University Physics Midterm 3 Mon Nov 21, 2016

Your name: SOLUTIONS

Please show your work, write neatly, and put a box or circle around each of your answers.
There are 5 pages with 5 problems, all with multiple parts.
Assume Earth's gravitational acceleration of g=9.8 m/s^2.
Remember to include units where appropriate!
(Yes, there are 110 possible points. That's intentional.)

1. **Fluid Mechanics; Stress/Strain** (20 total points)
   Dr. Todd has a personal submarine that has a total volume of 10 m^3. He gets in and dives 1000 m into a pure freshwater lake.
   (a) (10 points) What is the change in pressure on his submarine, in Pascals *and* in atmospheres?
   (b) (10 points) What is the bulk modulus of Dr. Todd's submarine, in Pascals, if its volume is reduced by 7.0×10^{-4} m^3 when it descends?

a) $\Delta p = \rho g h = (10^3\,kg/m^3)(9.8\,m/s^2)(10^3\,m) = \boxed{9.8\times10^6\,Pa = \Delta p}$

$1\,atm = 101325\,Pa$ $\Rightarrow \boxed{\Delta p = 96.7\,atm}$

b) $B \equiv \dfrac{-\Delta p}{(\Delta V/V_0)}$ $\dfrac{\Delta V}{V_0} = \dfrac{-7.0\times10^{-4}\,m^3}{10\,m^3} = -7.0\times10^{-5}$

$B = \dfrac{-(9.8\times10^6\,Pa)}{(-7.0\times10^{-5})} = \boxed{1.4\times10^{11}\,Pa = B}$

2. **Fluid Mechanics** (15 total points)

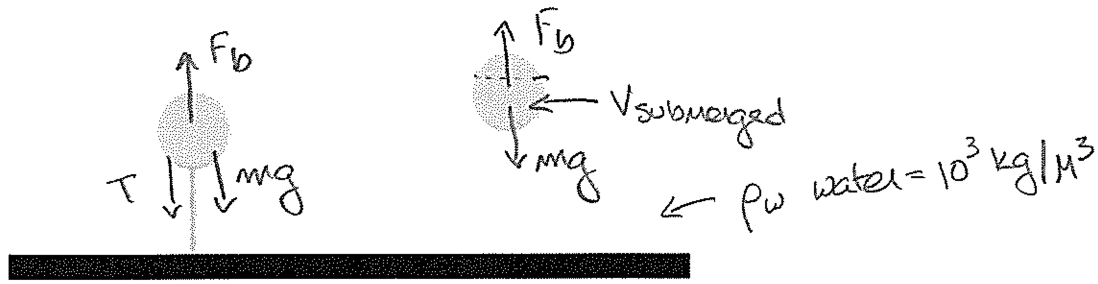


Figure 2: A sphere, submerged and floating, for problem 2.

Dr. Todd connects a solid sphere of volume 1.5 m^3 to an anchor with a light cord and throws it in a freshwater lake. The sphere is suspended in the water like the left image of Figure 2. The tension in the cord is 4200 N.

(a) (5 points) Calculate the buoyant force exerted on the sphere by the water, in Newtons.

(b) (5 points) What is the sphere's mass, in kg?

(c) (5 points) The cord breaks, and the sphere bobs to the top of the water like the right image of Figure 2. When the sphere comes to rest, what volume of the sphere is *above water*, in m^3?

a) Buoyant force is weight of same volume of water

$$F_b = \rho_w V g = (10^3\ kg/m^3)(1.5\ m^3)(9.8\ m/s^2) = \boxed{14700\ N = F_b}$$

b) $F_{net} = 0 = F_b - T - mg \Rightarrow m = \dfrac{F_b - T}{g} = \dfrac{(14700N - 4200N)}{(9.8\ m/s^2)}$

$$\Rightarrow \boxed{m = 1071\ kg} \qquad \rho = \frac{m}{V} = 714\ kg/m^3$$

c) $\underbrace{F_{net} = 0 = F_b - mg}$ New F_b! $\underbrace{F_b = \rho_w V_{submerged}\, g}$

$$F_b = m\cancel{g} = \rho_w V_{submerged}\cancel{g} \Rightarrow V_{submerged} = \frac{m}{\rho_w} = \frac{1071\ kg}{1000\ kg/m^3}$$

$$\Rightarrow V_{submerged} = 1.07\ m^3$$

$$\boxed{V_{above} = V - V_{submerged} = 0.43\ m^3}$$

3. **Angular Kinematics** (25 total points)
Dr. Todd has a flywheel that is shaped like a uniform solid cylinder of radius r=20 cm and mass m=3.0 kg that spins around the its central axis. It starts at rest, then starts spinning up with constant angular acceleration.

(a) (5 points) What is the moment of inertia of Dr. Todd's flywheel? Remember to include the proper units.

(b) (10 points) If the flywheel takes 0.5 s to make its ~~*third*~~ second complete revolution, how long did it take to make its *first* complete revolution (in seconds)?

(c) (5 points) What is the flywheel's angular acceleration, in rad/s^2?

(d) (5 points) What is the net torque on the flywheel as it is is accelerating, in N-m?

a) $I = \frac{1}{2}mr^2 = \frac{1}{2}(3.0\,kg)(0.20\,m)^2 = \boxed{0.06\,kg\text{-}m^2 = I}$

solid cylinder

b) First revolution: $\Delta\theta = \cancel{\omega_0 t_1} + \frac{1}{2}\alpha t_1^2 \Rightarrow 2\pi = \frac{1}{2}\alpha t_1^2$

Second " : $4\pi = \frac{1}{2}\alpha t_2^2$ $\Rightarrow t_2 = t_1\sqrt{2}$

t_1 is time from start for first turn

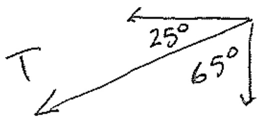
t_2 is time from start for two turns

We know $t_2 - t_1 = 0.5s \Rightarrow t_1\sqrt{2} - t_1 = 0.5s \Rightarrow \boxed{t_1 = \frac{0.5s}{\sqrt{2}-1} = 1.21s}$

c) $2\pi = \frac{1}{2}\alpha t_1^2 \Rightarrow \alpha = \frac{4\pi\,rad}{t_1^2} = \frac{4\pi\,rad}{(1.21s)^2} = \boxed{8.6\,rad/s^2 = \alpha}$

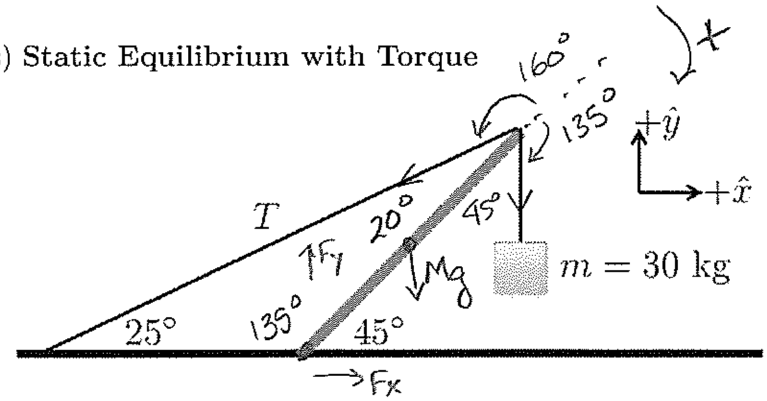
d) $\tau = I\alpha = (0.06\,kg\cdot m^2)(8.6\,rad/s^2) = \boxed{0.52\,N\cdot m = \tau}$

4. (25 points) **Static Equilibrium with Torque**



$T_y = -T\cos 65°$

$T_x = -T\sin 65°$



M = 8 kg

L = 2.5 m

Dr. Todd builds a small crane that supports a mass of 30 kg as shown above. The crane boom is 2.5 m long, has a uniformly distributed mass of 8 kg, and is inclined to the ground at a 45° angle. The crane is hinged to the ground at its base, and it is secured to the ground behind it with a rope that carries tension T.

(a) (10 points) What is the tension T (in Newtons) so Dr. Todd's crane is in static equilibrium?

(b) (10 points) What is the magnitude of the force that the ground exerts on the crane boom (in Newtons)?

(c) (5 points) What is the angle of the force that the ground exerts on the crane boom, in degrees from the $+\hat{x}$ direction?

a) Can be found from $\tau_{net} = 0$ around the support point

$$\tau_{net} = +Mg\,(L/2)\sin 135° + mgL\sin 135° - TL\sin 160° = 0$$

$$T = \frac{(M/2 + m)\,g\,\sin 135°}{\sin 160°} = \frac{(4\,\text{kg} + 30\,\text{kg})(9.8\,\text{m/s}^2)(0.707)}{0.342}$$

$$\boxed{T = 689\text{ N}}$$

b) $F_{y,net} = 0 = F_y - Mg - mg - T\cos 65°$

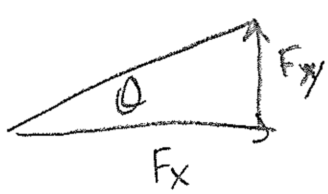
$\Rightarrow F_y = (M+m)g + T\cos 65° = (38\,\text{kg})(9.8\,\text{m/s}^2) + 689\,\text{N}(0.422)$

$F_y = 663$ N

$F_{x,net} = 0 = F_x - T\sin 65° \Rightarrow F_x = T\sin 65° = 624$ N

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(624\,\text{N})^2 + (663\,\text{N})^2} = \boxed{910\text{ N} = F}$$

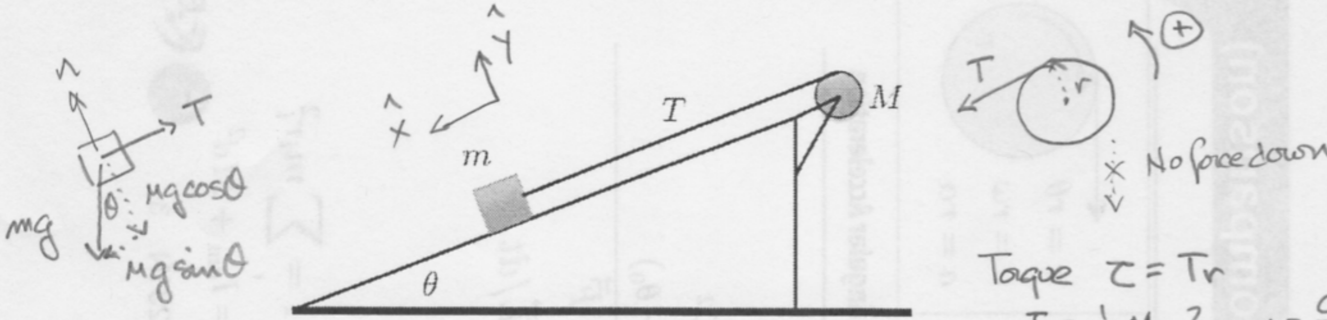
c)



$$\tan\theta = \frac{F_y}{F_x} = \frac{663\,\text{N}}{624\,\text{N}} = 1.063$$

$$\theta = \tan^{-1}(1.063) = \boxed{46.7° = \theta}$$

5. **Combining Rotational and Linear Motion** (25 total points)



Torque $\tau = Tr$

$I = \frac{1}{2}Mr^2 \quad \alpha = \frac{a}{r}$

Dr. Todd builds another one of his crazy demos, putting a block of m=3.0 kg on a *frictionless* inclined plane of angle $\theta = 27°$. He attaches the block to a freely spinning (frictionless) cylinder of mass M=1.0 kg and radius r=5 cm at the top of the ramp. Dr. Todd holds everything at rest, then lets go. The block moves down the plane with constant acceleration, and the cylinder turns with constant angular acceleration.

(a) (5 points) Write Newton's second law for each of the masses in terms of the provided quantities (m, M, r, θ, g).

(b) (5 points) What is the tension T in the rope, in Newtons?

(c) (5 points) What is the acceleration of the block down the plane, in m/s^2?

(d) (10 points) After 3.4 s, how many revolutions has the cylinder turned from its starting point?

a) $F_{net,x} = ma \Rightarrow \boxed{mg\sin\theta - T = ma \text{ for block } 1}$

$F_{net,y} = 0 \Rightarrow \boxed{-mg\cos\theta + n = 0 \text{ for block } 2}$

$\tau_{net} = I\alpha \Rightarrow T\cancel{r} = I\alpha = (\tfrac{1}{2}M r^{\cancel{2}})(a/\cancel{r}) \Rightarrow \boxed{T = \tfrac{1}{2}Ma \; 3 \text{ for cylinder}}$

b) 1 $a = g\sin\theta - T/m$ put into 3 $\rightarrow T = \frac{1}{2}M(g\sin\theta - T/m)$

$T = \frac{1}{2}Mg\sin\theta - TM/2m \Rightarrow T + TM/2m = \frac{1}{2}Mg\sin\theta = T(1 + M/2m)$

$T = \dfrac{Mg\sin\theta}{2(1+M/2m)} = \dfrac{(1.0\text{kg})(9.8\,\text{m/s}^2)\sin 27°}{2(1+(1.0\text{kg})/(2\times 3.0\text{kg}))} = \boxed{1.91\text{N} = T}$

c) 3 $\rightarrow a = \dfrac{2T}{M} = \dfrac{2(1.91\text{N})}{(1.0\text{kg})} = \boxed{3.81\,\text{m/s}^2 = a}$

d) $\alpha = \dfrac{a}{r} = \dfrac{(3.81\,\text{m/s}^2)}{(0.05\,\text{m})} = 76.2\,\text{rad/s}^2$

$\Delta\theta = \cancel{\omega_0 t}^{\,0} + \frac{1}{2}\alpha t^2 = \frac{1}{2}(76.2\,\text{rad/s}^2)(3.4\text{s})^2 = 440 \text{ rad}$

$\Delta\theta = 440\text{rad}\left(\dfrac{1\text{rev}}{2\pi\text{rad}}\right) = \boxed{70.1\text{rev} = \Delta\theta}$