Your name: _

Please show your work, write neatly, and put a box or circle around each of your answers. Assume Earth's gravitational acceleration ($g = 9.8 \text{ m/s}^2$) unless otherwise specified.

There are 6 problems. Some are quick and some are a bit more involved.

Remember to include units where appropriate!

(Yes, there are 110 possible points. That's intentional.)

1. Stress/Strain (15 total points)

Four steel legs, each of circular cross section with diameter 10 cm and length 1.0 m, support a large steel table of mass $m = 3 \times 10^4$ kg. The pressure on each beam is equal.

- (a) (5 points) What is the pressure on each leg of the table?
- (b) (10 points) How far does each leg of the table compress?

2. Fluid Mechanics (15 total points)

A slab of ice 2.0 m per side and 1.0 m thick floats on a pure freshwater lake.

- (a) (5 points) How much thickness of the ice slab is exposed above the lake's surface?
- (b) (5 points) A 75 kg woman gently steps onto the ice slab and stands on it, pushing the ice down into the water a bit. How much thickness of the ice slab is exposed above the lake's surface in this case?
- (c) (5 points) If the woman quickly jumps to accelerate upwards at 3.0 m/s^2 , does the ice slab bob below the surface of the water? Why or why not?

3. Angular Kinematics (30 total points)



A gyroscope is built from a solid uniform disk of radius R = 4 cm and mass M = 500 g, centered on an axle of radius r = 0.4 cm and mass m = 100 g. The system can spin around the center of the axle.

- (a) (5 points) What is the total rotational inertia $I_{\text{tot}} = I_{\text{disk}} + I_{\text{axle}}$ of the gyroscope when rotated around the axis centered on the axle?
- (b) (5 points) A massless string is wrapped around the axle and pulled with a constant tension of T = 0.1 N. What is the angular acceleration α of the gyroscope in units of rad/s²?
- (c) (10 points) If the gyroscope is at rest, then the string is pulled for 7 s, how many revolutions does the gyroscope make while the string is being pulled?
- (d) (10 points) What is the gyroscope's final angular velocity ω after the string is pulled, in units of rad/s?

4. Static Equilibrium (20 total points)



A small crane supports a mass of 10 kg as shown above. The crane boom is 3 m long, has a uniformly distributed mass of 5 kg, and is hinged to the ground at its base, and it is attached to the ground behind it with a rope of tension T.

- (a) (10 points) What is the tension T (in Newtons) such that this crane is in static equilibrium?
- (b) (5 points) What is the magnitude of the force that the ground exerts on the crane boom (in Newtons)?
- (c) (5 points) What is the angle of the force that the ground exerts on the crane boom, in degrees from the positive horizontal direction?

5. Rotational Motion on a Frictionless Ramp (20 total points)



A mass m=5.0 kg is accelerating down a *frictionless* ramp of angle $\theta = 30^{\circ}$ with acceleration a=1.0 m/s². It is attached to a *massless* string which is wound around a cylinder of mass M and radius r=20 cm. The string has tension T that is parallel to the surface of the ramp, and it turns the cylinder as the mass is sliding down the ramp.

- (a) (10 points) What is the tension T in the string?
- (b) (10 points) What is the mass M of the cylinder?

6. Torsional Spring (10 total points)



A very light (effectively massless) cylinder of radius R=50 cm turns on its axis, and is attached to a spring that exterts a torque τ on it that depends on the rotation of the cylinder θ , via the equation $\tau = -\kappa\theta$ where $\kappa=5$ N·m.

- (a) (5 points) A mass M=50 g is hung from the side of the cylinder with a massless string of tension T, and gently released so the system achieves static equilibrium. How far did the wheel turn?
- (b) (5 points) What is the tension T in the string (in Newtons) after the system is in static equilibrium?