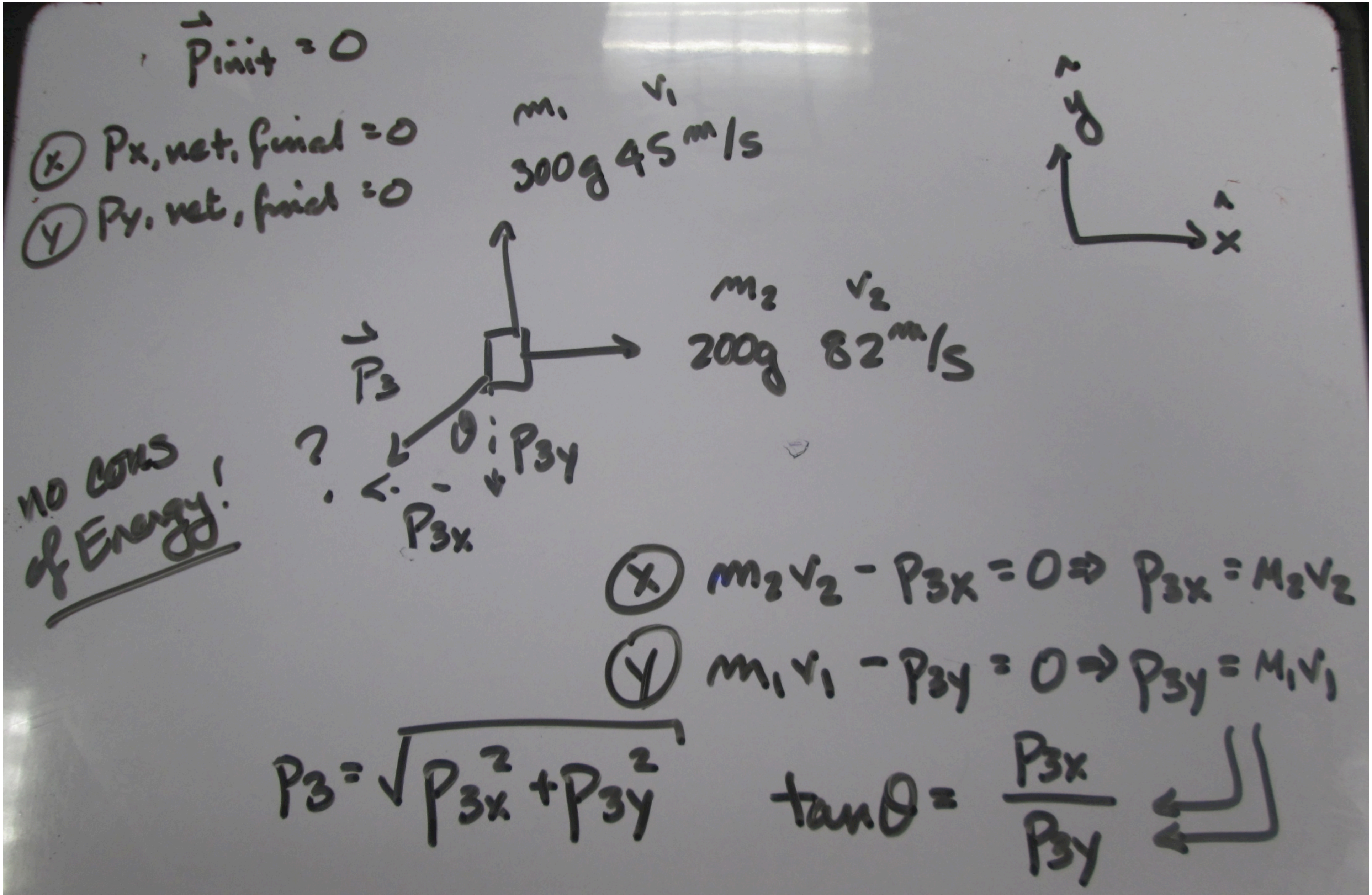
E. $K_A = 9K_B$




Initial
Final
v
M
3M
1/3 v
KE_i: 1/2 Mv^2
KE_2= 1/2 (3M)(1/3 v)^2
KE_2 = 1/3 · 1/2 Mv^2

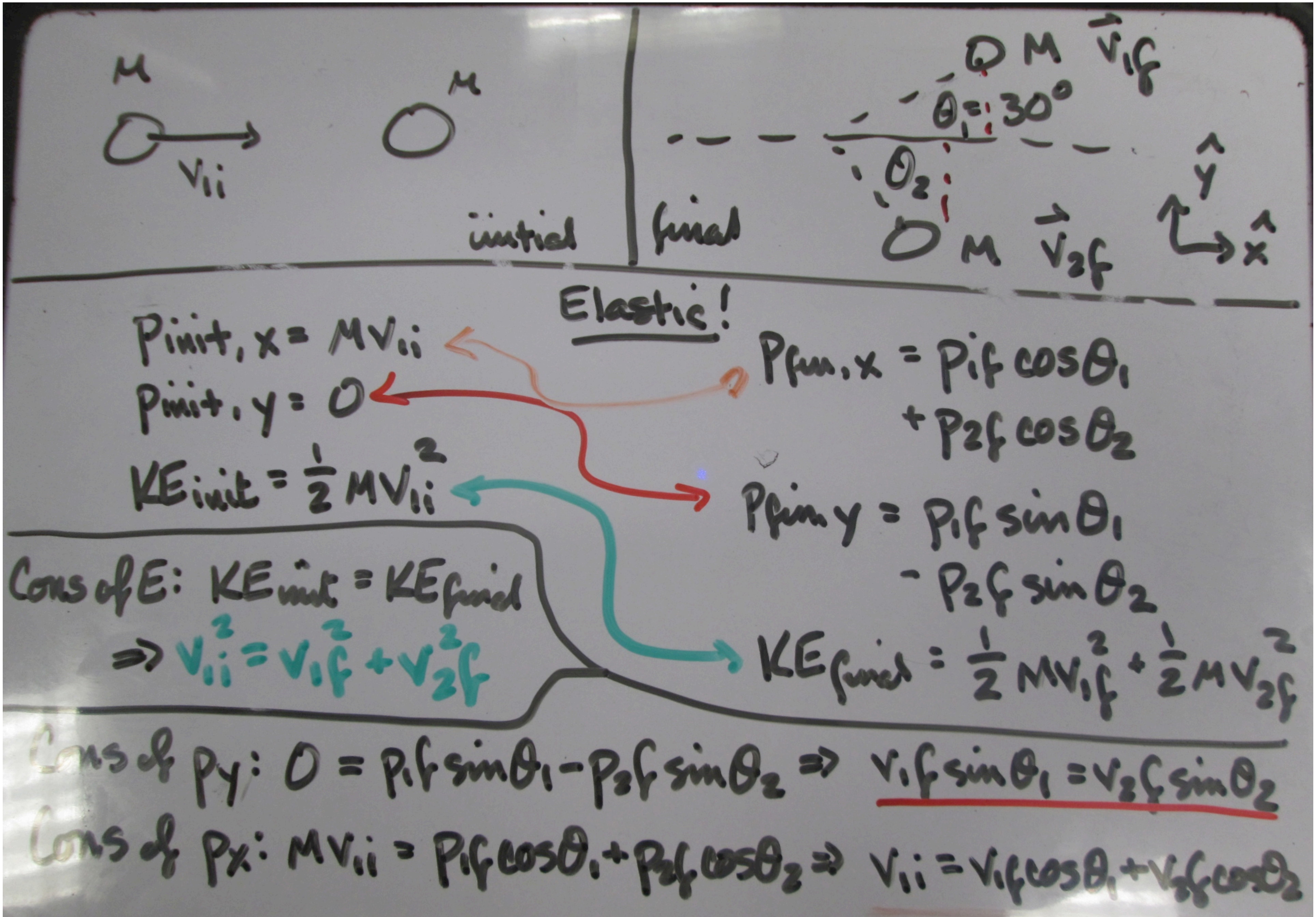
Conservation of Momentum Example 3

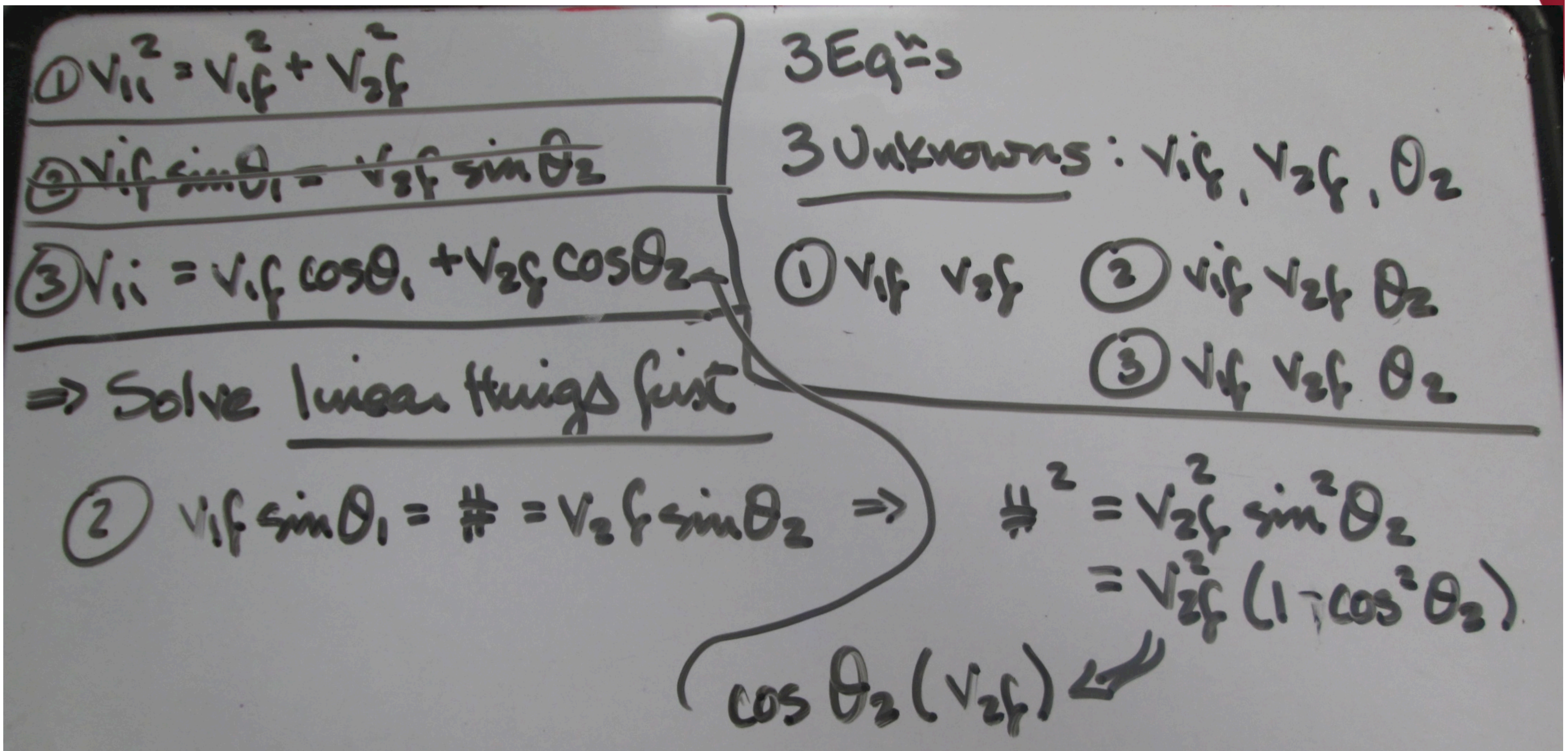
- A firecracker at rest blows up into three pieces. One has a mass of 200 g and flies off along the +*x*-axis with a speed of 82.0 m/s. A second piece has a mass of 300 g and flies off along the +*y*-axis with a speed of 45.0 m/s.
  - What are the magnitude and direction of the momentum of the third piece?


P init = 0
(x) Px, net, final = 0
(y) Py, net, final = 0
m1 v1
300g 45 m/s
y
x
m2 v2
200g 82 m/s
P3
?
θ
P3y
P3x
no cons of Energy!
(x) m2v2 - P3x = 0 ⇒ P3x = M2V2
(y) m1v1 - P3y = 0 ⇒ P3y = M1V1
P3 = √(P3x² + P3y²)
tanθ = P3x / P3y

Conservation of Momentum/Energy Example 4

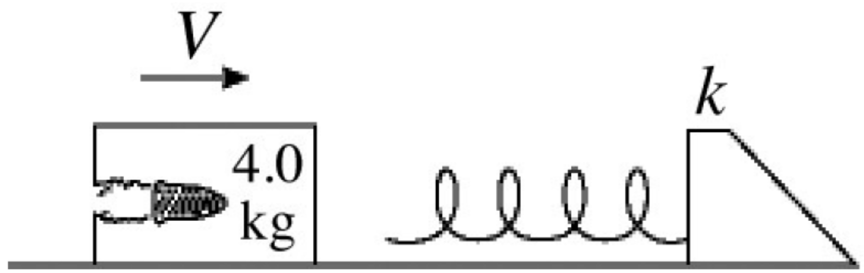
- A billiard ball traveling at 3.00 m/s collides elastically with an identical billiard ball initially at rest on a level table. The first billiard ball deflects 30 degrees from its original direction.
  - What are the speeds of both balls after the collision?
  - What is the direction of the second ball after the collision?


M
M
Vii
initial
final
M v1f
θ1 = 30°
θ2
M v2f
ŷ
x̂
Elastic!
Pinit,x = MVii
Pinit,y = 0
KEinit = 1/2 MVii²
Pfin,x = P1f cosθ1 + P2f cosθ2
Pfin,y = P1f sinθ1 − P2f sinθ2
KEfinal = 1/2 MV1f² + 1/2 MV2f²
Cons of E: KEinit = KEfinal
⇒ Vii² = V1f² + V2f²
Cons of Py: 0 = P1f sinθ1 − P2f sinθ2 ⇒ V1f sinθ1 = V2f sinθ2
Cons of Px: MVii = P1f cosθ1 + P2f cosθ2 ⇒ Vii = V1f cosθ1 + V2f cosθ2


① $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$
② $v_{1f} \sin\theta_1 = v_{2f} \sin\theta_2$
③ $v_{1i} = v_{1f} \cos\theta_1 + v_{2f} \cos\theta_2$
⇒ Solve linear things first
② $v_{1f} \sin\theta_1 = \# = v_{2f} \sin\theta_2$ ⇒
$\#^2 = v_{2f}^2 \sin^2\theta_2$
$= v_{2f}^2 (1 - \cos^2\theta_2)$
$\cos\theta_2 (v_{2f})$
3 Eq^{n}s
3 Unknowns: v_{1f}, v_{2f}, θ_2
① v_{1f} v_{2f}
② v_{1f} v_{2f} θ_2
③ v_{1f} v_{2f} θ_2

Conservation of Momentum Example 5

- An 8.0-g bullet is shot into a 4.0-kg block, at rest on a frictionless horizontal surface. The bullet remains lodged in the block. The block moves into an ideal massless spring and compresses it by 8.7 cm. The spring constant of the spring is 2400 N/m.
  - What is the initial velocity of the bullet?



Conservation of Momentum and Relativity

- Momentum is conserved regardless of your frame of reference
  - Remember that **momentum** is defined as $\vec{p} = m\vec{v}$
  - Velocity depends on how you and the object are moving **relative** to each other
- As long as you do not change your velocity between "before" and "after", conservation of momentum still works
  - You are adding or subtracting the total mass of the system times your extra velocity to both "before" and "after" total momenta
- This observation has deep and subtle connections to Einstein's theory of special relativity

Momentum Conservation: Newton's Cradle

- Newton's Cradle
  - https://www.youtube.com/watch?v=JadO3RuOJGU

- Pendulum Waves
  - https://www.youtube.com/watch?v=V87VXA6gPuE

Center of mass

- We can restate the principle of conservation of momentum in a useful way by using the concept of center of mass.
- Suppose we have several particles with masses m_1, m_2, and so on.
- We define the center of mass of the system as the point at the position given by:

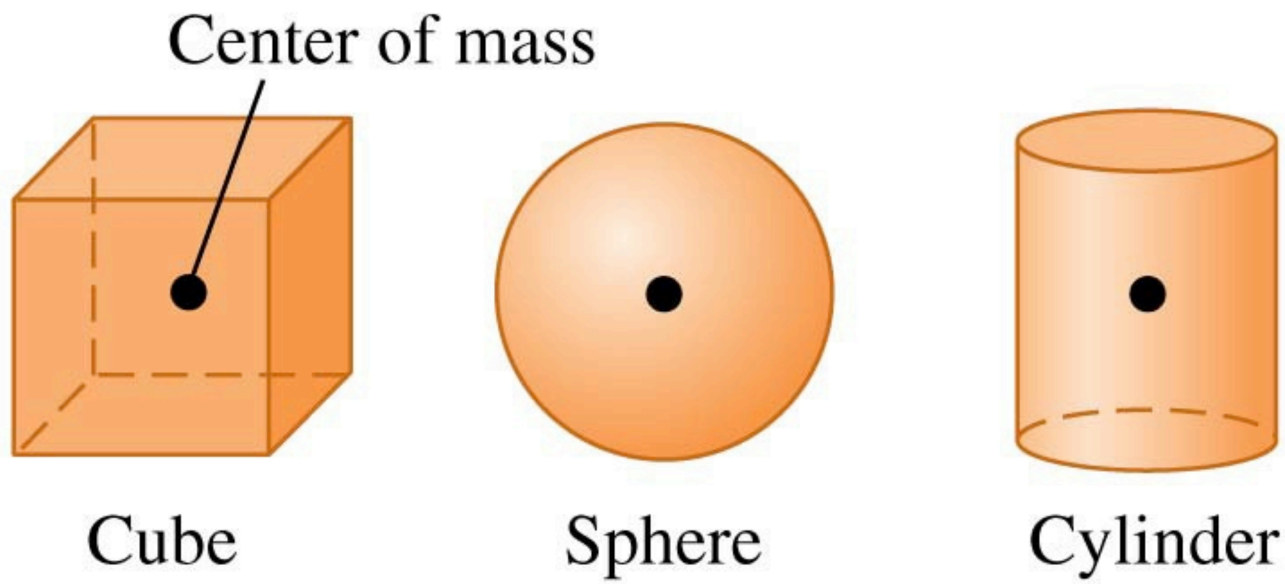
Position vectors of individual particles

Position vector of **center of mass** of a system of particles

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

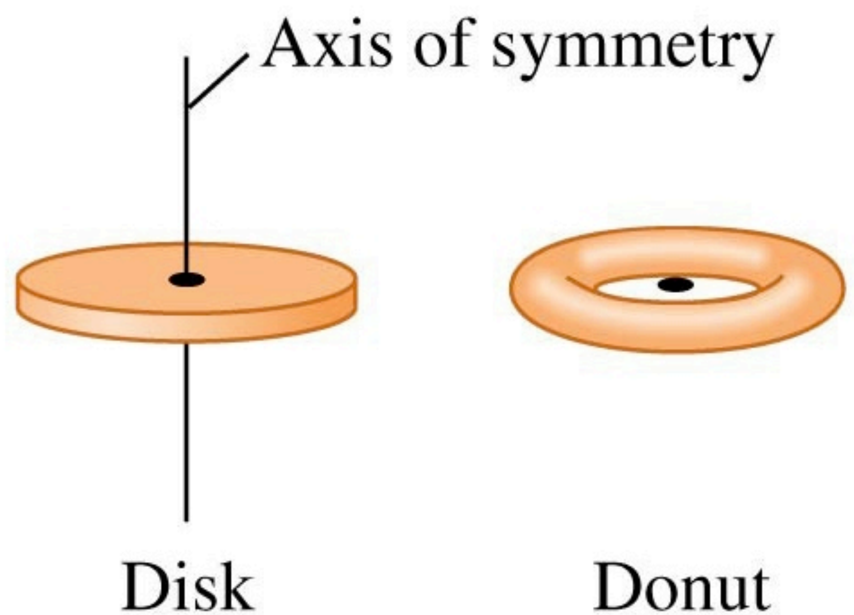
Masses of individual particles

Center of mass of symmetrical objects



If a homogeneous object has a geometric center, that is where the center of mass is located.

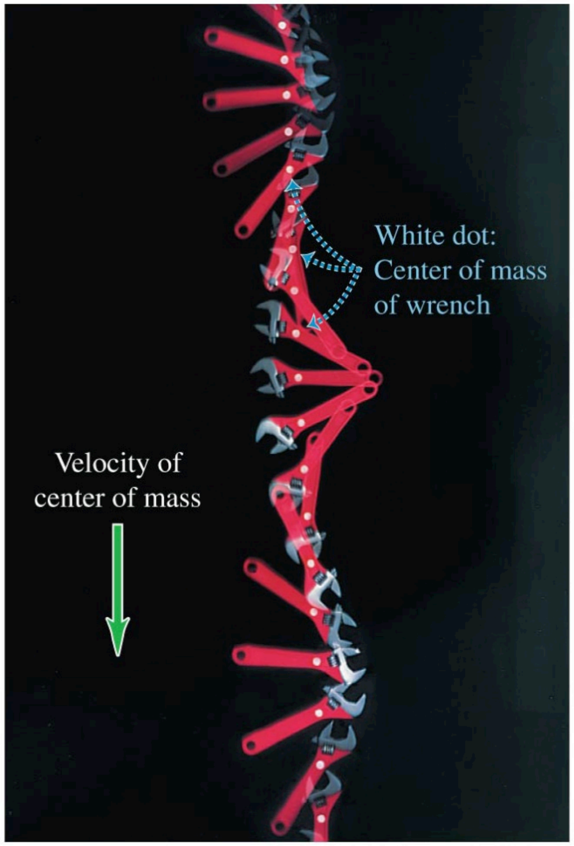
Center of mass of symmetrical objects



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

Motion of the center of mass

- The total momentum of a system is equal to the total mass times the velocity of the center of mass.
- The center of mass of the wrench at the right moves as though all the mass were concentrated there.



Total mass of a system of particles

Momenta of individual particles

$$M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = \vec{P}$$

Velocity of center of mass

Total momentum of system

External forces and center-of-mass motion

- When a body or a collection of particles is acted on by external forces, the center of mass moves as though all the mass were concentrated at that point and it were acted on by a net force equal to the sum of the external forces on the system.