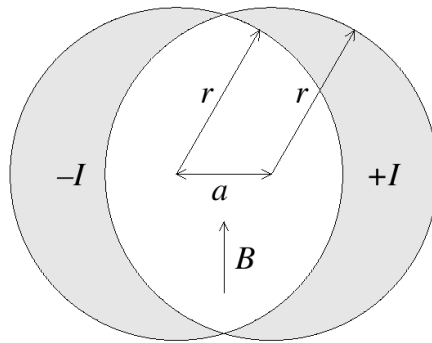


USPAS Graduate Accelerator Physics Homework 2

Due date: Wednesday January 18, 2017

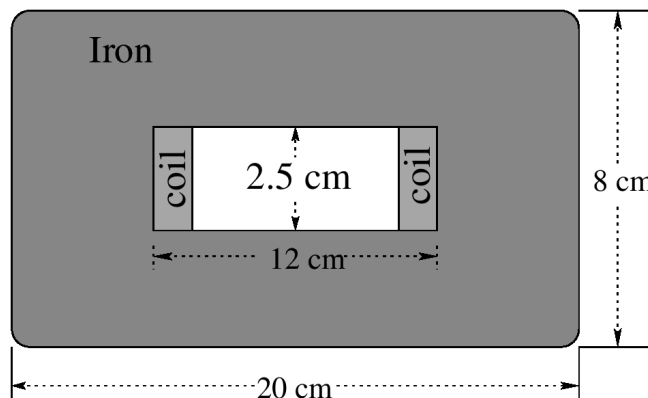
1 C-M 4-8: A Simple $\cos\theta$ Magnet

(10 points) A magnet is constructed from conductors with a transverse cross section as shown in the figure, with uniform current density in the shaded areas. Show that there is a uniform magnetic field in the intersection region of the two circles, and calculate its direction and strength. (Hint: Use linear superposition of two current-carrying cylinders.)



2 C-M 4-11: Designing A Window Frame Magnet

Consider a 1m long window-frame dipole magnet with the cross section shown in the figure below.



- (a) (4 points) Estimate the number of ampere-turns necessary to achieve a 0.6 T field in the gap. Assume that the iron is not saturated, so the relative permeability is roughly $\mu_r \equiv (\mu/\mu_0) = 5000$.

- (b) (4 points) Air-cooled copper coils can carry as much as 1.5 A/mm^2 , while water-cooled copper coils can carry almost 10 times as much current density. However water-cooling also adds the potential for water leaks and is more expensive, so it is not done unless necessary. For the given magnet dimensions, would you recommend water-cooled or air-cooled magnets? How much horizontal space would be available between the coils?
- (c) (3 points) If the magnet is to be powered by a power supply with a maximum current of 1000 A, how many turns should be used in the coil?
- (d) (3 points) What is the stored energy in the gap?
- (e) (3 points) Assuming constant field in the iron, estimate the additional energy stored in the iron yoke.
- (f) (3 points) Estimate the inductance of the magnet.

3 C-M 3-4: Beamline Matrix Identities

The usual representation for Pauli spin matrices of $SU(2)$ is:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

You can see that the symplectic form S is related to $i\sigma_y$.

- (a) (5 points) Given M for some beamline, show that $\check{M} \equiv \sigma_z M^{-1} \sigma_z$ is the transfer matrix for the s-direction mirror image of the beamline.
- (b) (5 points) Recall that the Pauli matrices satisfy the cyclic relation $\sigma_j \sigma_k = i\epsilon_{jkl} \sigma_l$. Using this, show that the following relations hold:

$$\begin{aligned} M^T &= \sigma_x \check{M} \sigma_x \\ M^{-1} &= \sigma_y M^T \sigma_y \end{aligned}$$

4 C-M 3-8: Solenoid Matrix

- (a) (5 points) Following the methods of Section 3.7 of the text, and starting from the Hamiltonian (CM:3.70), find the linear equations of motion for a particle in a uniform magnetic field along the s-axis, with the vector potential

$$\vec{A} = \left(-\frac{B_0}{2}y, \frac{B_0}{2}x, 0\right) \quad \text{for } 0 < s < l \quad \vec{A} = 0 \quad \text{otherwise}$$

Be sure to use a canonical system of coordinates inside the magnet.

- (b) (10 points) Find the generator G and use it to obtain the linear transformation matrix

$$\mathbf{M} = \begin{pmatrix} \frac{1+\cos\phi}{2} & r \sin\phi & \frac{\sin\phi}{2} & r(1-\cos\phi) \\ -\frac{\sin\phi}{4r} & \frac{1+\cos\phi}{2} & -\frac{1-\cos\phi}{4r} & \frac{\sin\phi}{2} \\ -\frac{\sin\phi}{2} & -r(1-\cos\phi) & \frac{1+\cos\phi}{2} & r \sin\phi \\ \frac{1-\cos\phi}{4r} & -\frac{\sin\phi}{2} & -\frac{\sin\phi}{4r} & \frac{1+\cos\phi}{2} \end{pmatrix},$$

for the transformation through a solenoid magnet in the hard-edge approximation, with $r = p_s/(qB_0)$, and $\phi = l/r$.