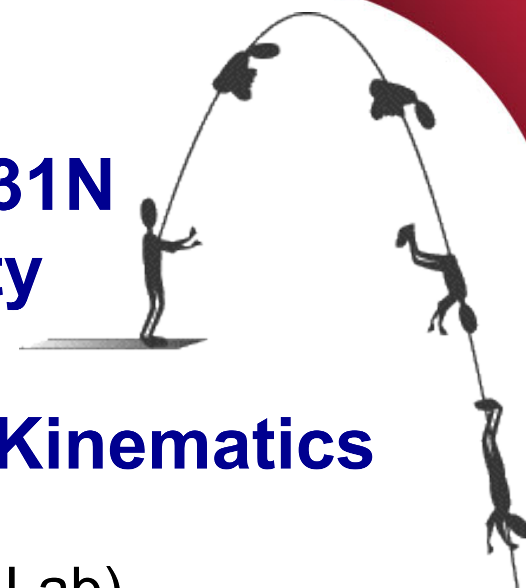
University Physics 226N/231N
Old Dominion University



Ch. 9: Rotational Motion, Angular Kinematics

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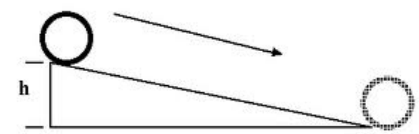


Monday, October 24, 2016

Reminder: The Third Midterm will be Mon Nov 21 2016

Second Midterms will be returned on Wednesday

Happy Birthday to Antonie van Leeuwenhoek, Drake, Kevin Kline, Wilhelm Weber, and Pierre-Gilles de Gennes (1991 Nobel)!

Happy United Nations Day and National Bologna Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

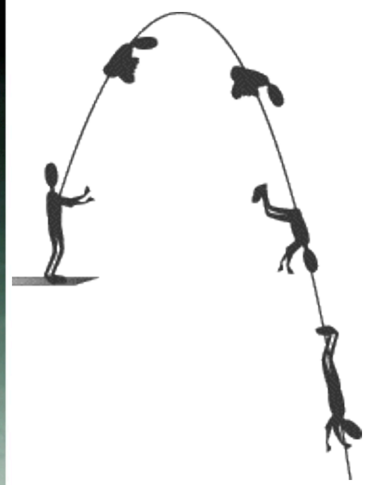
Coming Up…

- This Wednesday
  - I'll have all midterms to return
  - Will review common misconceptions
  - Will email everyone with grades to date, rank in class, and **estimated** letter grade
  - In plenty of time for drop deadline of Tue Nov 8

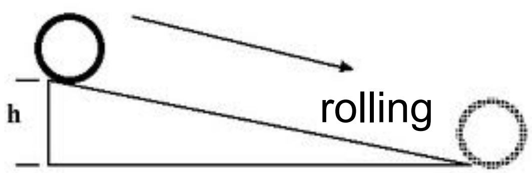
- Halloween next Monday!
  - I'll bring candy and I **might** dress up

- Substitute lecturer in two weeks!
  - Dr. Todd Goes To Vancouver
  - Dr. Balsa Terzic will be standing in on Mon Nov 7
  - No class (just homework and posted notes) Wed Nov 9!

This Week: Rotational Motion



- Describe the rotational motion of rigid bodies
  - We'll develop an analogy between new quantities describing rotational motion and familiar quantities from one-dimensional linear kinematics
- Calculate the rotational inertias of objects made of discrete and continuous distributions of matter
  - Rotational inertia is the rotational analog of mass
  - Gyroscopes!!
- We'll start to handle more interesting problems involving both linear and rotational motion
- We'll describe rolling motion



Quick One-Dimensional Kinematics Review

- We're going to draw explicit analogies between angular motion quantities and our old friends **position, velocity,** and **acceleration** from one-dimensional kinematics
- Time for a bit of review
  - Definitions of velocity and acceleration

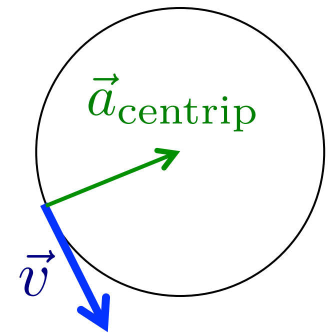
$$\text{velocity} \quad v \equiv \frac{dx}{dt} \qquad\qquad \text{acceleration} \quad a \equiv \frac{dv}{dt}$$

  - Constant acceleration motion

$$x - x_0 = v_0 t + \frac{1}{2} a t^2 \qquad\qquad v = v_0 + at$$

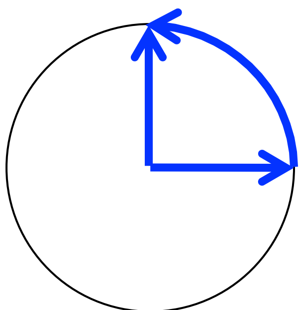
  - Centripetal acceleration related to tangential velocity

$$a_{\text{centrip}} = \frac{v^2}{r}$$

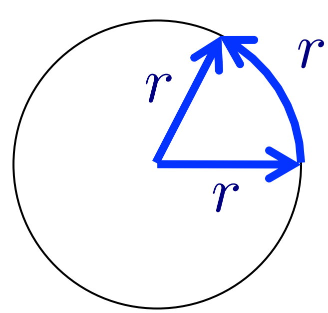


Angular Position

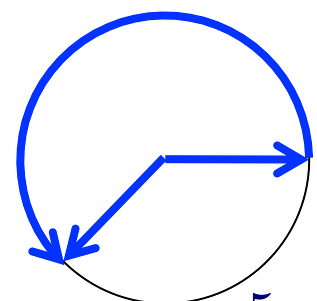
- What do we use for position in angular motion problems?
  - The angle that an object is from a reference angle $\theta = 0$
  - The sign convention is **usually that clockwise is positive**
  - The $\theta = 0$ location, like x=0, is usually defined by the problem
    - We care more about **angular distances**, $\Delta\theta = \theta(t_2) - \theta(t_1)$
  - We also always use **radians** where $\boxed{2\pi \text{ rad} = 360^\circ}$
    - 1 rad is the angle where the arc length is equal to the circle radius



$$\Delta\theta = 90^\circ = \frac{\pi}{2} \text{ rad}$$



$$\Delta\theta = 57.3^\circ = 1 \text{ rad}$$



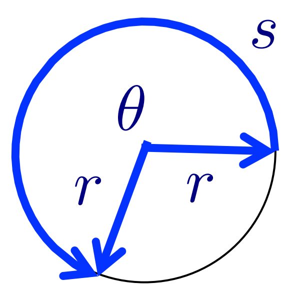
$$\Delta\theta = 225^\circ = \frac{5\pi}{4} \text{ rad}$$

- A conversion example: $225^\circ \left(\frac{2\pi \text{ rad}}{360^\circ}\right) = \frac{5}{8}(2\pi) \text{ rad} = \frac{5\pi}{4} \text{ rad}$

Angular Position: Importance of Radians

- That radians thing? Yeah, that's important…
  - If we write angles in radians, we can write a **tremendously useful equation** that relates the actual distance around the arc s to angles and radii:

$$\boxed{s = r\theta}$$



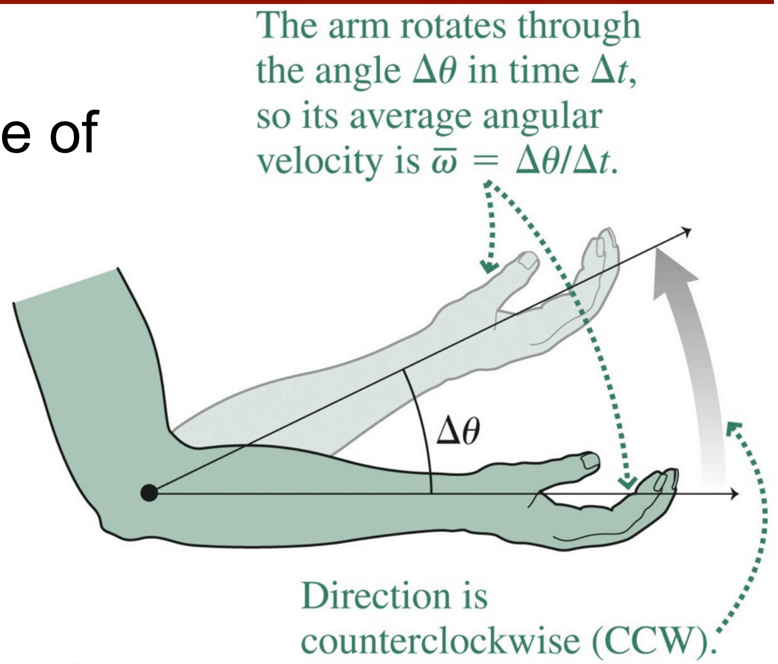
  - Here s is the distance around the arc. This formula lets us switch between real distances (like s and r, which are in distance units like meters) and angular distances (which are in radians)
  - Example: $\theta = 30^\circ = \frac{\pi}{6}\ \mathrm{rad}, \quad r = 2\ \mathrm{m} \quad \Rightarrow \quad s = \frac{\pi}{3}\ \mathrm{m} \approx 1.05\ \mathrm{m}$
- **Warning**: This equation (and most others we'll derive from now on) only work if the angle θ is in radians!
  - Since a radian is a ratio, it really is technically "dimensionless"

Angular Velocity

- **Angular velocity** ω [rad/s] is the rate of change of angular position with time

$$\text{Average}: \quad \bar{\omega} \equiv \frac{\Delta\theta}{\Delta t}$$

$$\text{Instantaneous}: \quad \omega \equiv \frac{d\theta}{dt}$$



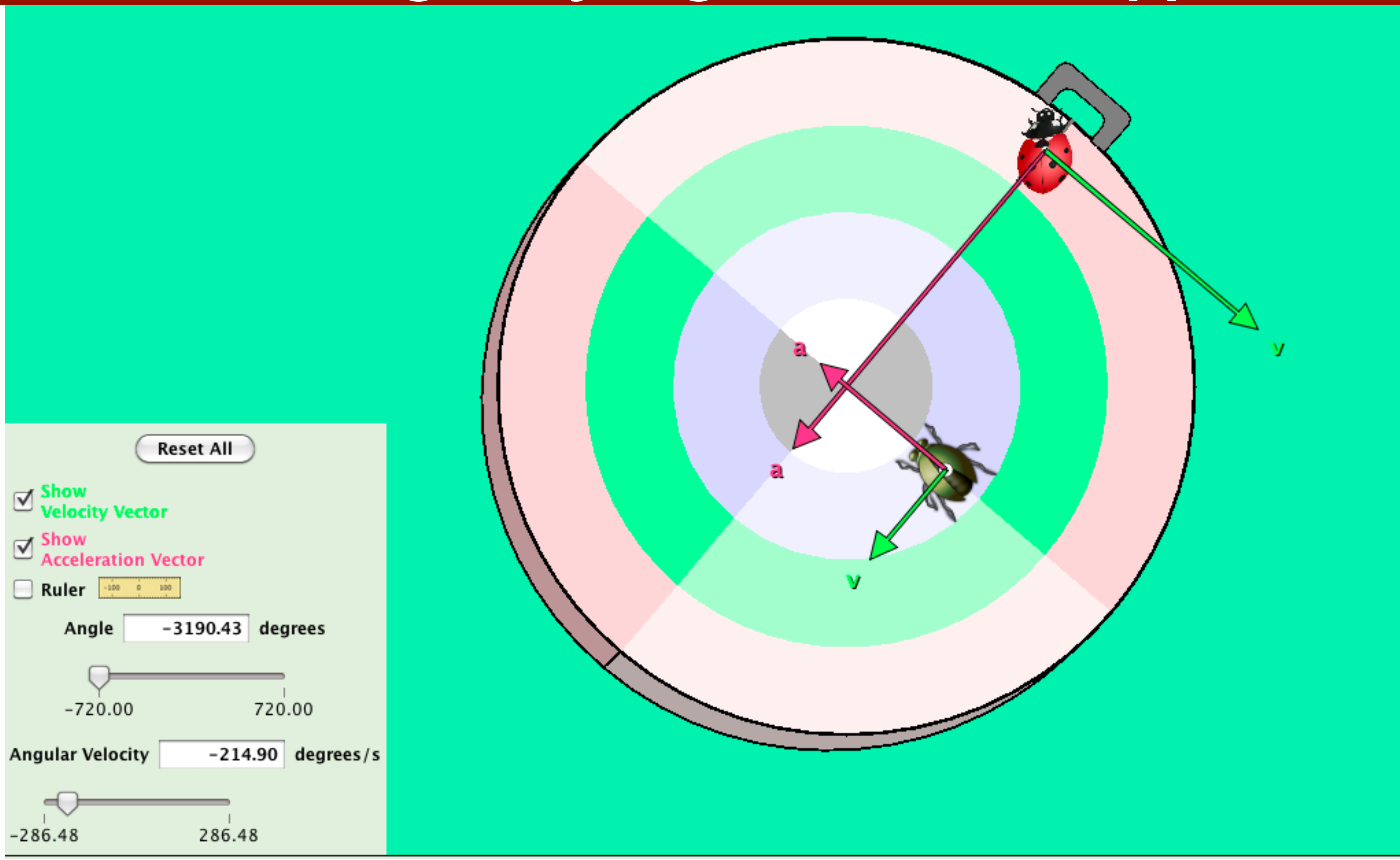
- Angular velocity ω is related to linear velocity v at a particular radius r from the rotation axis

$$\boxed{v = \omega\, r}$$

Warning: This is only true if the angular velocity distance units are radians!!!

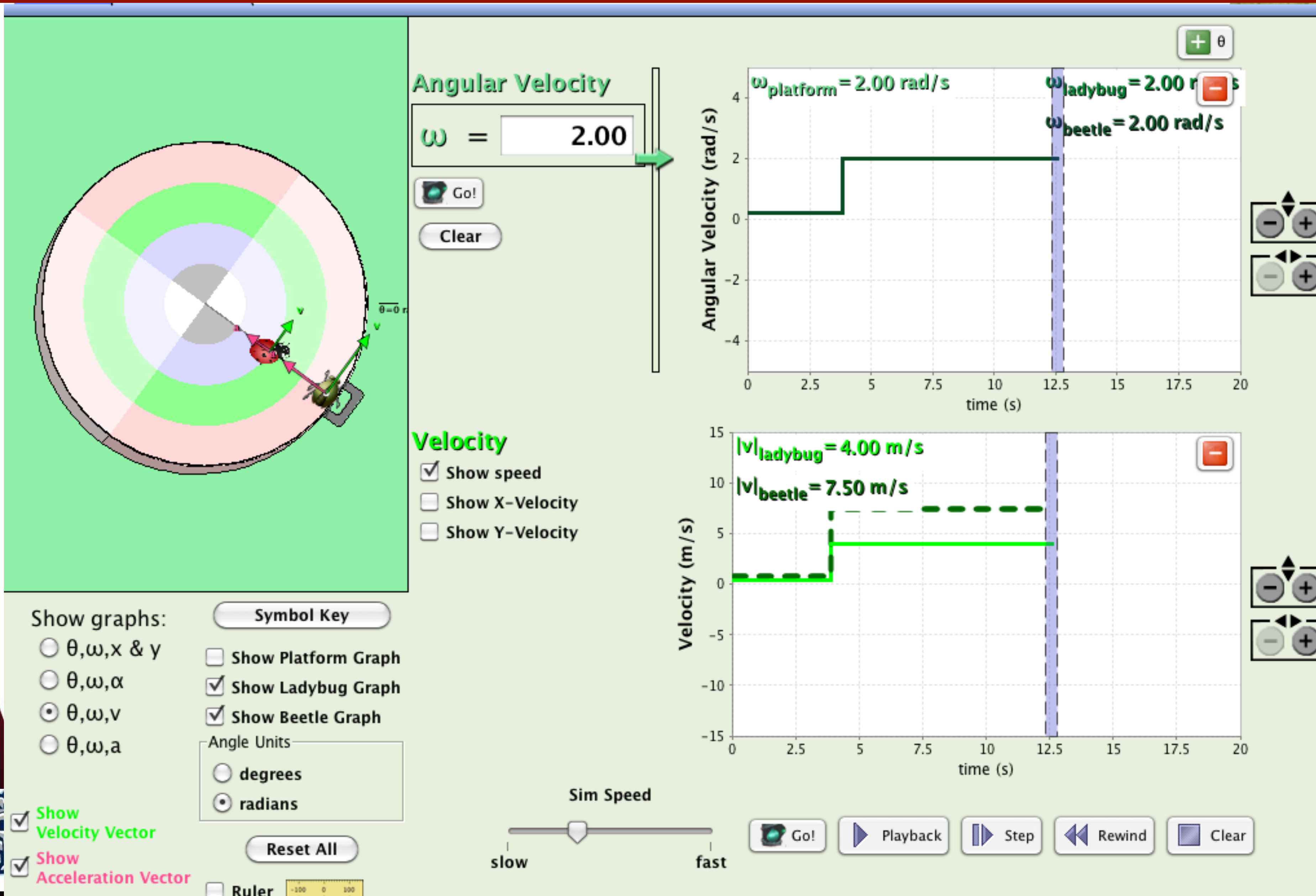
all radii rotate at 45 rpm but different v!

Revisiting Ladybug Revolution Applet



- http://phet.colorado.edu/en/simulation/rotation
- See differences in magnitudes **and** directions of **linear velocity and centripetal acceleration**
- Click on the "Rotation" tab

Revisiting Ladybug Revolution



Revisit an old Ponderable (10 minutes)

- Todd gets nostalgic and spins up his old **45 RPM** (revolutions per minute) record collection. Each record has a **7 inch total diameter**.
  - How fast is the outermost edge of the album moving in inches/sec?
  - How many "gees" of acceleration does a bug on the edge feel? (g=32 feet/s^2=384 inches/s^2)

Working the problem…

(1)

$$\omega = 45\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)\left(\frac{1\,\text{min}}{60\,\text{sec}}\right) = \frac{3}{2}\pi\,\frac{\text{rad}}{\text{s}}$$

$$v = \omega r = \left(\frac{3}{2}\pi\,\frac{\text{rad}}{\text{s}}\right)\left(3.5''\right) = \boxed{5.25\pi\ \text{in}/\text{s} = v}$$

$$r = 7''/2 = 3.5''$$

$$a = \frac{v^2}{r} = \frac{(5.25\pi\ \text{in}/\text{s})^2}{(3.5\,\text{in})} = 77.7\ \text{in}/\text{s}^2$$

$$= \underline{0.2\ \text{gees}}$$

Angular Acceleration

- **Angular acceleration** α [rad/s^2] is the rate of change of angular velocity with time

$$\text{Average}: \quad \bar{\alpha} \equiv \frac{\Delta\omega}{\Delta t}$$

$$\text{Instantaneous}: \quad \alpha \equiv \frac{d\omega}{dt}$$

- Angular acceleration α is related to **tangential** linear acceleration a_t at a particular radius r from the rotation axis

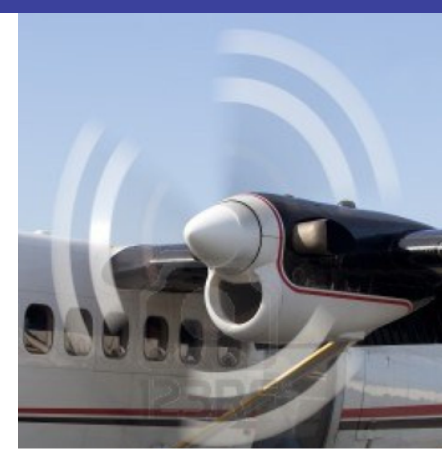
$$\boxed{a_t = \alpha\, r}$$

Recall there is also centripetal (radial) acceleration

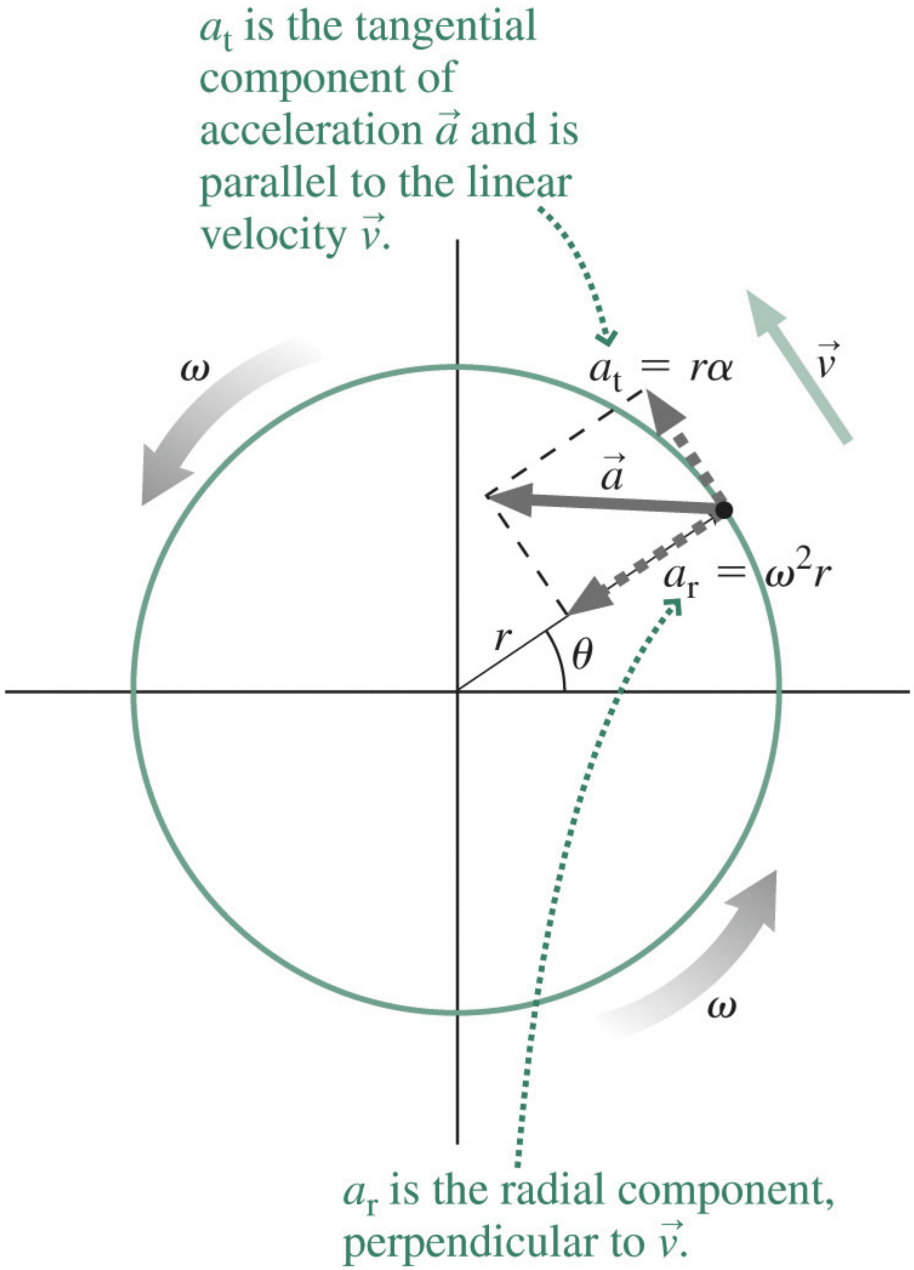
$$a_r = a_{\text{centrip}} = \frac{v^2}{r} = \omega^2 r$$

Think of an amusement park ride or propeller or engine spinning up – or spinning down.

That's angular acceleration.



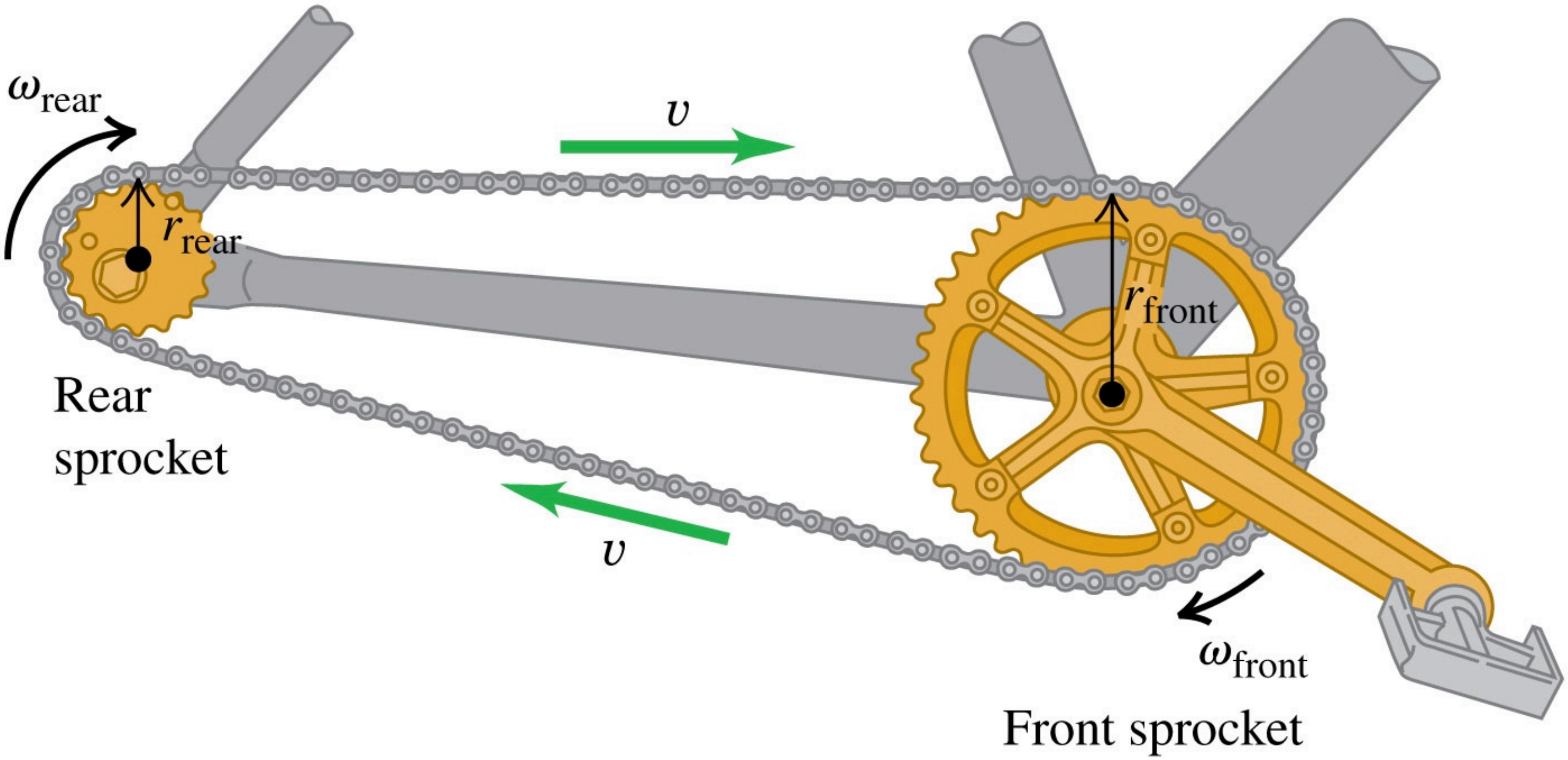
Angular Acceleration Picture (from text)



$$\boxed{a_t = \alpha\, r}$$

$$\boxed{a_r = a_{\text{centrip}} = \frac{v^2}{r} = \omega^2 r}$$

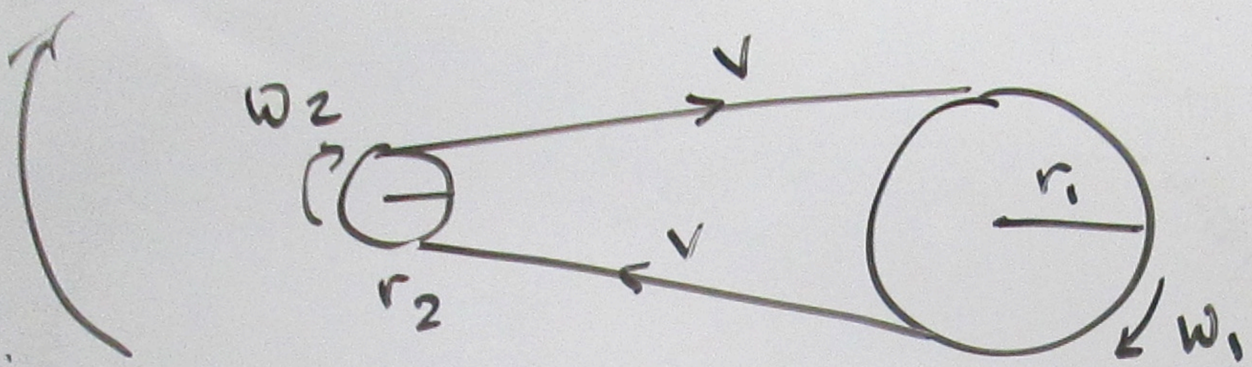
How do angular velocities compare?



The chain moves at the same linear velocity on both gears.
How do the angular velocities of both gears compare?
How do the angular accelerations of both gears compare?

Discussion

2



$$s = \theta r \Rightarrow v = \omega r$$

$$v = \omega_1 r_1$$
$$v = \omega_2 r_2$$
$$\Rightarrow \omega_1 r_1 = \omega_2 r_2$$

$$\frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$$

$$a = \alpha r$$

$$\frac{r_1}{r_2} = \frac{\alpha_2}{\alpha_1}$$

Constant Angular Acceleration

- Problems with constant angular acceleration are exactly analogous to similar problems involving linear motion in one dimension.
  - The **exact** same equations apply, with $x \to \theta, \quad v \to \omega, \quad a \to \alpha$

Table 10.1 Angular and Linear Position, Velocity, and Acceleration

Linear Quantity	Angular Quantity
Position x	Angular position θ
Velocity $v = \dfrac{dx}{dt}$	Angular velocity $\omega = \dfrac{d\theta}{dt}$
Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$

Equations for Constant Linear Acceleration		Equations for Constant Angular Acceleration	
$\bar{v} = \frac{1}{2}(v_0 + v)$	(2.8)	$\bar{\omega} = \frac{1}{2}(\omega_0 + \omega)$	(10.6)
$v = v_0 + at$	(2.7)	$\omega = \omega_0 + \alpha t$	(10.7)
$x = x_0 + v_0 t + \frac{1}{2}at^2$	(2.10)	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(10.8)
$v^2 = v_0^2 + 2a(x - x_0)$	(2.11)	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10.9)

Example Problem

- A record on a turntable is accelerated from rest to an angular velocity of 33.3 revolutions/minute in 2 secs. Find the average angular acceleration.

  - Solution: The initial angular velocity is zero.

$$\omega_0 = 0 \text{ rad/s}$$

The final angular velocity is

$$\omega = (33 \text{ rev/min}) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 3.46 \text{ rad/s}$$

Then we have

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t} = \frac{3.46 \text{ rad/s}}{2 \text{ s}} = 1.73 \text{ rad/s}^2$$

Example Problem 2

- A fan is started from rest and after 5.0 s has reached its maximum rotational velocity of 60 rad/s. Find the average angular acceleration and how many revolutions the fan makes while accelerating, assuming constant acceleration.

Solution:

$$\omega_0 = 0 \text{ rad/s}$$
$$\omega = 60 \text{ rad/s}$$
$$\Delta t = 5.0 \text{ s}$$

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t} = \frac{60 \text{ rad/s}}{5.0 \text{ s}} = 12 \text{ rad/s}$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2 \qquad \text{like} \quad \Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\Delta\theta = \frac{1}{2}(12 \text{ rad/s})(5.0 \text{ s})^2 = 150 \text{ rad}$$

or converting to revolutions:

$$\Delta\theta = 150 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 23.9 \text{ revolutions}$$

Example Problem 3 – book 9.13

- A turntable rotates with a constant 2.25 rad/s^2 angular acceleration. After 4.00 s it has rotated through an angle of 30.0 rad. What is the angular velocity of the wheel at the beginning of the 4.00 s interval?

Working through problem 9.13

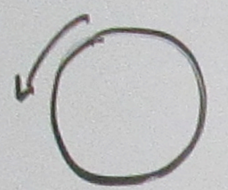
③

9.13 book)

$\alpha = 2.25\,\mathrm{rad/s^2}$

$t = 4.00\,\mathrm{s}$ $\qquad \Delta\theta = 30.0\,\mathrm{rad}$

$\omega_0 = ?$



$\Delta\omega = \alpha t$

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\times\ \omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

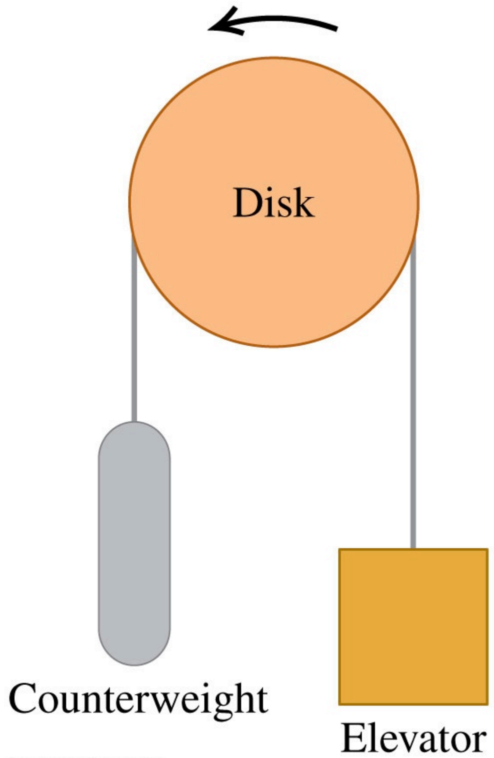
$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$\omega_0 = \frac{\Delta\theta}{t} - \frac{1}{2}\alpha t$$

$$= \frac{(30.0\,\mathrm{rad})}{(4.00\,\mathrm{s})} - \frac{1}{2}(2.25\,\mathrm{rad/s^2})(4.00\,\mathrm{s})$$

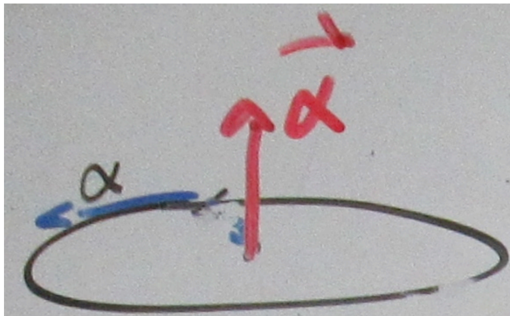
$$= 7.5\,\mathrm{rad/s} - 4.5\,\mathrm{rad/s} = \boxed{3.0\,\mathrm{rad/s} = \omega_0}$$

Example Problem 4 – book 9.18



- An elevator is constructed like the image to the left, with a rotating disk 2.50 m in diameter. The cable does not slip on the disk.
  - How many RPM must the disk turn to raise the elevator at 25.0 cm/s?
  - To start the elevator moving, it must be accelerated at 1/8 g. What must be the angular acceleration of the disk, in rad/s^2?
  - Through what angle has the disk turned when it has raised the elevator 3.25 m between floors?

Angular Acceleration as a Vector



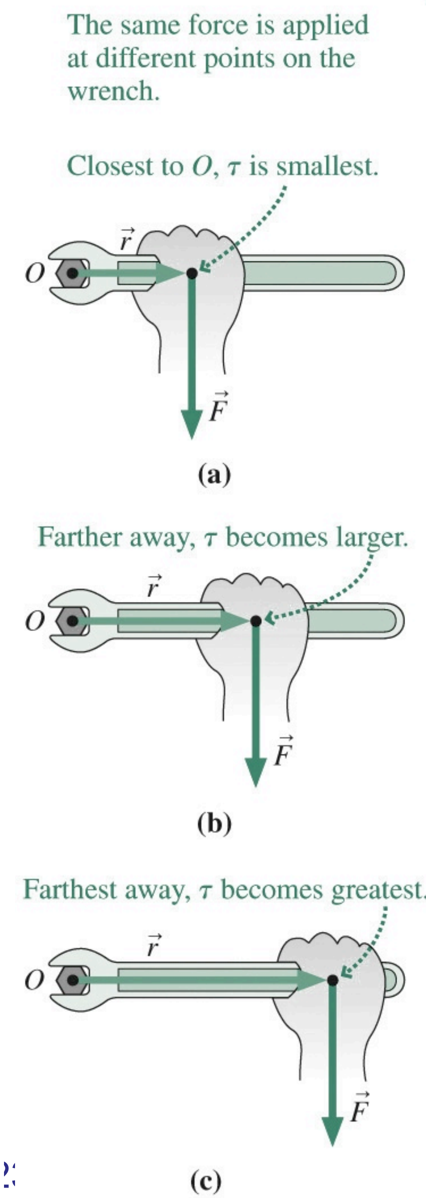
rotational motion
in a circle
with ang. acceleration α

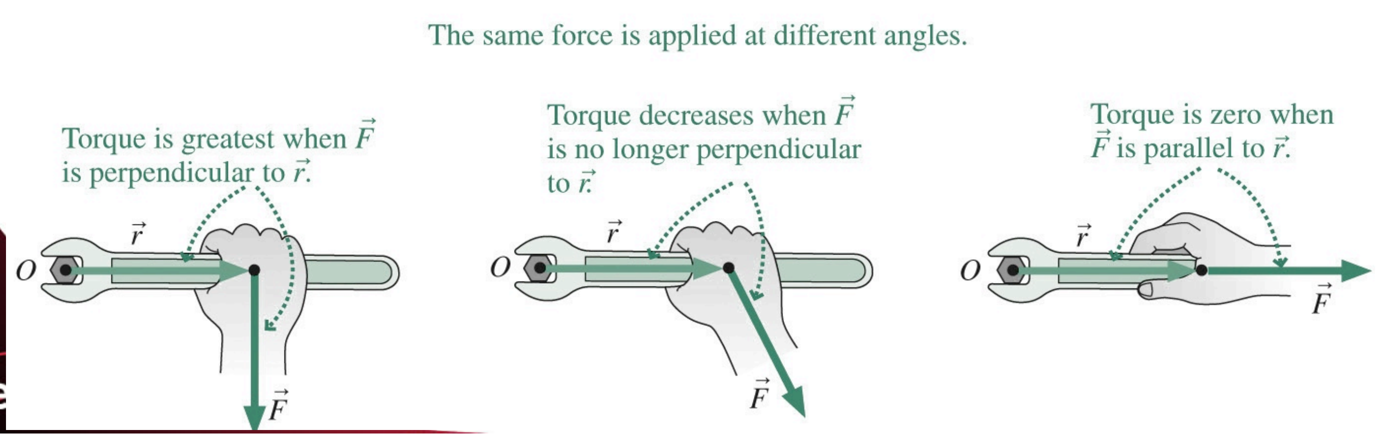
<u>ang</u> acceleration is a <u>vector</u>
points $\perp$ to plane of circle
direction $\Rightarrow$ right hand rule

Torque

- Torque τ is the rotational analog of force, and results from the application of one or more forces.
- Torque is relative to a chosen rotation axis.
- Torque depends on:
  - the distance from the rotation axis to the force application point.
  - the magnitude of the force $\vec{F}$
  - the orientation of the force relative to the displacement $\vec{r}$ from axis to force application point:

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \tau = rF\sin\theta$$





Torque as a Vector