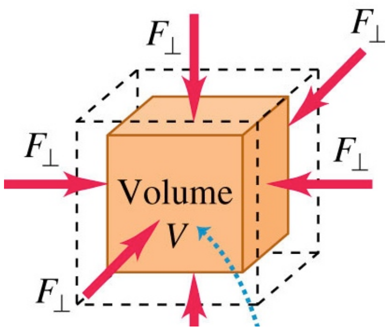

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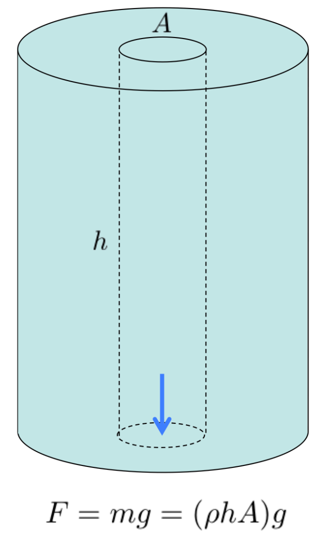
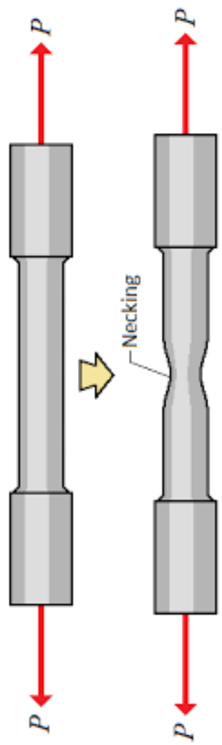


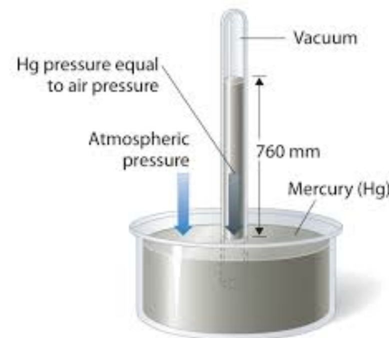


Ch 11: Finish Stress/Strain/Shear
Ch 12: Start Fluid Mechanics

Dr. Todd Satogata (ODU/Jefferson Lab)

satogata@jlab.org

http://www.toddsatogata.net/2016-ODU



Monday, November 14, 2016

Reminder: The Third Midterm will be Mon Nov 21 2016

Happy Birthday to Condoleeza Rice, Yanni, Buckwheat Zydeco, Boutros Boutros-Ghali, Aaron Copland, and Claude Monet!
Happy National Spicy Guacamole Day!

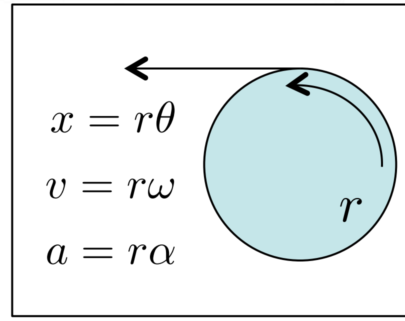
Please set your cell phones to “vibrate” or “silent” mode. Thanks!

Homework and Midterm #3

- **Homework:**
  - Apologies for last week's homework getting waylaid
  - I'll be assigning some homework for review for Midterm #3, due next Monday morning before the midterm
  - This homework covers items that will be on the midterm
- **Midterm #3:**
  - Rotational kinematics and rotational motion
    - Including conservation of energy and angular momentum
  - Rotational equilibrium
  - Stress/Strain/Shear
  - Fluid mechanics
  - I will prepare a handout with all cheat sheat materials
    - Includes tables of material constants, moments of inertia, etc
  - I'll also post something like a sample midterm

(Cheat Sheet: Linear and Angular Summary Comparison)

Linear Quantity	Angular Quantity
Position x	Angular position θ
Velocity $v = \dfrac{dx}{dt}$	Angular velocity $\omega = \dfrac{d\theta}{dt}$
Acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$



Equations for Constant Linear Acceleration	Equations for Constant Angular Acceleration
$\bar{v} = \frac{1}{2}(v_0 + v)$	$\bar{\omega} = \frac{1}{2}(\omega_0 + \omega)$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$ "movie equation"	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$ "equation that doesn't involve time"	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Forces

Torques: $\vec{\tau} = \vec{r} \times \vec{F}$

Newton's 2nd Law: $\vec{F}_{\text{net}} = m\vec{a} = d\vec{p}/dt$

$\vec{\tau}_{\text{net}} = I\vec{\alpha} = d\vec{L}/dt$

Momentum: $\vec{p} = m\vec{v}$

$\vec{L} = I\vec{\omega}$

Kinetic Energy: $\text{KE} = \frac{1}{2}mv^2$

$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2$

(inertial) Mass: m

Moment of Inertia: $I = \sum m_i r_i^2$

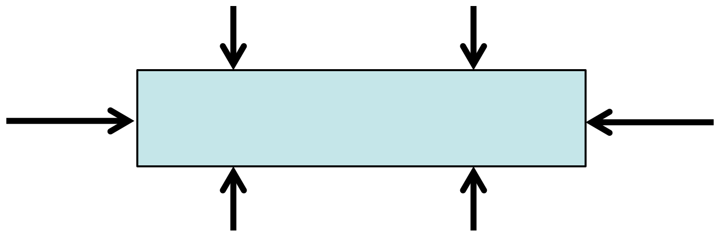
Parallel Axis Theorem: $I = I_{\text{cm}} + Md^2$

Review: Strain, stress, and elastic moduli

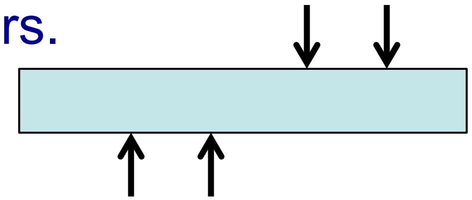
- Three examples and types of **stress**:
  a) Guitar strings under **tensile stress**, being stretched by forces acting at their ends.

(also linear compressive stress)

  b) A diver under **bulk stress**, being squeezed from all sides by forces due to water pressure.



  c) A ribbon under **shear stress**, being deformed and eventually cut by forces exerted by the scissors.



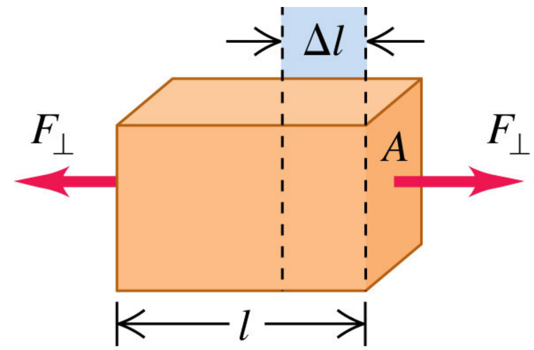
Review: Elastic Moduli Definitions

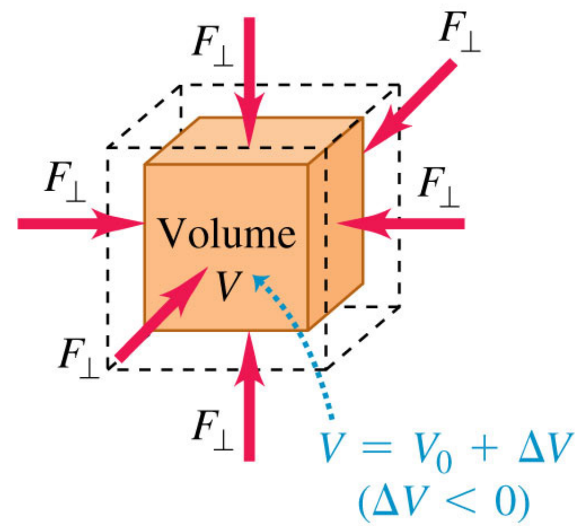
- Tensile stress: $F_\perp / A$
- Tensile strain: $\Delta l / l_0$
- Young's modulus: $Y \equiv \dfrac{F_\perp / A}{\Delta l / l_0}$

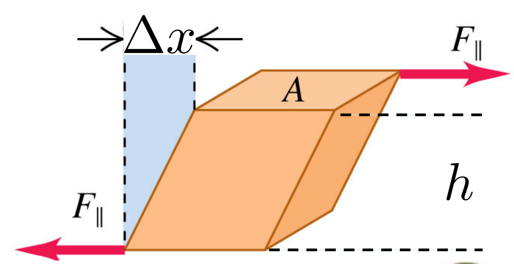
- **Pressure** or volume stress: $p = F_\perp / A$
- Volume strain: $\Delta V / V_0$
- Bulk modulus: $B \equiv -\Delta p / (\Delta V / V_0)$

- Shear stress: $F_\parallel / A$
- Shear strain: $\Delta x / h$
- Shear modulus: $S \equiv \dfrac{F_\parallel / A}{(\Delta x / h)}$

- All moduli are **material properties**
  - All moduli assume **elastic deformation**
  - All moduli have units of pressure [Pa]







Some values of approximate elastic moduli

Larger modulus means larger force required for deformation

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	B/Y	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	1.07	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	0.66	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}		4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	0.76	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	2.56	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}		7.8×10^{10}
Silicone rubber	0.001×10^{10}	0.2×10^{10}	200!	0.0002×10^{10}
Steel	20×10^{10}	16×10^{10}		7.5×10^{10}
Tendon (typical)	0.12×10^{10}	—		—

Larger B/Y ratio: Stress doesn't change volume much
Material necks or squeezes out sides
Smaller B/Y ratio: Stress changes volume more
Material doesn't neck or squeeze out

Problem: Post Compression

- **Problem 11.30**: A vertical, solid steel post 25 cm in diameter and 2.50 m long is required to support a load of 8000 kg. Ignore the weight of the post. What are the stress, strain, and change in the post's length when the load is applied?

Problem: Post Compression (corrected for cylindrical post!)

- **Problem 11.30**: A vertical, solid steel post 25 cm in diameter and 2.50 m long is required to support a load of 8000 kg. Ignore the weight of the post. What are the stress, strain, and change in the post's length when the load is applied?

$$Y(\text{steel}) = 20 \times 10^{11} \text{ Pa} \qquad Y \equiv \frac{F_\perp / A}{\Delta l / l_0}$$

$$r = 0.125 \text{ m} \qquad A = \pi (0.125 \text{ m})^2 = 0.049 \text{ m}^2$$

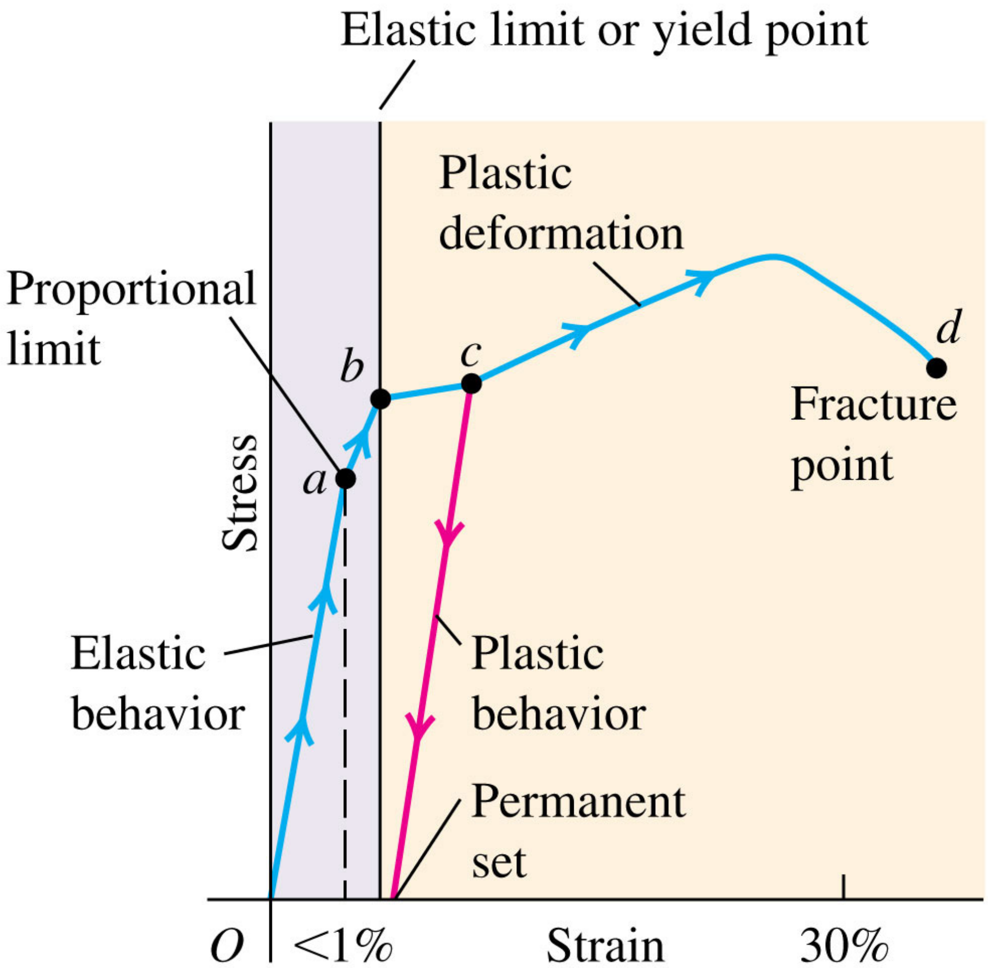
$$l = 2.50 \text{ m}$$

$$\text{stress} = \frac{F_\perp}{A} = \frac{(8000 \text{ kg})(9.8 \text{ m/s}^2)}{0.049 \text{ m}^2} = 1.6 \times 10^6 \text{ Pa}$$

$$\text{strain} = \frac{\text{stress}}{Y} = \frac{(1.6 \times 10^6 \text{ Pa})}{(20 \times 10^{11} \text{ Pa})} = 8 \times 10^{-7}$$

$$\Delta l = l \times \text{strain} = (2.50 \text{ m})(8.0 \times 10^{-7}) = 2 \times 10^{-6} \text{ m}$$

Elasticity and Plasticity: Hooke's Law



- Here is a typical stress-strain diagram for a ductile metal, such as copper or soft iron, under tension.
- The linear relationship of stress and strain is called **Hooke's Law**
  - It is not really a law!
  - Only applies for ideal springs and **modest** material deformations

The Limits of Hooke's Law

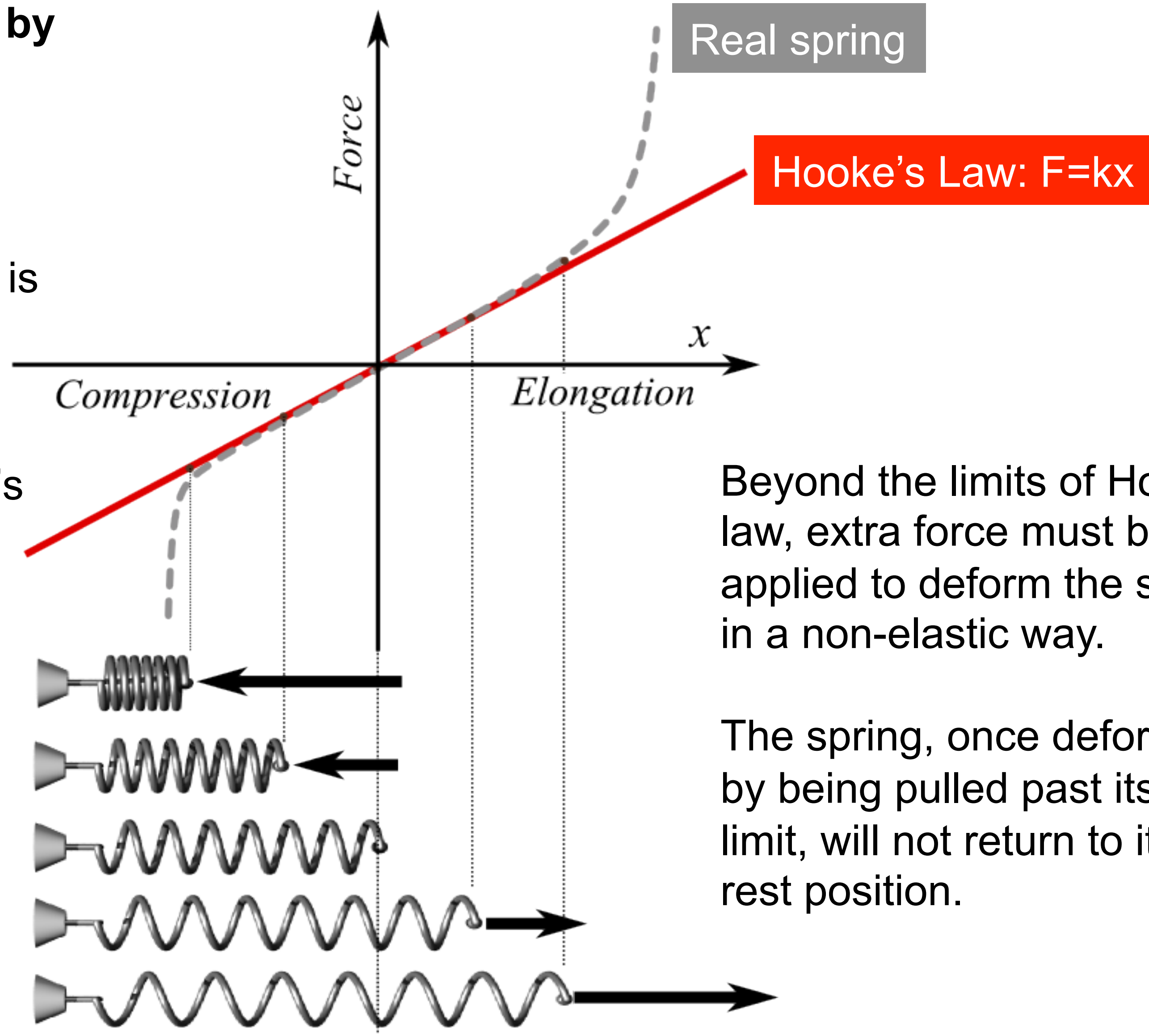
Recall force exerted **by** a spring:

$$F = -kx$$

So force **to** stretch a spring in elastic limit is

$$F = kx$$

(because of Newton's Third law)

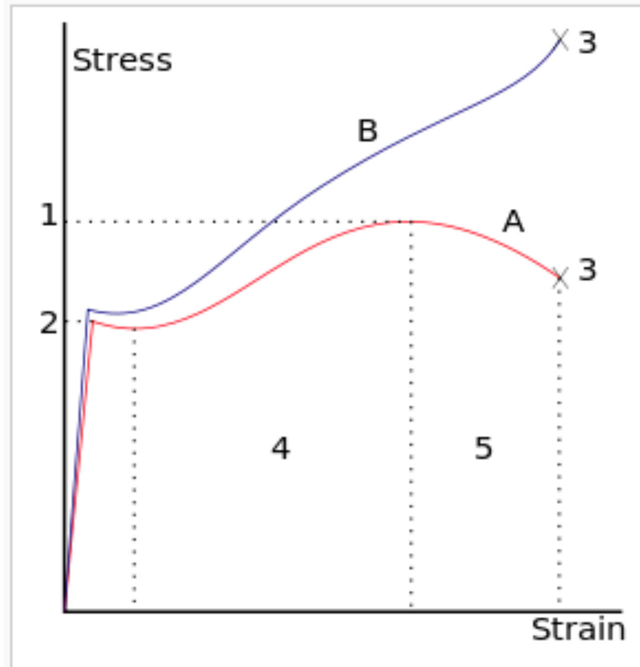


Beyond the limits of Hooke's law, extra force must be applied to deform the spring in a non-elastic way.

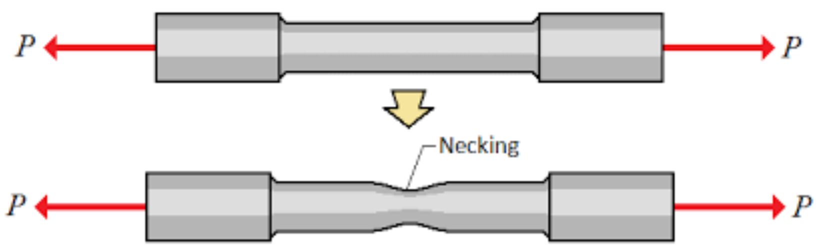
The spring, once deformed by being pulled past its elastic limit, will not return to its initial rest position.

Engineering Complications

- This treatment of material stress, strain, and modulus makes many simplifying assumptions
  - E.g. cross section stays constant as material is deformed
- Proper treatment involves many details of the specific material involved
  - Dynamics of deformation also change due to material deformation
  - Example: Necking changes cross sectional area



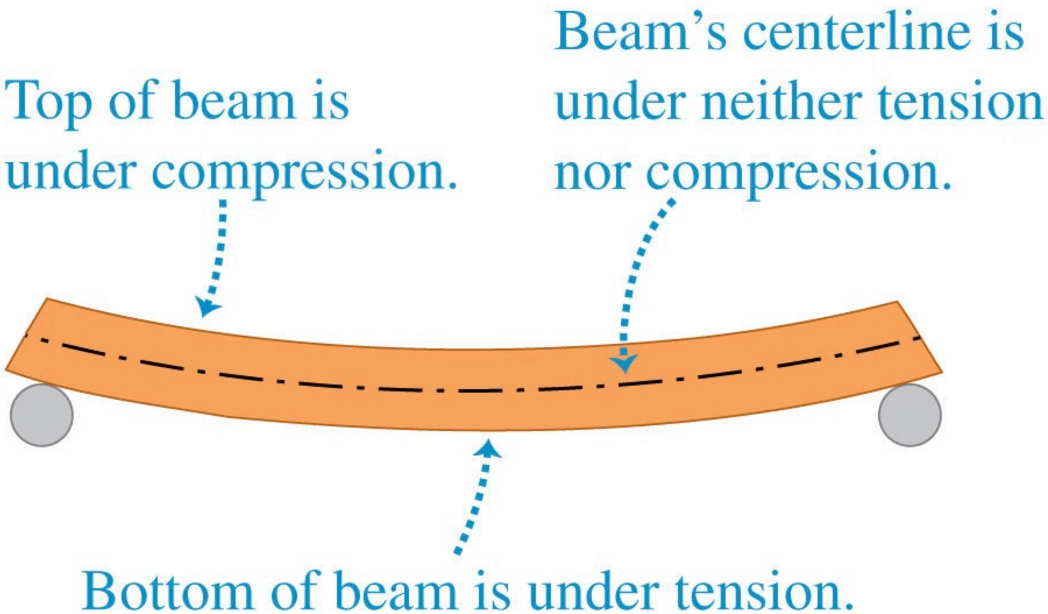
"Engineering" (red) and "true" (blue) stress–strain curve typical of structural steel.
1: **Ultimate strength**
2: Yield strength (yield point)
3: Rupture
4: Strain hardening region
5: Necking region
A: Apparent stress (F/A_0)
B: Actual stress (F/A)



[Wikipedia]

Compression and tension

- In many situations, objects can experience both tensile and compressive stresses at the same time.
- For example, a horizontal beam supported at each end sags under its own weight.



- Which engineering materials are stronger under compression? Which are stronger under tension?

Compressibility

- The reciprocal of the bulk modulus is called the **compressibility** and is denoted by *k*:

$$k \equiv \frac{1}{B} = -\frac{\Delta V/V_0}{\Delta p} = -\frac{1}{V_0}\frac{\Delta V}{\Delta p}$$

  - Higher compressibility means **smaller** force required for deformation

Liquid	Compressibility, k	
	Pa^{-1}	atm^{-1}
Carbon disulfide	93×10^{-11}	94×10^{-6}
Ethyl alcohol	110×10^{-11}	111×10^{-6}
Glycerine	21×10^{-11}	21×10^{-6}
Mercury	3.7×10^{-11}	3.8×10^{-6}
Water	45.8×10^{-11}	46.4×10^{-6}

  - Liquids are not truly incompressible, but it's a good approximation!
  - **Hydraulics** work because liquids are very nearly incompressible
    - **Pneumatics** work through compressibility of gases

Problem: Deep Seawater Compression

- **Problem 11.34**: In the Challenger Deep of the Marianas Trench, the depth of seawater is 10.9 km and the pressure is 1.16 x 10^8 Pa. If a cubic meter of water is taken from the surface to this depth, what is its relative change in volume?

$$B \equiv -\Delta p/(\Delta V/V_0)$$

$$B(\text{water}) = 2.18 \times 10^9 \text{ Pa}$$

$$p(1 \text{ atm}) = 1.0 \times 10^5 \text{ Pa}$$

Problem: Deep Seawater Compression

- **Problem 11.34**: In the Challenger Deep of the Marianas Trench, the depth of seawater is 10.9 km and the pressure is 1.16 x 10^8 Pa. If a cubic meter of water is taken from the surface to this depth, what is its relative change in volume?

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$$B(\text{water}) = 2.18 \times 10^9 \text{ Pa}$$

$$p(1 \text{ atm}) = 1.0 \times 10^5 \text{ Pa}$$

$$\Delta p = 1160 \times 10^5 \text{ Pa} - 1.0 \times 10^5 \text{ Pa} = 1159 \times 10^5 \text{ Pa}$$

$$\frac{\Delta V}{V_0} = \left(-\frac{\Delta p}{B}\right) = -0.053$$

Water under the pressures at the bottom of the ocean only changes its volume by about 5%!

Chapter 12: Fluid Mechanics (Statics and Dynamics)

- We consider fluids that are **nearly incompressible**
  - So we assume that fluid volume doesn't change under pressure
  - We just saw this is a reasonable approximation for water

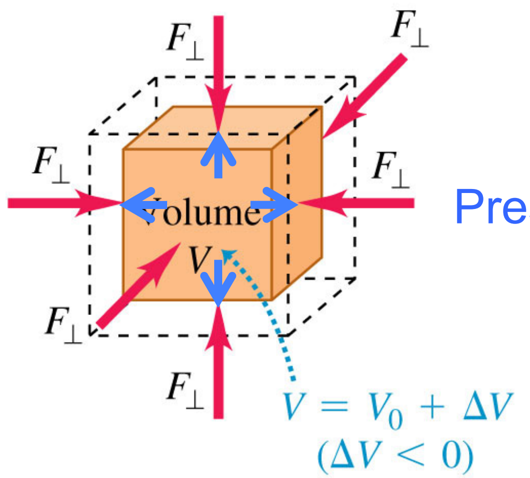
- This is equivalent to assuming that the fluid has nearly constant **density** ρ

$$\rho \equiv \frac{m}{V}$$

  - Density is a material property for solids and incompressible liquids
  - For gases and compressible liquids, it can vary by quite a lot

Pressure In A Fluid

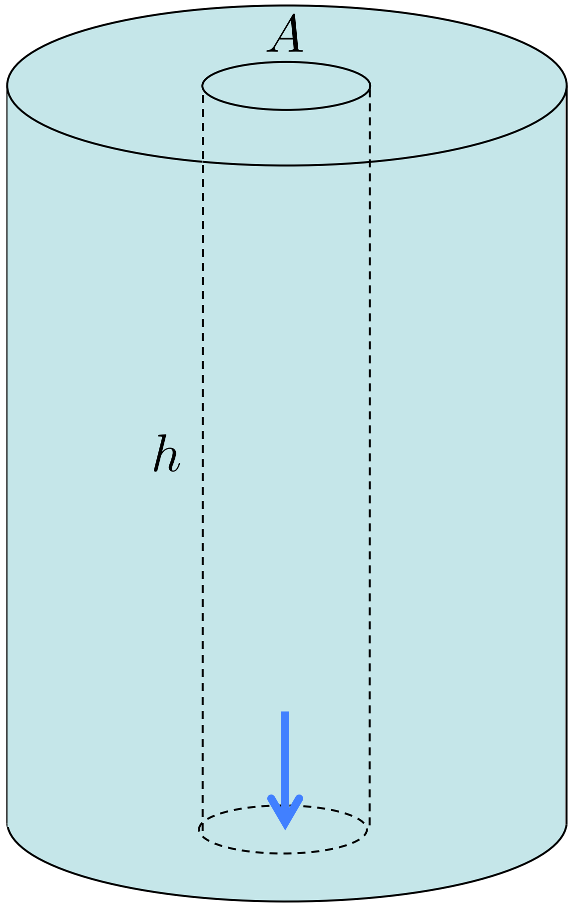
- A fluid exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.
  - This is an example of Newton's Third Law



Pressure $$p \equiv \frac{F_\perp}{A}$$

- Pressure in a small volume of fluid is constant over all surfaces of the volume
  - Pressure certainly varies with height in a fluid column though
- Pressure is a scalar! (Always perpendicular to the area *A*)

Example: Fluid Column



$$F = mg = (\rho h A)g$$

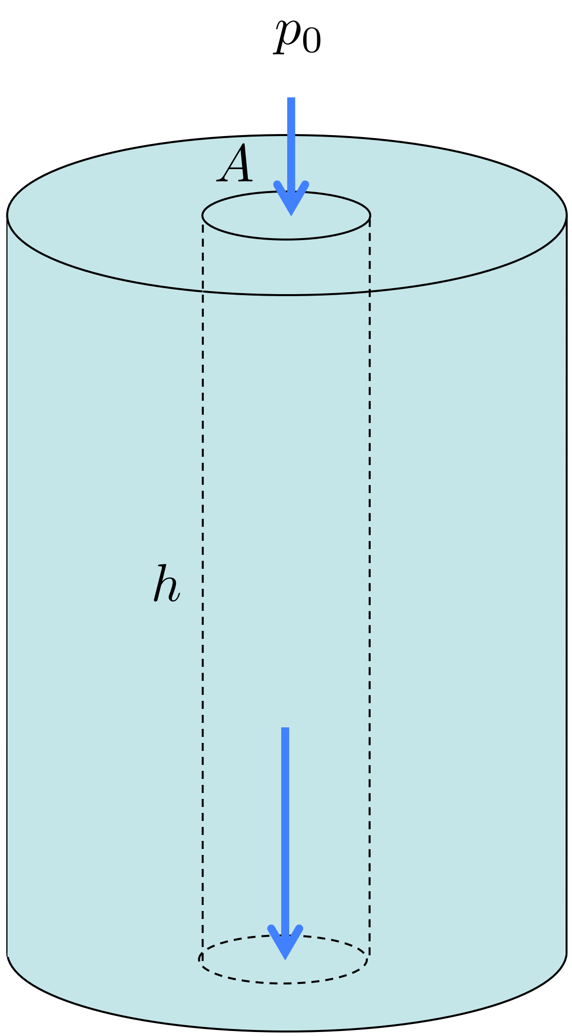
- Consider a column of fluid of cross sectional area *A* and height *h*
- The force at the bottom of the column is the weight of the fluid, or *mg*
- The mass of the fluid in the column is related to its density and volume:

$$m = \rho h A$$

- So additional pressure is

$$p = \frac{F_\perp}{A} = \rho g h$$

Example: Fluid Column



$$F = p_0 A + mg = p_0 A + \rho h A g$$

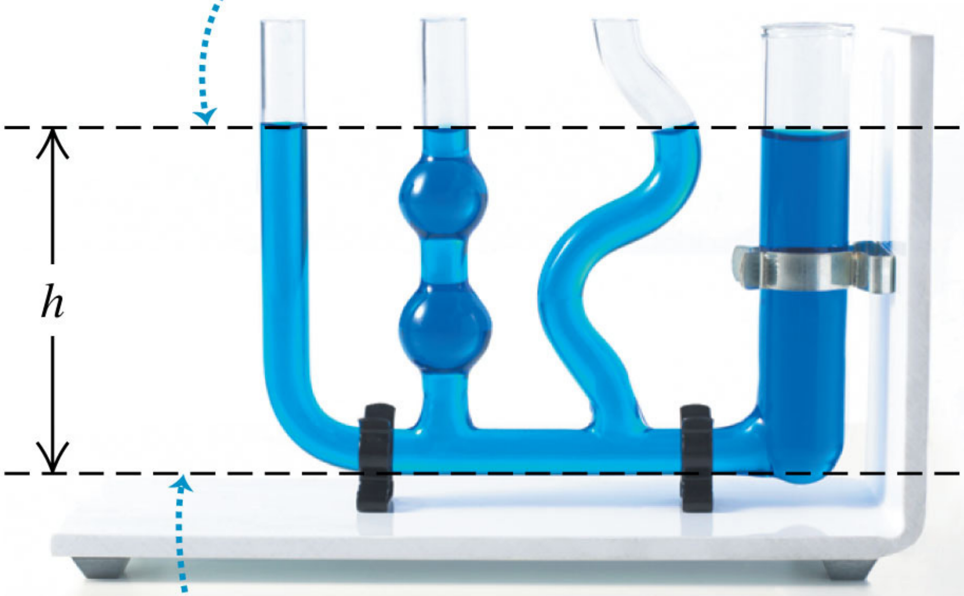
- In atmosphere, there is additional air pressure p_0 acting at the top of the liquid column

- So the total pressure at a depth h below the surface of the fluid is

$$p = p_0 + \rho g h$$

Pressure at depth in a fluid

The pressure at the top of each liquid column is atmospheric pressure, p_0.



The pressure at the bottom of each liquid column has the same value p.

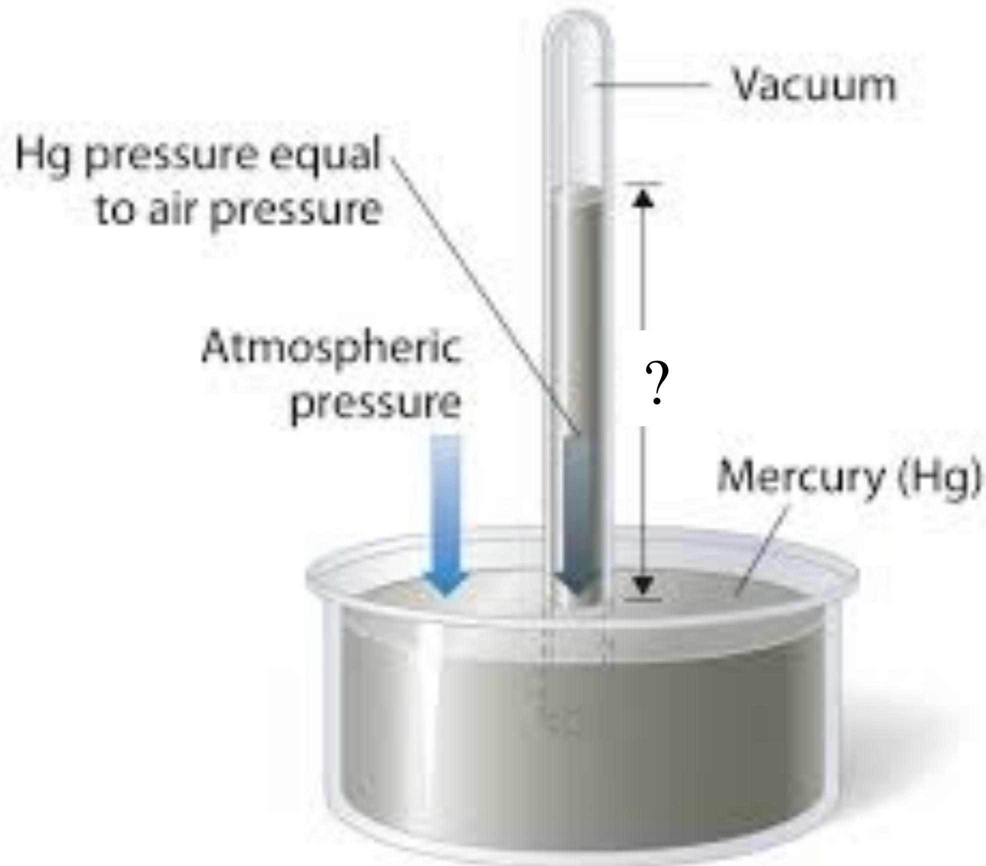
The difference between p and p_0 is $\rho g h$, where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

- Each fluid column has the same height, no matter what its shape.

Example: Barometric Pressure and Mercury

- What is the height of a column of mercury under typical atmospheric pressure (1.01×10^5 Pa)?

$$\rho_{\mathrm{Hg}} = 1.36 \times 10^4 \ \mathrm{kg/m^3}$$



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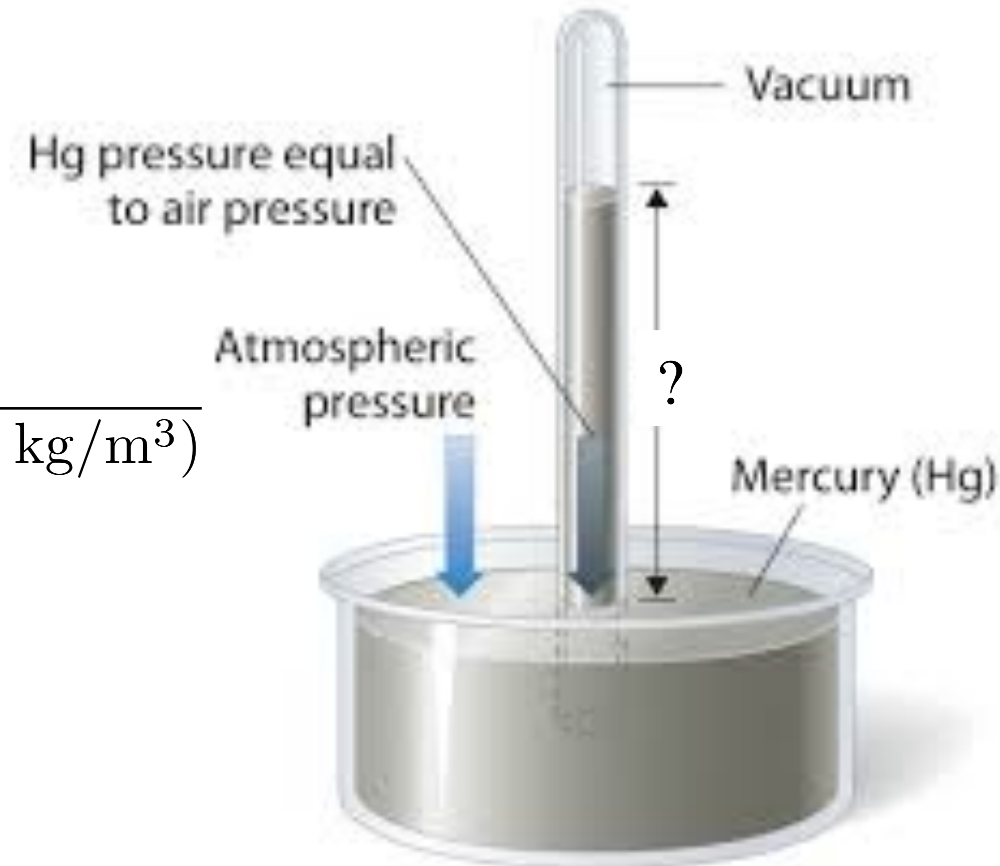
$$p = \rho g h$$

$$h = \frac{p}{g\rho} = \frac{1.01 \times 10^5 \ \mathrm{Pa}}{(9.8 \ \mathrm{m/s^2})(1.36 \times 10^4 \ \mathrm{kg/m^3})}$$

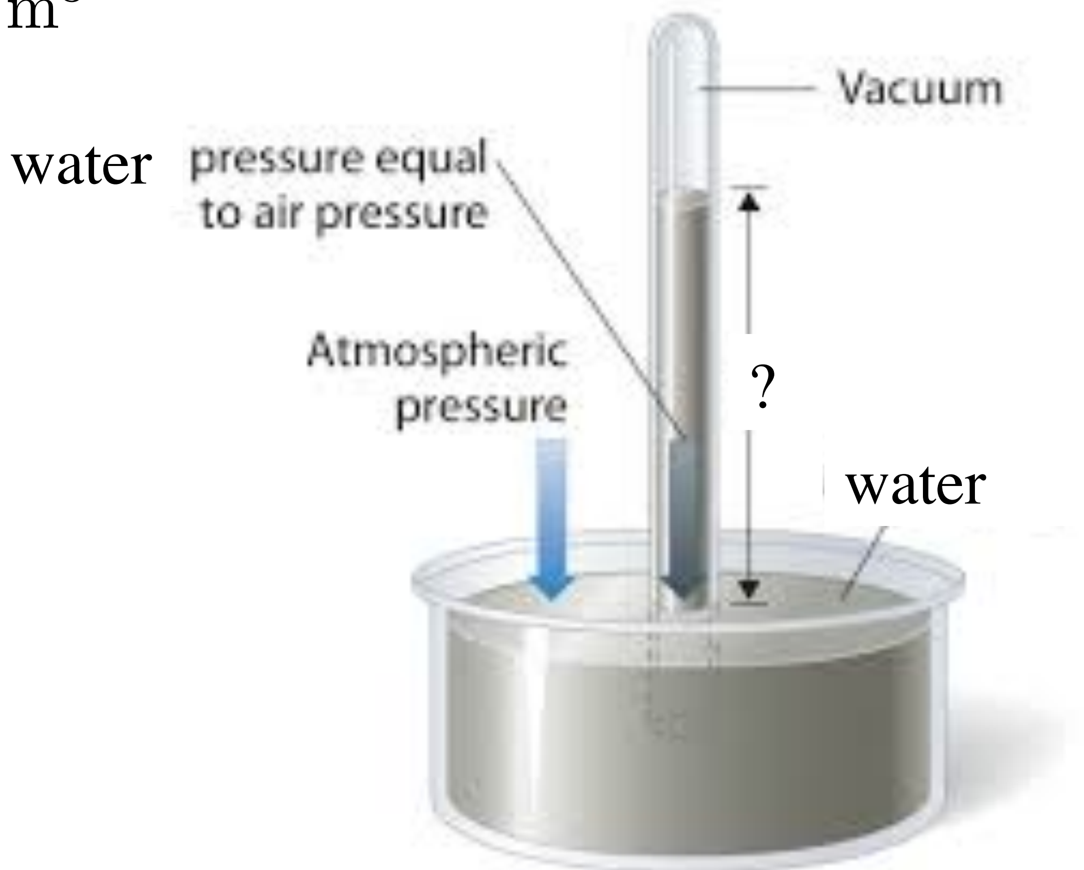
$$h = 758 \ \mathrm{mm}$$



Example: Barometric Pressure and Water

- What is the height of a column of **water** under typical atmospheric pressure (1.01 x 10^5 Pa)?

$$\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$$



Example: Barometric Pressure and Water

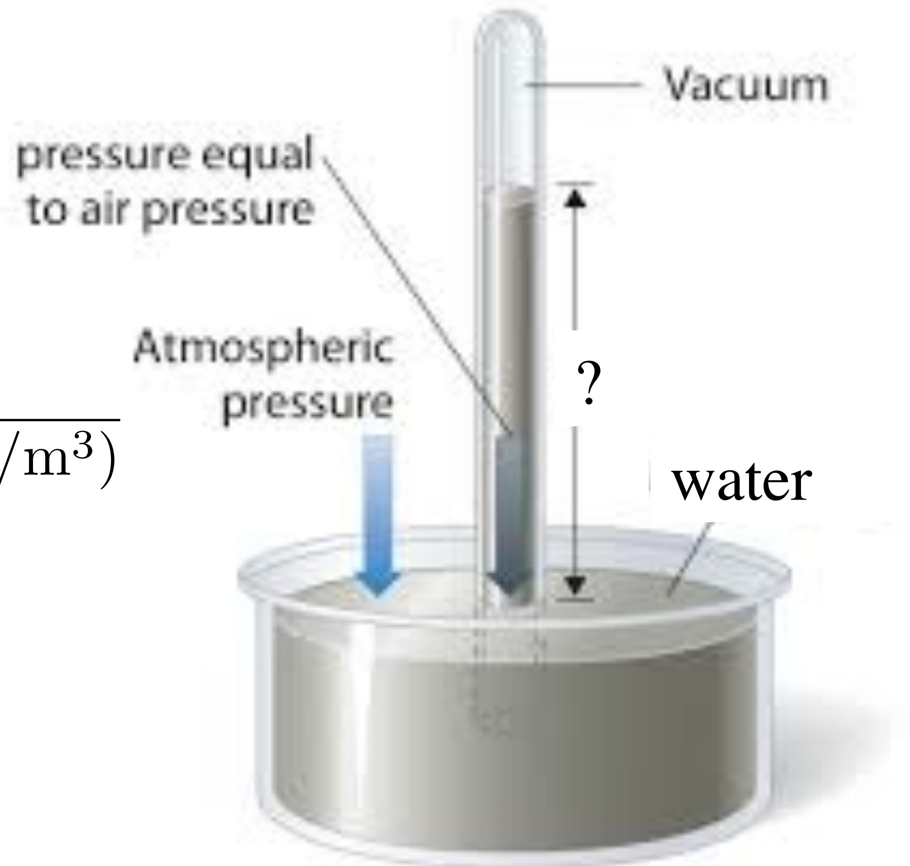
- What is the height of a column of **water** under typical atmospheric pressure (1.01×10^5 Pa)?

$$\rho_{\text{water}} = 1.00 \times 10^3 \text{ kg/m}^3$$

$$p = \rho g h$$

$$h = \frac{p}{g\rho} = \frac{1.01 \times 10^5 \text{ Pa}}{(9.8 \text{ m/s}^2)(1.00 \times 10^3 \text{ kg/m}^3)}$$

$$h = 10.3 \text{ m!!}$$

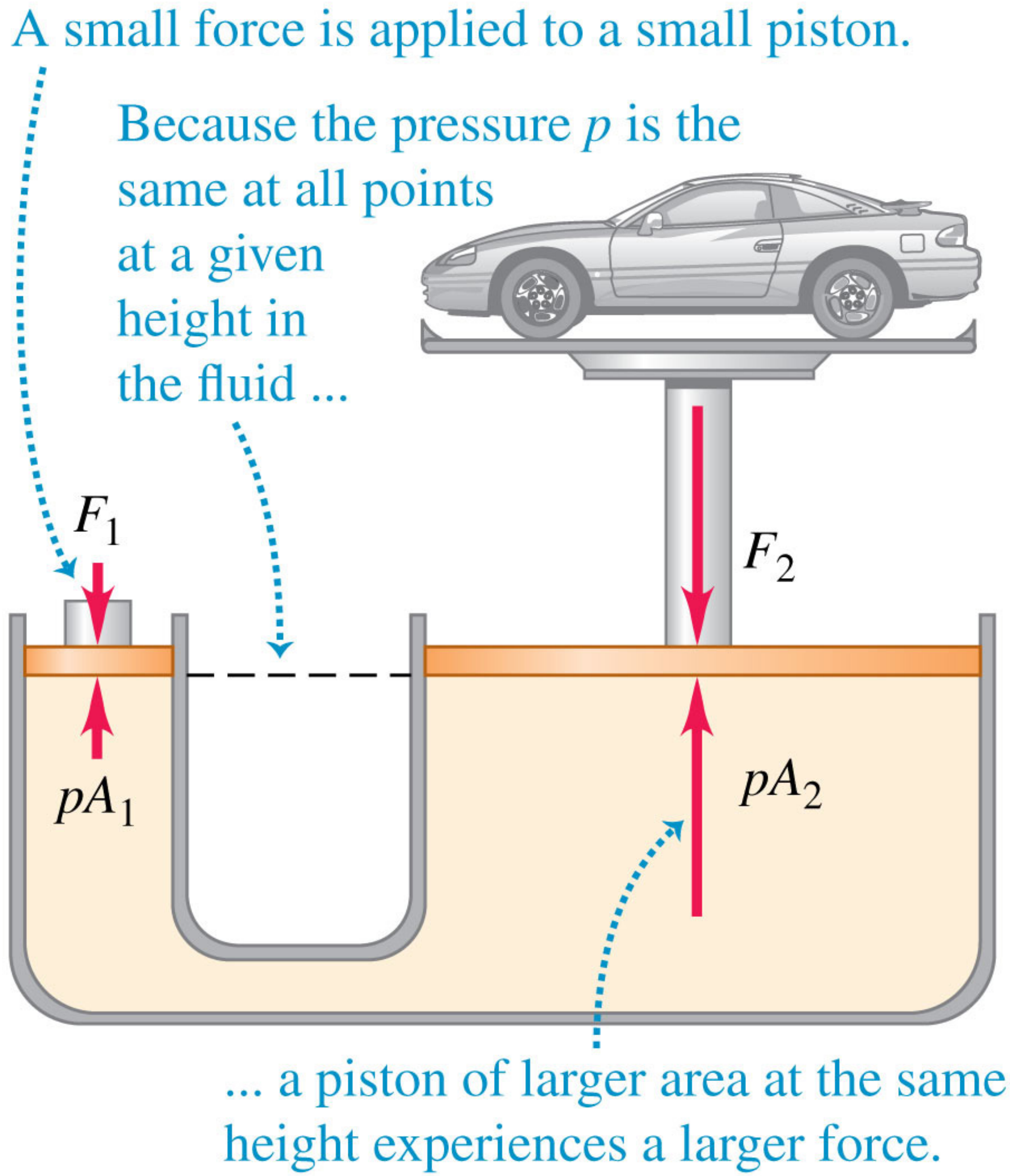


Pascal's law

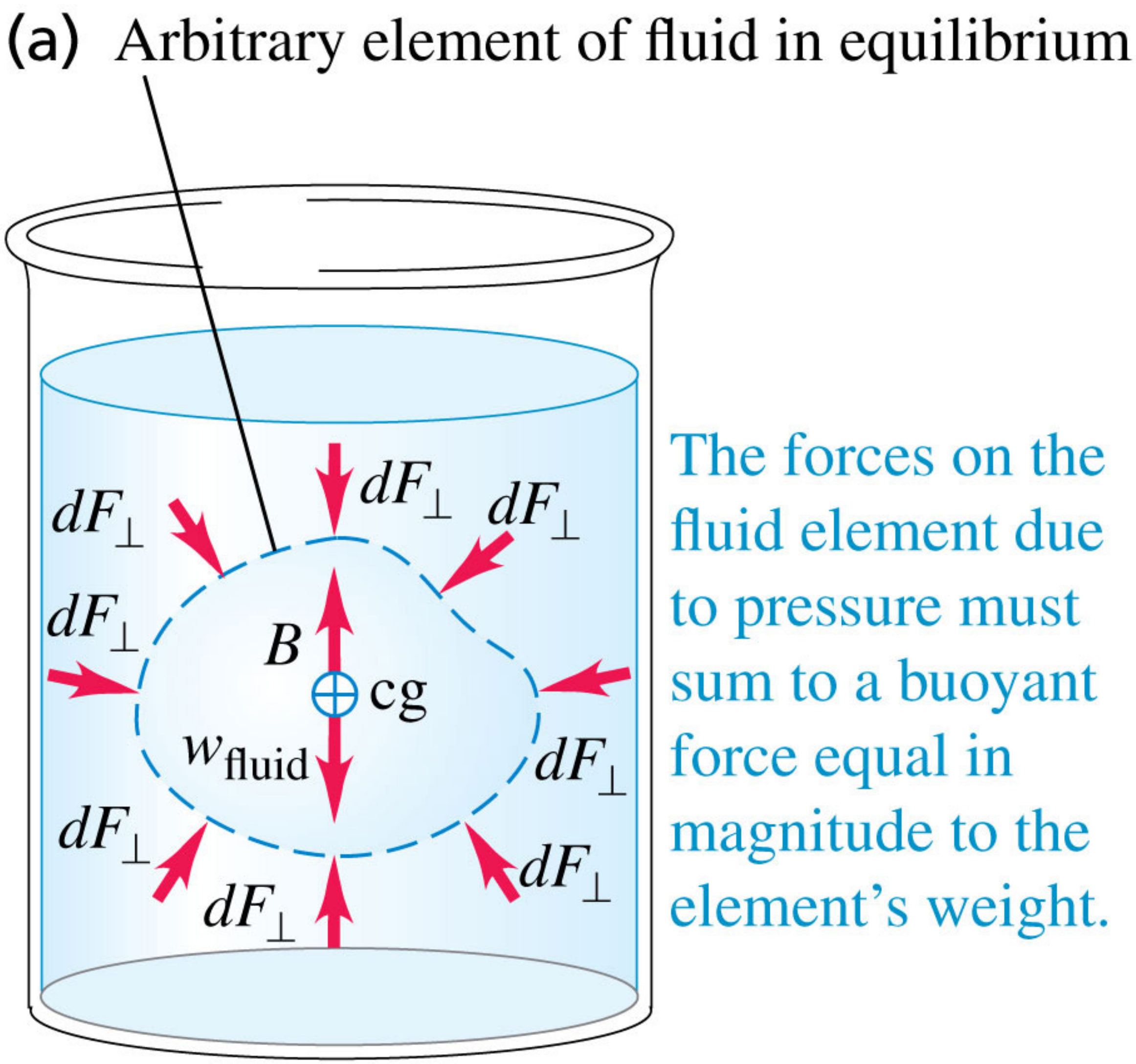
- Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

- Why hydraulics work
- Acts like a liquid lever
- Energy (force times distance) is conserved

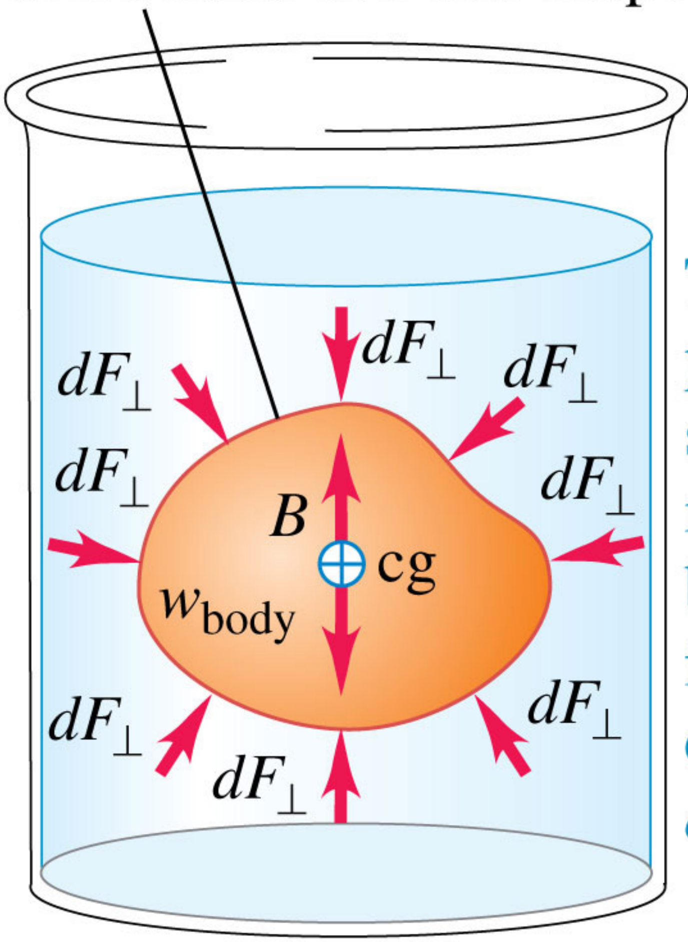


Archimedes's principle: Proof step 1



Archimedes's principle: Proof step 2

(b) Fluid element replaced with solid body of the same size and shape



The forces due to pressure are the same, so the body must be acted upon by the same buoyant force as the fluid element, *regardless of the body's weight.*

Archimedes’s principle

- A body immersed in water seems to weigh less than when it is in air.
- When the body is less dense than the fluid, it floats.
- The human body usually floats in water, and a helium-filled balloon floats in air.
- These are examples of buoyancy, a phenomenon described by *Archimedes’s principle*:
  - **When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.**

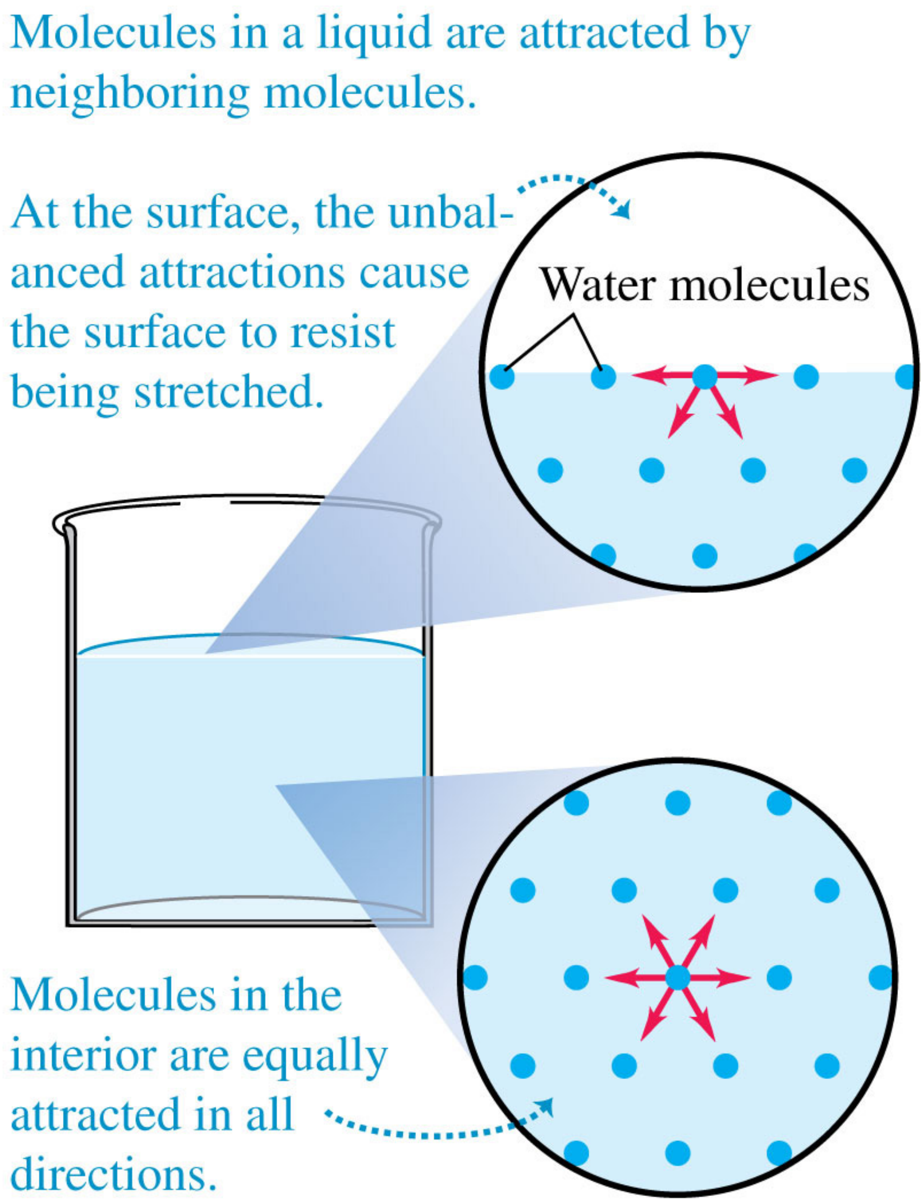
(End of Mon Nov 14 lecture)

Surface tension

- The surface of the water acts like a membrane under tension, allowing this water strider to “walk on water.”

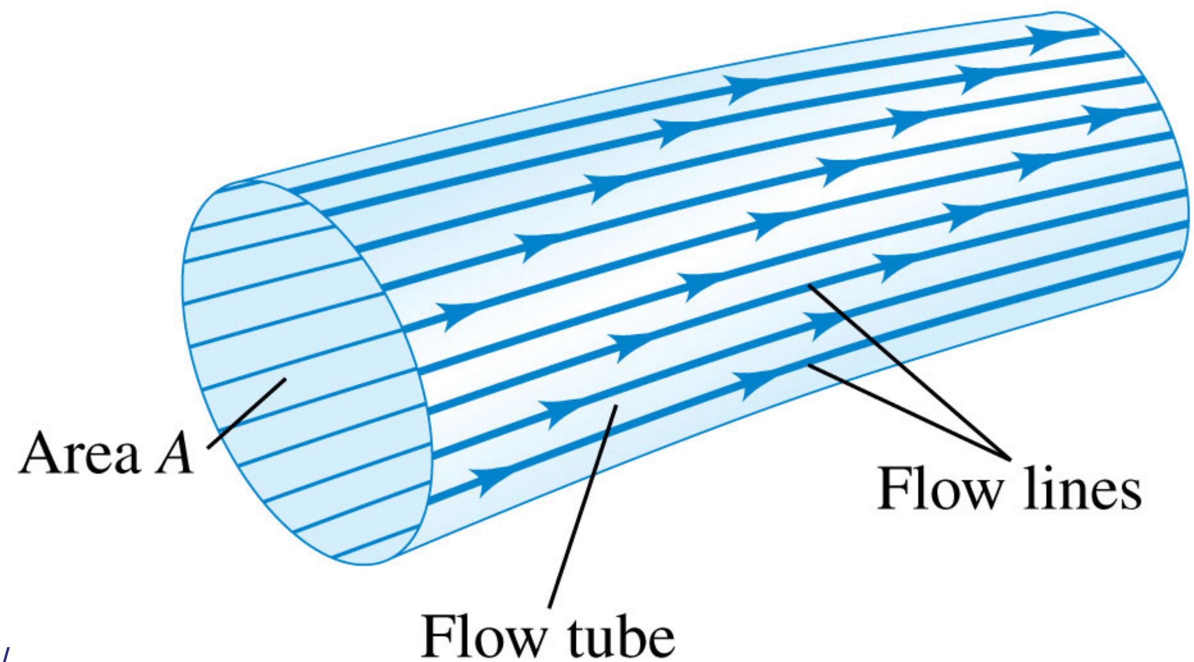
Surface tension

- A molecule at the surface of a liquid is attracted into the bulk liquid, which tends to *reduce* the liquid's surface area.



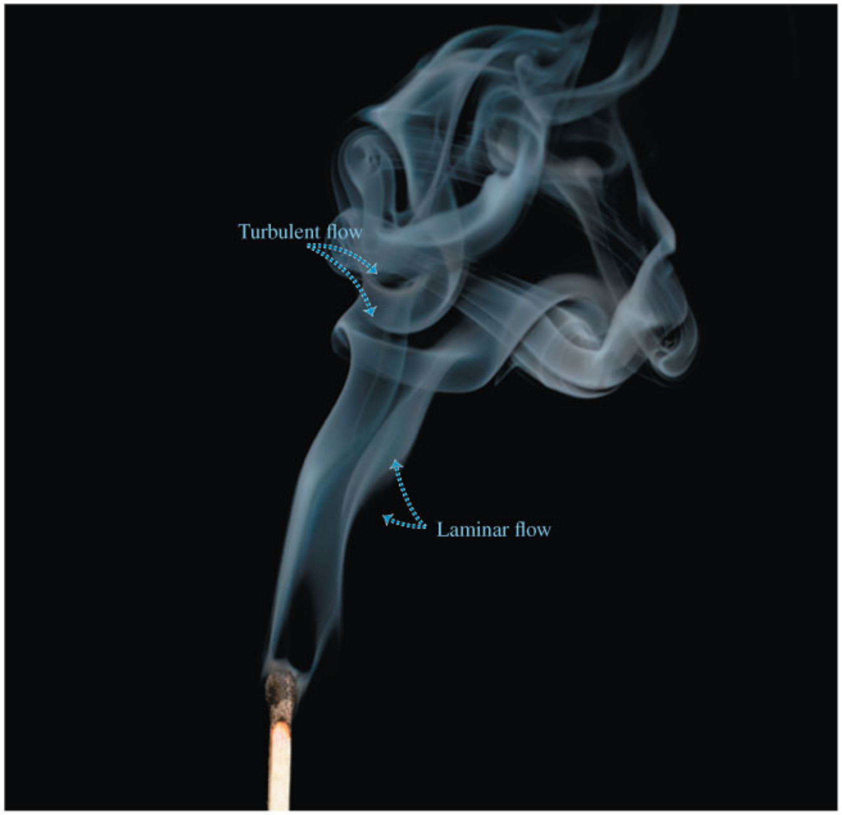
Fluid flow

- The path of an individual particle in a moving fluid is called a **flow line**.
- In **steady flow**, the overall flow pattern does not change with time, so every element passing through a given point follows the same flow line.
- In steady flow no fluid can cross the side walls of a given flow tube.



Fluid flow

- In laminar flow, adjacent layers of fluid slide smoothly past each other and the flow is steady.
- At sufficiently high flow rates, the flow can become turbulent.
- In turbulent flow there is no steady-state pattern; the flow pattern changes continuously.



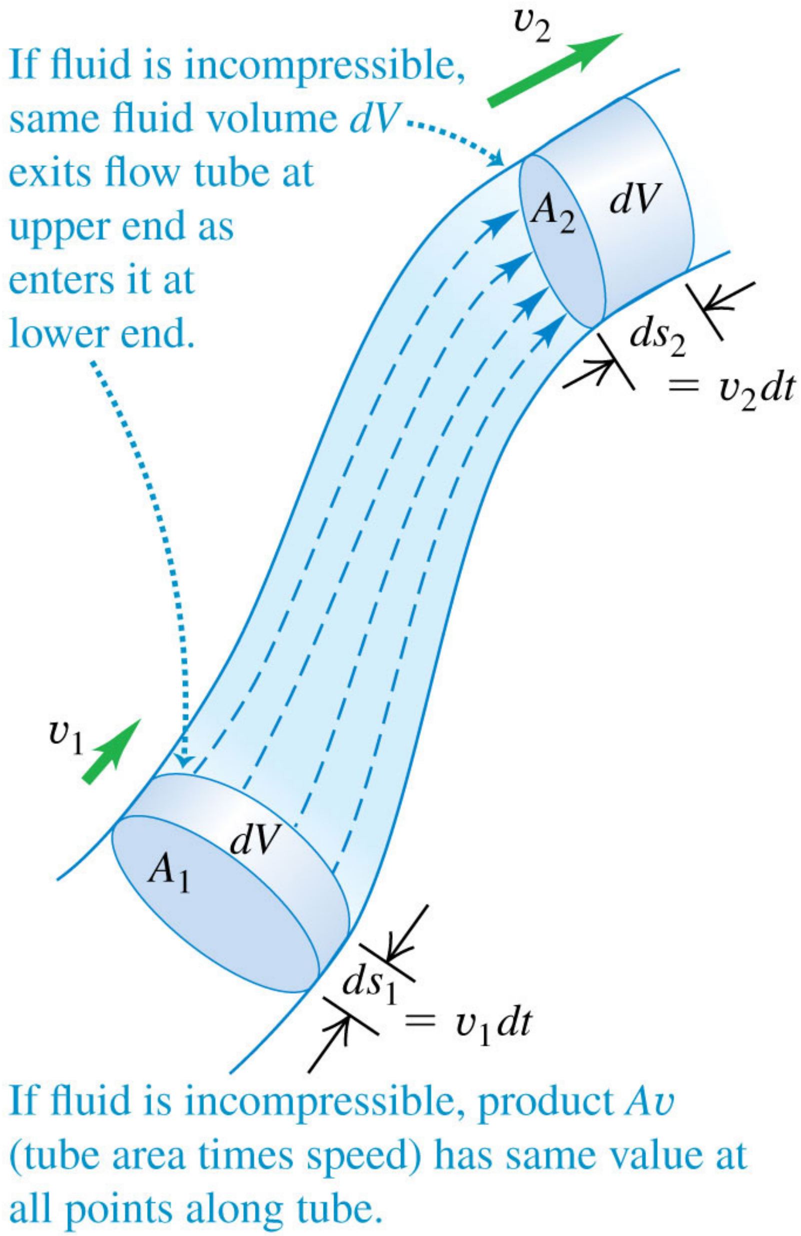
The Continuity Equation

- The figure at the right shows a flow tube with changing cross-sectional area.
- The **continuity equation** for an incompressible fluid is

$$A_1 v_1 = A_2 v_2$$

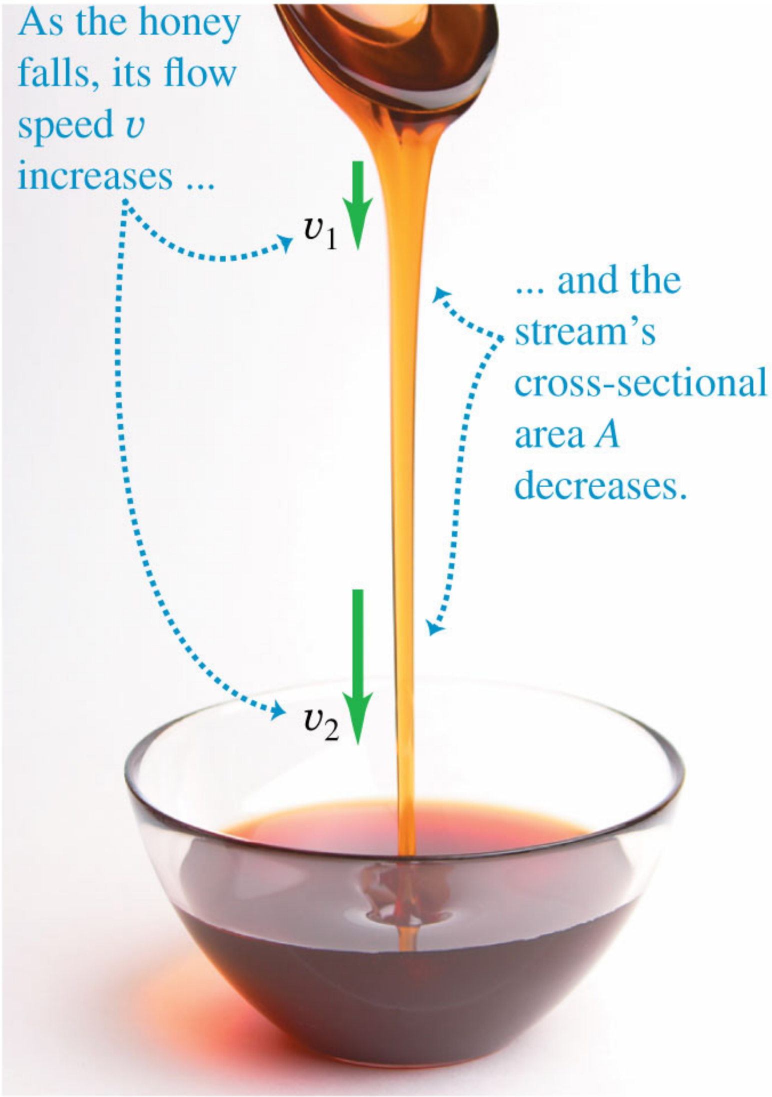
- The **volume flow rate** is

$$\frac{dV}{dt} = Av$$



The continuity equation

- The continuity equation helps explain the shape of a stream of honey poured from a spoon.



The volume flow rate $dV/dt = Av$ remains constant.