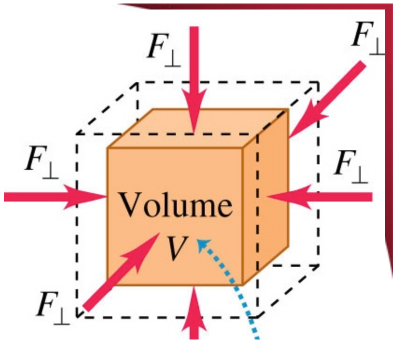
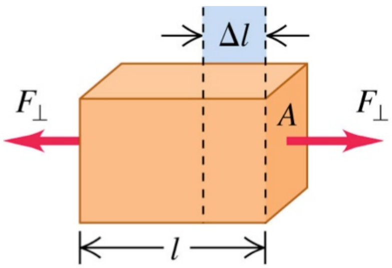
University Physics 226N/231N
Old Dominion University



Ch 10-11: Review Rotational Equilibrium
Stresses, Strains, Shears

Dr. Balša Terzić for Dr. Todd Satogata (Thanks Balša!)



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Monday, November 7, 2016

Reminder: The Third Midterm will be Mon Nov 21 2016

Happy Birthday to Marie Curie (1903 and 1911 Nobels), Lise Meitner, C.V. Raman (1930 Nobel), Lorde, Billy Graham, and Albert Camus!
Happy National Bittersweet Chocolate With Almonds Day!

Please set your cell phones to “vibrate” or “silent” mode. Thanks!

Review: Equilibrium And Statics

- Recall that in translational statics, we studied systems that have all forces acting on them balanced so the objects do not accelerate

$$\vec{F}_{\text{net}} = 0$$

- We can also apply this to potentially rotating systems where the forces aren't all acting on the object's center of mass
  - All the torques have to balance to keep the object from rotating or angularly accelerating too

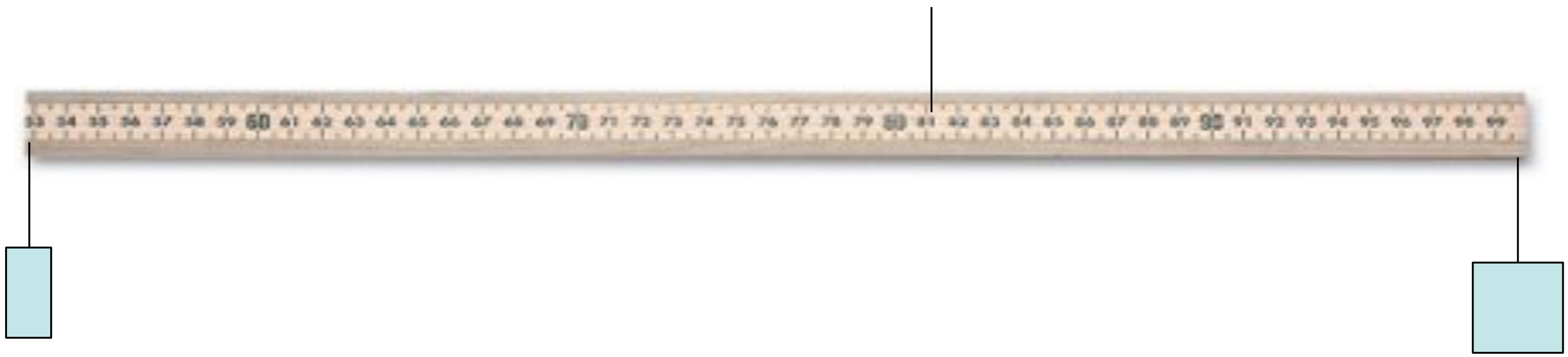
$$\vec{\tau}_{\text{net}} = 0$$

recall $\vec{\tau} \equiv \vec{r} \times \vec{F} = rF \sin\theta (\text{between r, F})$

- Later today we'll look at what happens when objects are deformed by tension, compression, shear, and stress too
  - But to start with, we review some applications of rotational equilibrium

Rotational Equilibrium 1: Mobile/Seesaw Balance

- You are building a simple mobile, starting by hanging two objects of masses m_1 and m_2 from the ends of a uniform stick of mass M and length L.
  - Where should you attach the string to hang your mobile?



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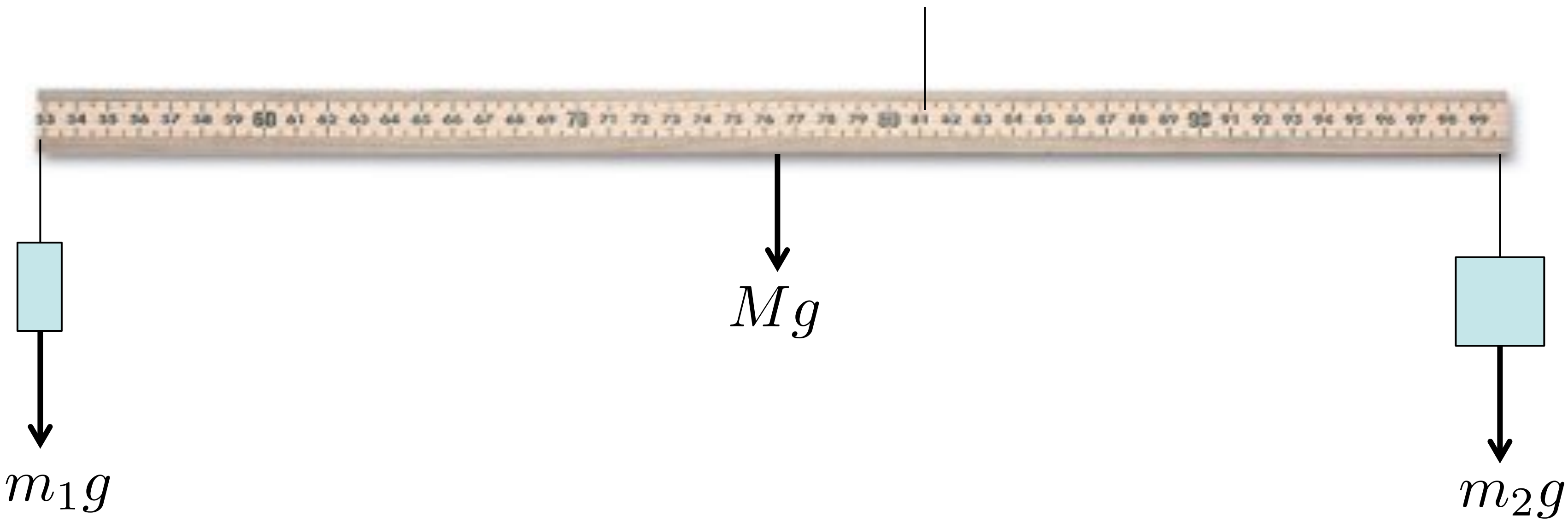

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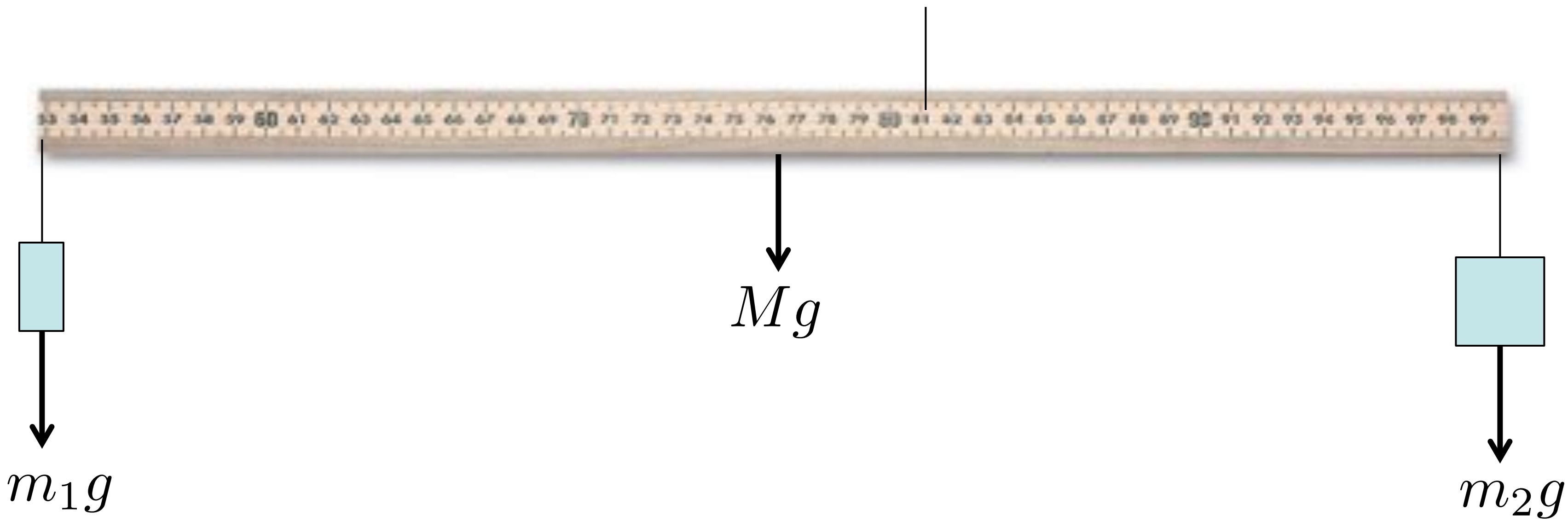

  - The tension in the support string is pretty clearly $(m_1+m_2+M)g$ to keep everything from falling downwards
  - Also need to balance torques for rotational equilibrium

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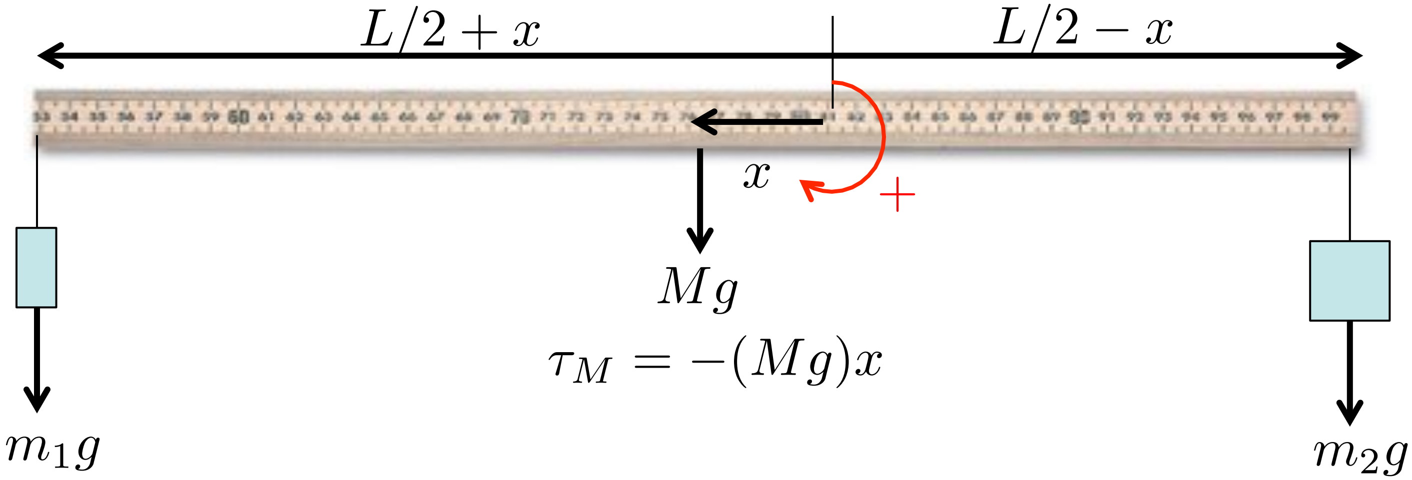

$$\tau_M = -(Mg)x$$

$$\tau_1 = -(m_1 g)(L/2 + x) \qquad \tau_2 = +(m_2 g)(L/2 - x)$$

For equilibrium: $\tau_{\text{net}} = 0$

$$0 = (m_2 g)(L/2 - x) - Mgx - (m_1 g)(L/2 - x)$$

$$\boxed{x = \frac{L(m_1 - m_2)}{2(m_1 - m_2 - M)}}$$

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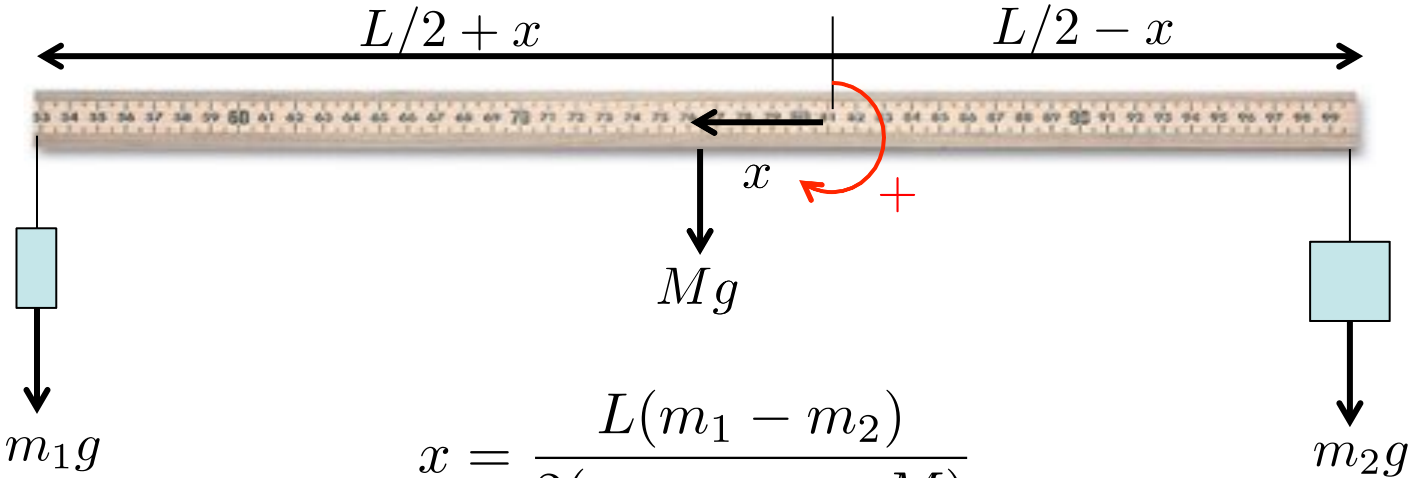


$$x = \frac{L(m_1 - m_2)}{2(m_1 - m_2 - M)}$$

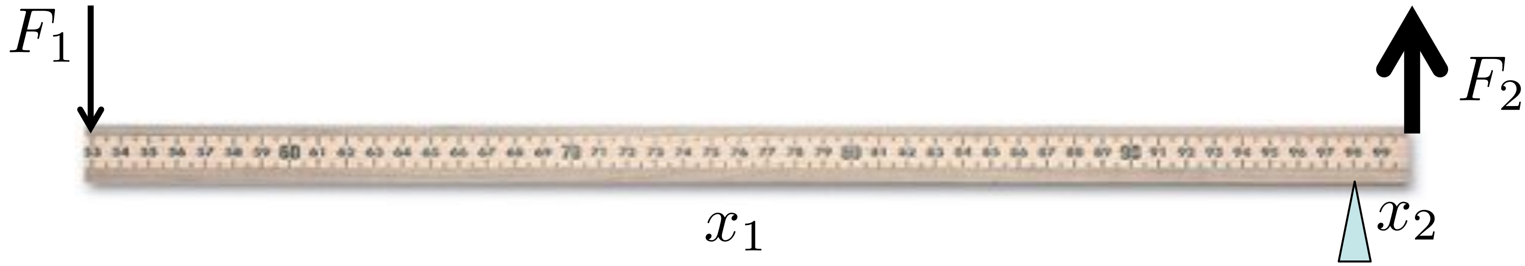
Evaluate: Does this answer make sense…

when $m_1=m_2$? (x=0 m)

when M is much larger than m_1 and m_2? (x is very close to 0 m)

when $m_1=0$ kg and $m_2=M$? (x = -L/4)

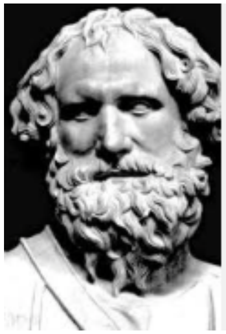
Rotational "Equilibrium" 2: Levers



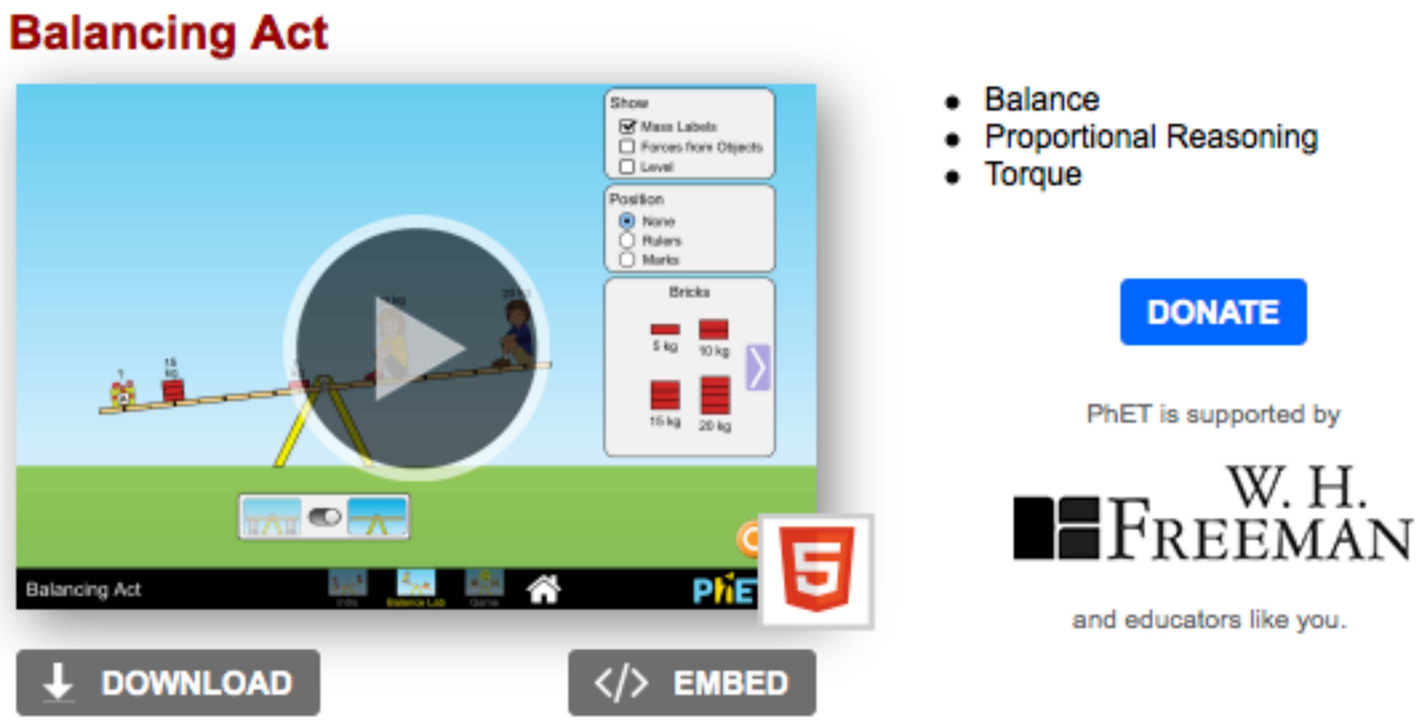
- The previous example illuminates why **levers** are considered good examples of simple machines
  - A small force F exerted with a large lever arm (radius) can produce very large magified forces: torques are the same

$$\tau_1 = \tau_2 \qquad \Rightarrow \qquad F_2 = \frac{x_1}{x_2} F_1$$

  - Archimedes: "Give me a lever long enough and a fulcrum on which to place it, and I shall move the world."
    - The fulcrum has to support the force F_2 to move the world though!!



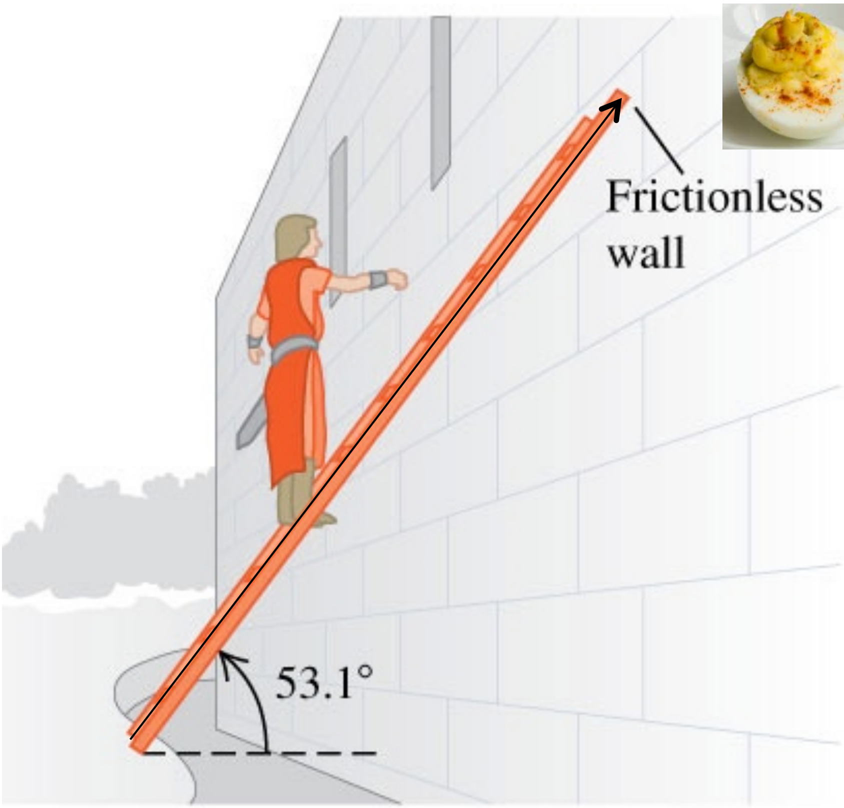
PHeT Simulation for simple seesaw torques



- PhET has a nice demo of seesaw balancing to play with simple torques and seesaw/mobile balancing
  - https://phet.colorado.edu/en/simulation/balancing-act
- Also see amazing balancing act on YouTube:
  - https://www.youtube.com/watch?v=-KVPA-9hofw

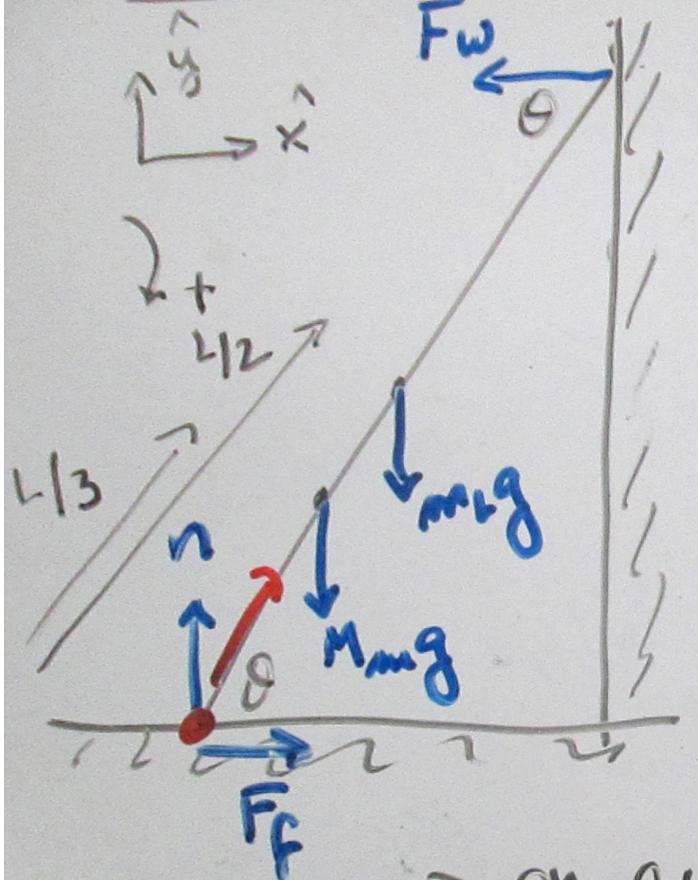
(Example: Lancelot and the Ladder)

- Sir Lancelot weighs 800N, and is climbing a 53.1 degree ladder that is 5.0m long and weighs 180N to raid a castle for its deviled eggs. The top of the ladder leans against a frictionless wall. The bottom of the ladder encounters friction on the ground. Lancelot is 1/3 of the way up the ladder.
  - Draw the force diagrams
  - Find normal and friction forces on the bottom of the ladder
  - Find the minimum coefficient of friction necessary to keep the ladder from slipping.



(Lancelot and the Ladder)

Ladder & wall



L = length of ladder

M_L = mass of ladder

M_M = mass of man

Draw forces ON LADDER ☺

Newton: linear

$$F_{net,x} = 0 = F_f - F_W \Rightarrow \boxed{F_f = F_W} \quad (= \mu_s n)$$

$$F_{net,y} = 0 = n - m_m g - m_L g$$

$$\Rightarrow \boxed{n = (M_m + M_L) g}$$

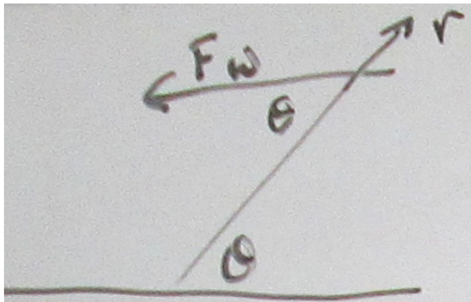
Newton: angular

$$\tau_{net} = 0 = \tau_M + \tau_L - \tau_W$$

Def^{n} of torque: $\vec{\tau} = \vec{r} \times \vec{F}$ or $\tau = rF\sin\theta(r \& F)$

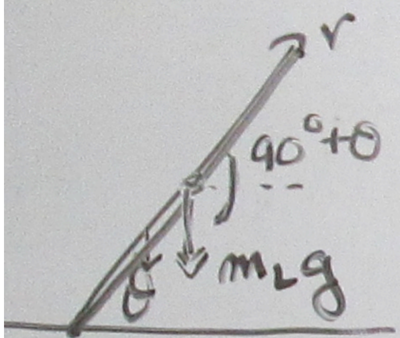
$$\Rightarrow 0 = \frac{L}{3} M_M g \cos\theta + \frac{L}{2} M_L g \cos\theta - L F_W \sin\theta$$

(Lancelot and the Ladder)



$$\tau_{wall} = L F_w \sin\theta \text{ (ladder \& wall)}$$

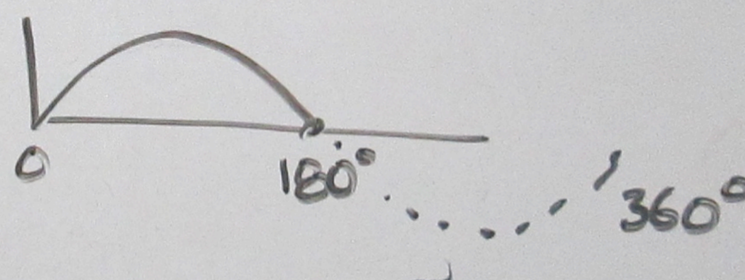
$$= L F_w \sin(180° - \theta) \quad \text{ccw}$$



$$\tau_{Ladder} = \left(\frac{L}{2}\right)(m_L g)\sin(90° + \theta)$$

$$\tau_{Man} = \left(\frac{L}{3}\right)(M_m g)\sin(90° + \theta)$$

sin



$$\Rightarrow \sin(180° - \theta) = \sin\theta$$

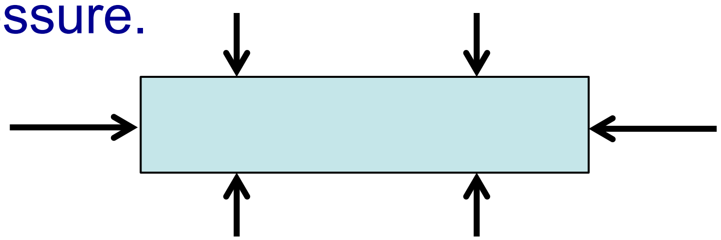
$$\sin(90° + \theta) = \cos\theta$$

Strain, stress, and elastic moduli

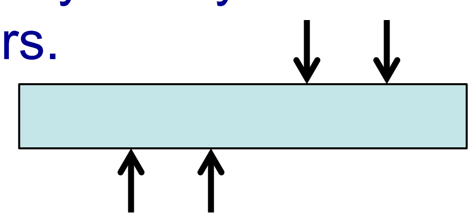
- Three examples and types of **stress**:
  a) Guitar strings under tensile stress, being stretched by forces acting at their ends.

(also linear compressive stress)

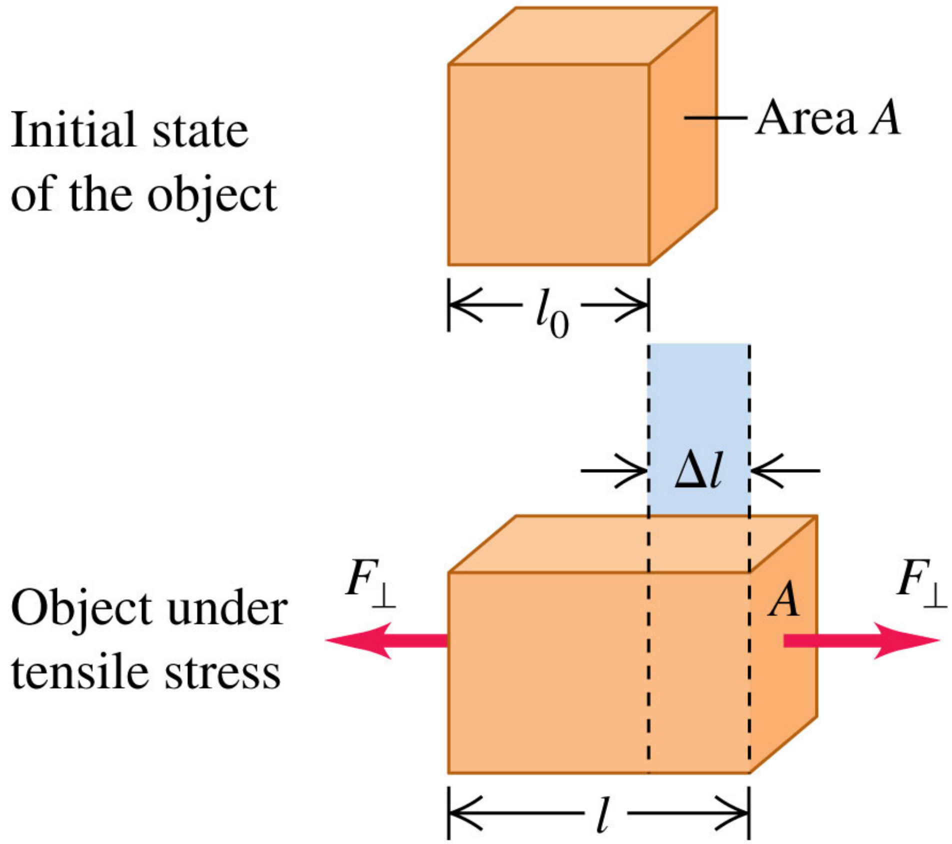
  b) A diver under bulk stress, being squeezed from all sides by forces due to water pressure.



  c) A ribbon under shear stress, being deformed and eventually cut by forces exerted by the scissors.



Quantifying Tensile Stress and Strain



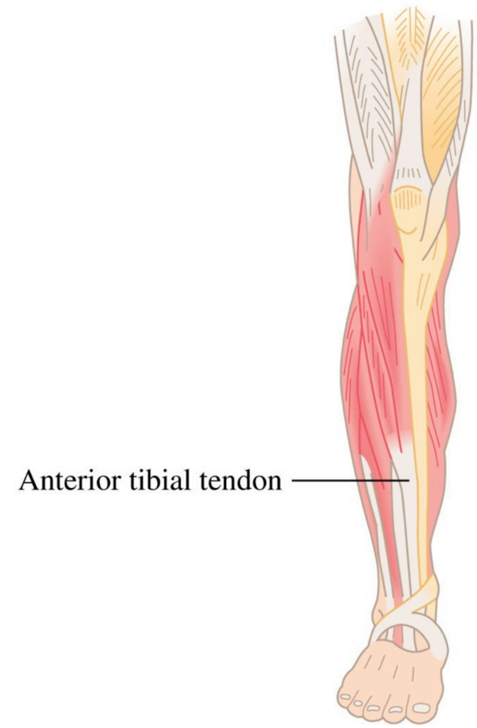
$$\text{Tensile stress} = \frac{F_\perp}{A} \qquad \text{Tensile strain} = \frac{\Delta l}{l_0}$$

- An object in tension.
- The net force on the object is zero, but the object deforms.
- The tensile stress produces a tensile strain.

- The units of **tensile stress is force per unit area**.
- **Tensile strain** is unitless: it is a **proportional** quantity.

Young's modulus

- Experiment shows that for a sufficiently small tensile stress, stress and strain are proportional.
- The corresponding elastic modulus is called **Young's modulus**.
- A human anterior tibial tendon has a Young's modulus of 1.2×10^9 Pascals
  - 1 Pascal is defined as 1 N/m^2, force per unit area

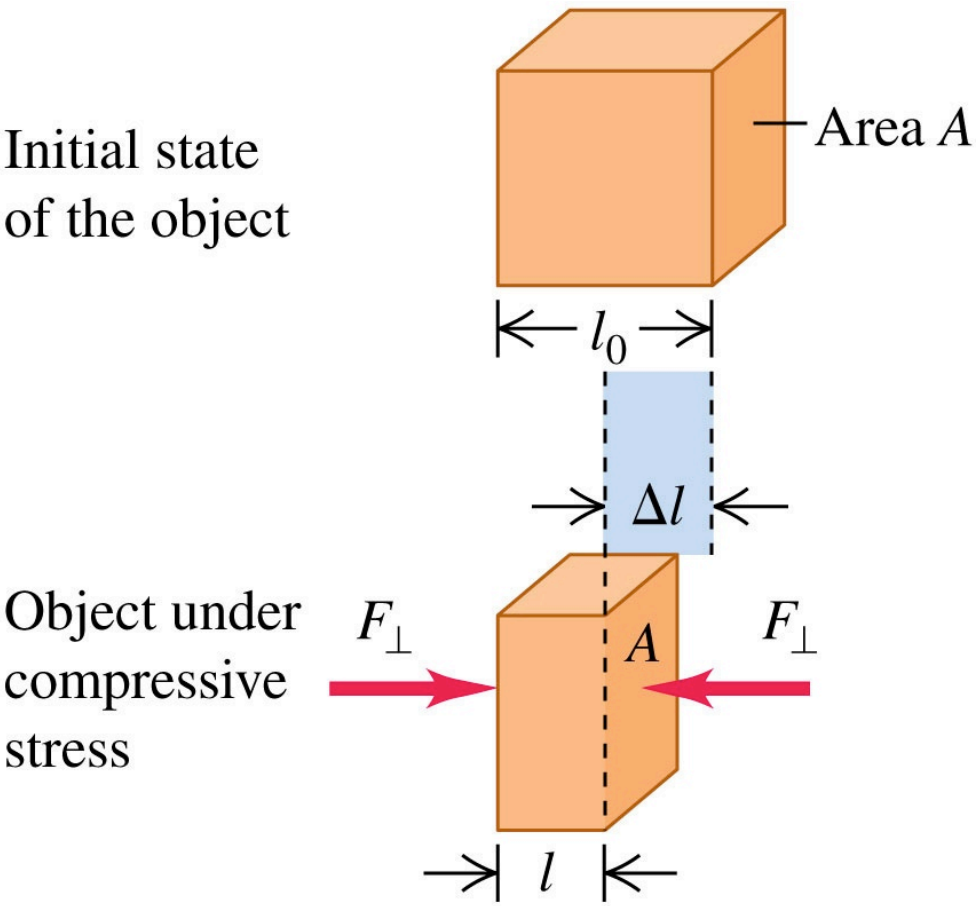


Young's modulus for tension

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp / A}{\Delta l / l_0} = \frac{F_\perp}{A} \frac{l_0}{\Delta l}$$

- $F_\perp$: Force applied perpendicular to cross section
- l_0: Original length (see Fig. 11.14)
- A: Cross-sectional area of object
- Δl: Elongation (see Fig. 11.14)

Compressive stress and strain



- An object in compression.
- The compressive stress and compressive strain are defined in the same way as tensile stress and strain, except that Δl now denotes the distance that the object contracts.

$$\text{Compressive stress} = \frac{F_\perp}{A} \qquad \text{Compressive strain} = \frac{\Delta l}{l_0}$$

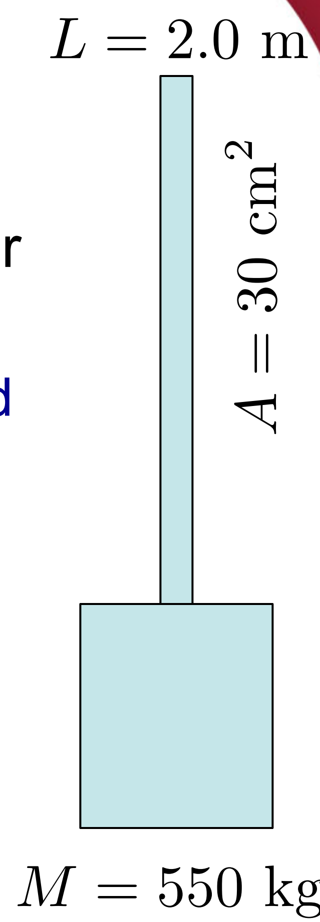
Some values of approximate elastic moduli

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Silicone rubber	0.001×10^{10}	0.2×10^{10}	0.0002×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}
Tendon (typical)	0.12×10^{10}	—	—

A much bigger table is at https://en.wikipedia.org/wiki/Young%27s_modulus

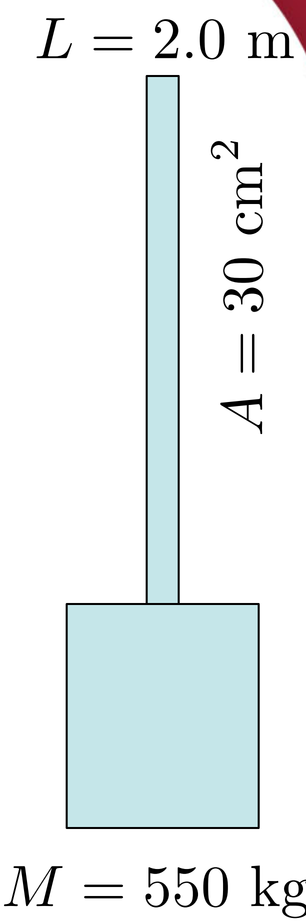
Stress Example

- A steel rod 2.0 m long has a cross-sectional area of 30 cm^2. It is hung by one end from a support, and a large machine of mass 550 kg is hung from the other end.
  - Find the stress on the rod, and the resulting strain and elongation.



Stress Example

- A steel rod 2.0 m long has a cross-sectional area of 0.30 cm². It is hung by one end from a support, and a large machine of mass 550 kg is hung from the other end.
  - Find the stress on the rod, and the resulting strain and elongation.



$$F = Mg = (550\text{ kg})(9.8\text{ m/s}^2) = 5390\text{ N}$$

$$\text{Young's modulus for steel}: Y = 20 \times 10^{10}\text{ Pa}$$

$$\text{Tensile stress}: = \frac{F_\perp}{A} = \frac{5390\text{ N}}{3.0 \times 10^{-5}\text{ m}^2} = 1.8 \times 10^8\text{ Pa}$$

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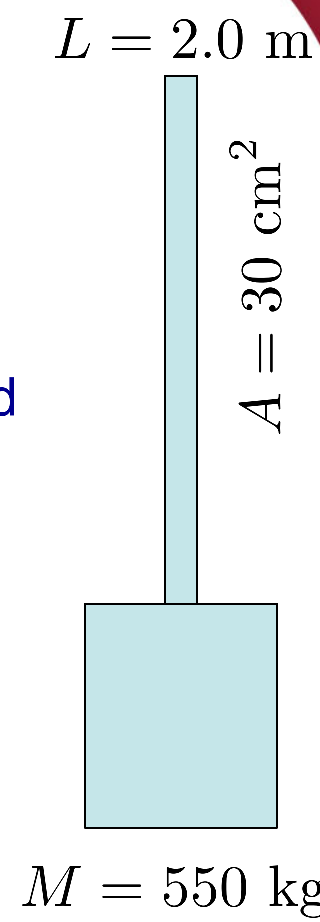

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$$\text{Young's modulus for steel}: Y = 20 \times 10^{10} \text{ Pa}$$

$$\text{Elongation} = \Delta L = (9.0 \times 10^{-4})(2.0 \text{ m}) = 1.8 \text{ mm}$$

The rod would compress by the same amount if the machine placed on top of it and compressed it rather than stretched it. It would probably be tough to balance this way though!

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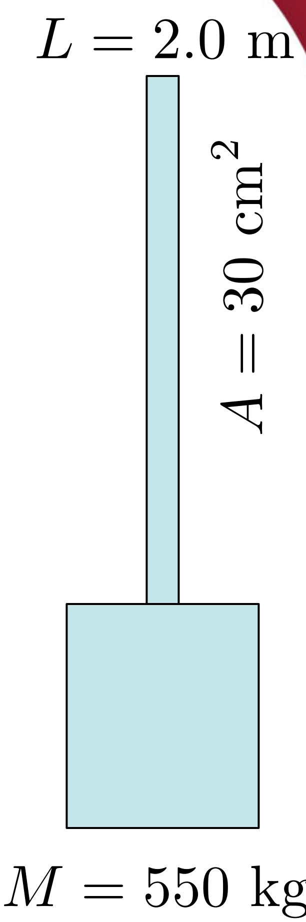

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$$\text{Strain} = \frac{\Delta L}{L} = \frac{\text{Stress}}{Y} = \frac{(1.8 \times 10^{8}\ \mathrm{Pa})}{(20 \times 10^{10}\ \mathrm{Pa})} = 9.0 \times 10^{-4}$$

$$\text{Elongation} = \Delta L = (9.0 \times 10^{-4})(2.0\ \mathrm{m}) = 1.8\ \mathrm{mm}$$

Stress Example 2

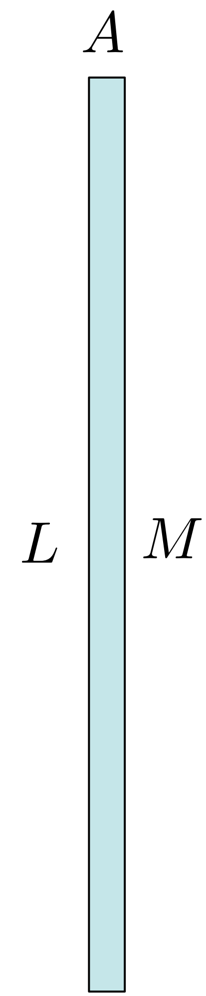
- A uniform steel rod of mass M=100 kg, length L=5m, and square cross section of 5 cm on each side is hanging from one end.
  - What is its elongation under its own weight?

$$A = (0.05\ \text{m})^2 = 2.5 \times 10^{-3}\ \text{m}^2$$

  - Note that different parts of the bar support more or less of the bar, so different parts of the bar have different **stress** and therefore different **strain**

Now depends on location in bar

$$\text{Tensile stress} = \frac{F_\perp}{A} \qquad \text{Tensile strain} = \frac{\Delta l}{l_0}$$



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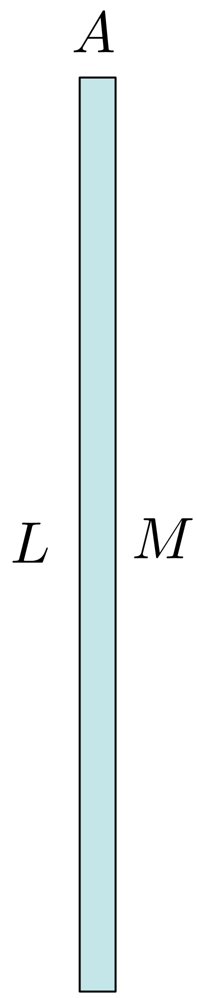
  - We can do this problem with calculus, integrating the strain throughout the bar
    - Doing it this way, you will find that the strain depends linearly on the location in the bar – zero near the bottom, and supporting nearly the full weight at the top.
  - Since this problem is linear, we can also work through a perfectly reasonable argument
    - On **average**, the bar stretches as though it supports its weight at its center of mass (halfway down the bar)
    - Closer to the top it stretches more; near the bottom it stretches less



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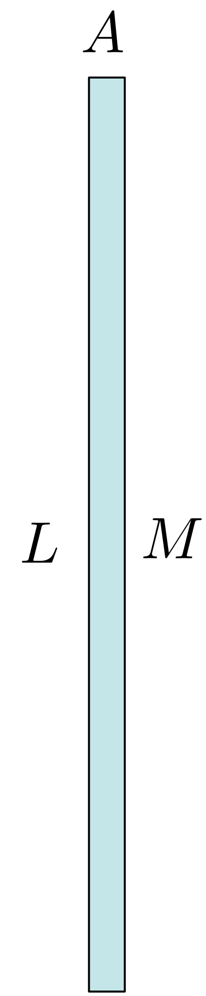
  - This ends up producing about what you would expect; the overall elongation is given by



$$\Delta L = \frac{(L/2)(Mg/A)}{Y}$$

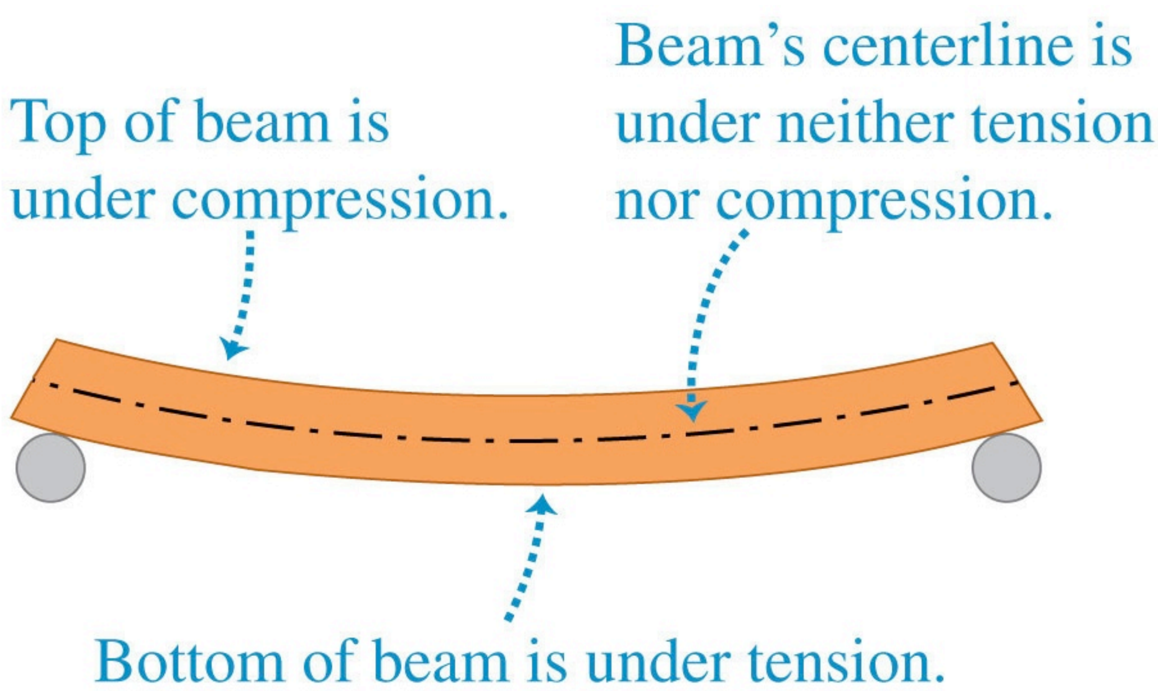
Note the L/2!

$$\Delta L = \frac{(2.5 \text{ m})(100 \text{ kg})(9.8 \text{ m/s}^2)/(2.5 \times 10^{-3} \text{ m}^2)}{20 \times 10^{10} \text{ Pa}}$$

$$\Delta L = 4.9 \times 10^{-6} \text{ m!}$$

Compression and tension

- In many situations, objects can experience both tensile and compressive stresses at the same time.
- For example, a horizontal beam supported at each end sags under its own weight.



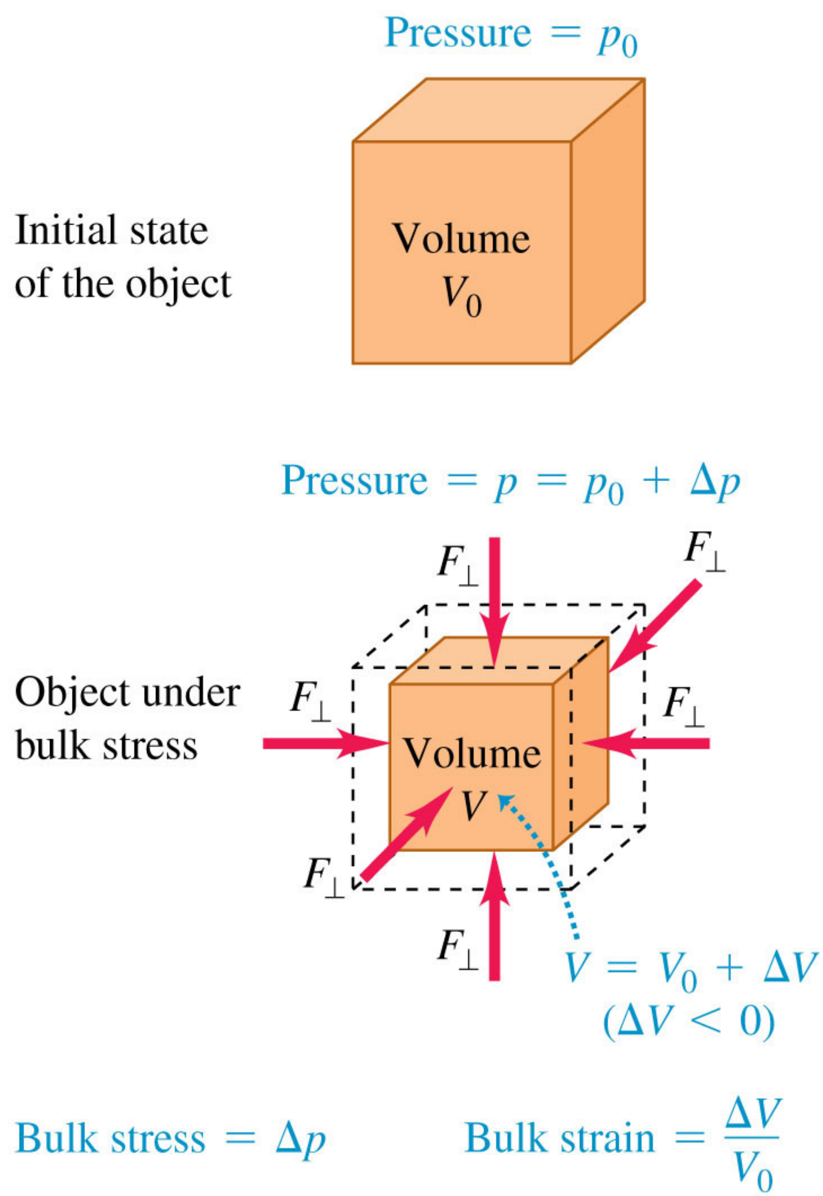
Bulk stress and strain

- Pressure in a fluid is force per unit area

$$p = F_{\perp}/A.$$

- **Bulk modulus** is bulk stress divided by bulk strain and is given by

$$B = -\Delta p/(\Delta V/V_0).$$



Bulk stress on an anglerfish

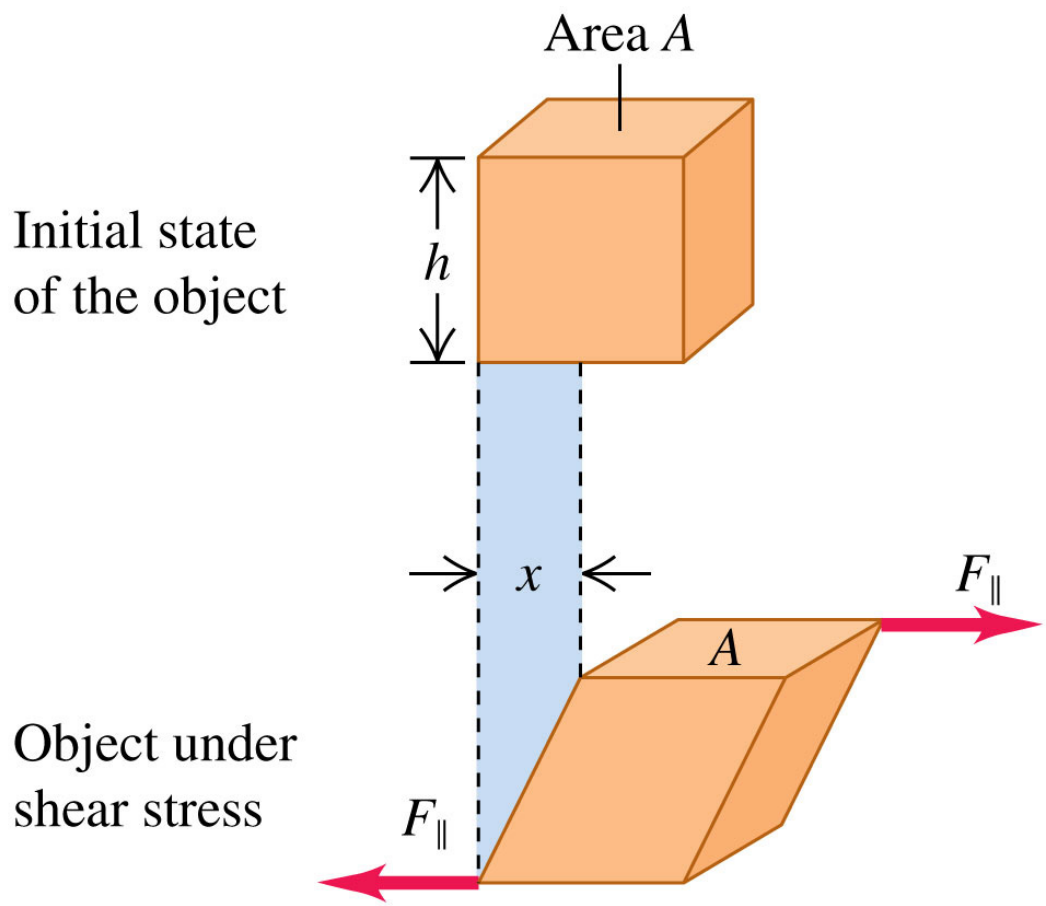
- The anglerfish is found in oceans throughout the world at depths as great as 1000 m, where the pressure (that is, the bulk stress) is about 100 atmospheres.

- Anglerfish are able to withstand such stress because they have no internal air spaces, unlike fish found in the upper ocean, where pressures are lower.
- The largest anglerfish are about 12 cm (5 in) long.

Sheer stress and strain

- **Sheer modulus** is sheer stress divided by sheer strain, and is given by

  $S = (F_{\|}/A)(h/x)$.



$$\text{Shear stress} = \frac{F_{\|}}{A} \qquad \text{Shear strain} = \frac{x}{h}$$

Some values of approximate elastic moduli

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
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Tendon (typical)	0.12×10^{10}	—	—

Compressibility

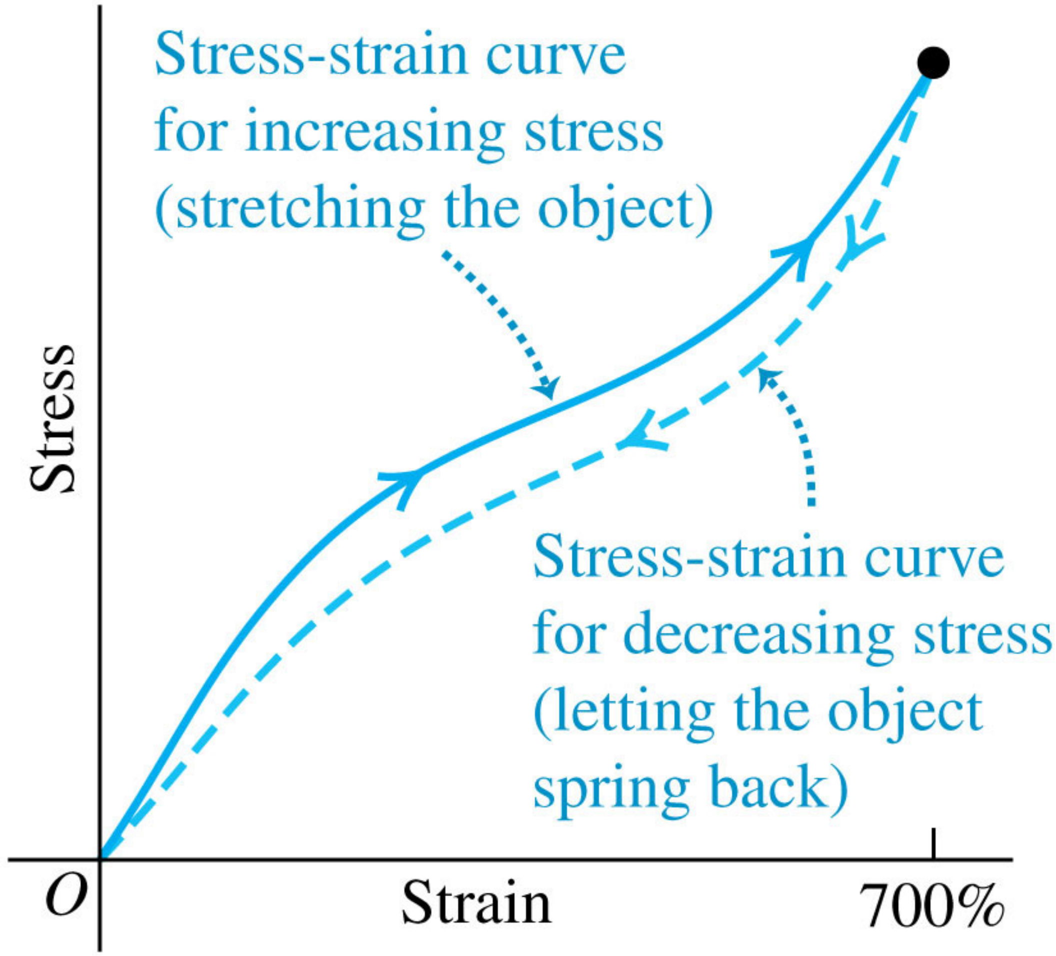
- The reciprocal of the bulk modulus is called the **compressibility** and is denoted by *k*.

$$k = \frac{1}{B} = -\frac{\Delta V/V_0}{\Delta p} = -\frac{1}{V_0}\frac{\Delta V}{\Delta p} \qquad \text{(compressibility)}$$

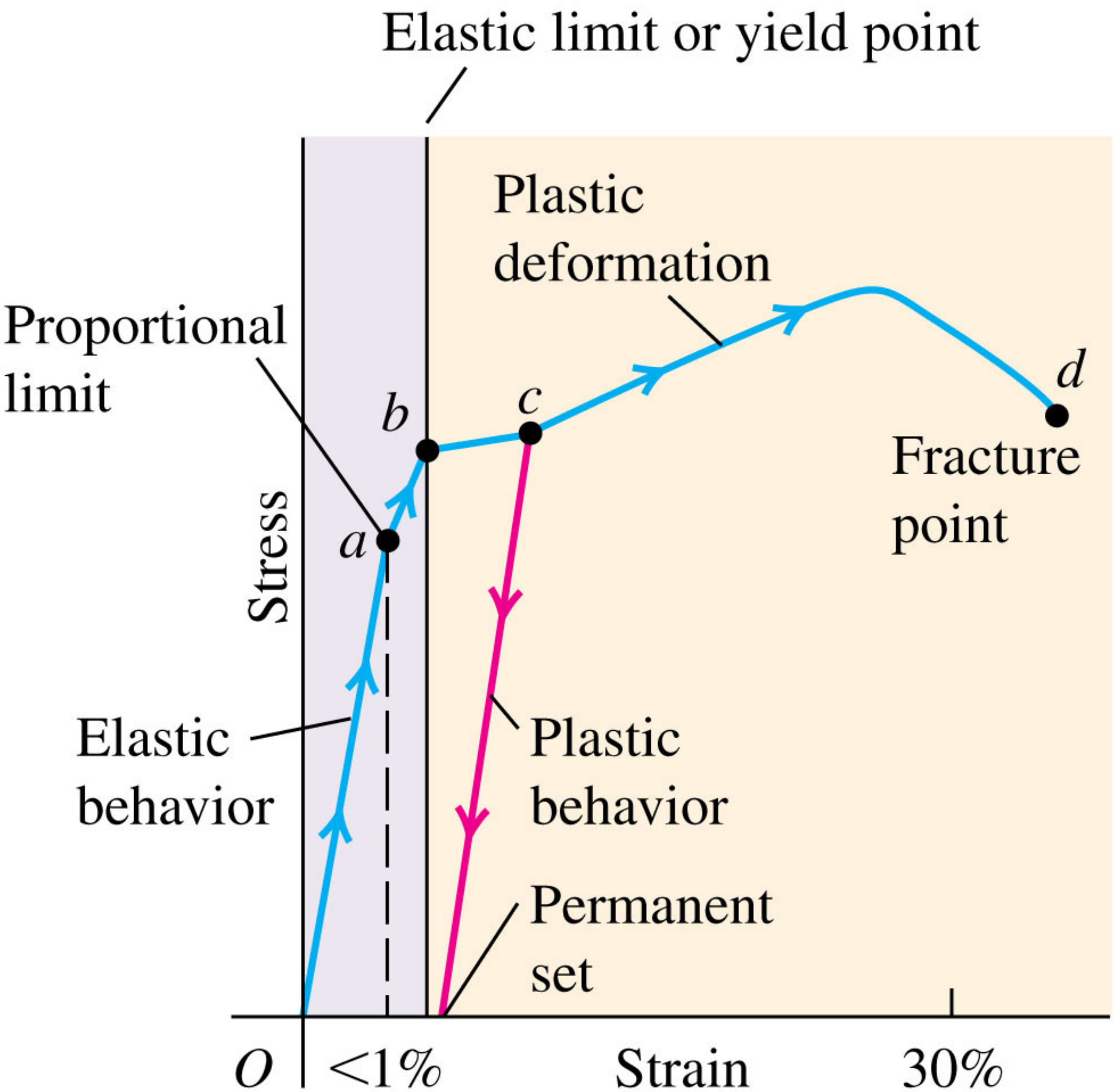
	Compressibility, *k*	
Liquid	$\mathbf{Pa^{-1}}$	$\mathbf{atm^{-1}}$
Carbon disulfide	93×10^{-11}	94×10^{-6}
Ethyl alcohol	110×10^{-11}	111×10^{-6}
Glycerine	21×10^{-11}	21×10^{-6}
Mercury	3.7×10^{-11}	3.8×10^{-6}
Water	45.8×10^{-11}	46.4×10^{-6}

Elasticity and plasticity

- Hooke's law—the proportionality of stress and strain in elastic deformations—has a limited range of validity.
- Shown is a typical stress-strain diagram fc vulcanized rubber, illustrating a phenomenon called **elastic hysteresis**.



Elasticity and plasticity



- Here is a typical stress-strain diagram for a ductile metal, such as copper or soft iron, under tension.

Approximate breaking stresses

- The stress required to cause actual fracture of a material is called the **breaking stress**.
- Table 11.3 gives typical values of breaking stress for several materials in tension:

Material	Breaking Stress (Pa or N/m^2)
Aluminum	2.2×10^8
Brass	4.7×10^8
Glass	10×10^8
Iron	3.0×10^8
Steel	$5-20 \times 10^8$
Tendon (typical)	1×10^8