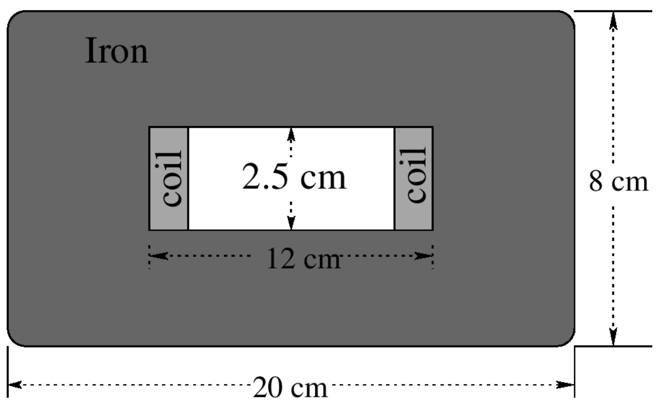
Introduction To Accelerator Physics Homework 3

Due date: Thursday February 15, 2017

1 C-M 4-11: Designing A Window Frame Magnet

Consider a 1m long window-frame dipole magnet with the cross section shown in the figure below.



(a) (4 points) Estimate the number of ampere-turns necessary to achieve a 0.6 T field in the gap. Assume that the iron is not saturated, so the relative permeability is roughly $\mu_r \equiv (\mu/\mu_0) = 5000$.

(b) (4 points) Air-cooled copper coils can carry as much as 1.5 A/mm^2, while water-cooled copper coils can carry almost 10 times as much current density. However water-cooling also adds the potential for water leaks and is more expensive, so it is not done unless necessary. For the given magnet dimensions, would you recommend water-cooled or air-cooled magnets? How much horizontal space would be available between the coils?

(c) (3 points) If the magnet is to be powered by a power supply with a maximum current of 1000 A, how many turns should be used in the coil?

(d) (3 points) What is the stored energy in the gap?

(e) (3 points) Assuming constant field in the iron, estimate the additional energy stored in the iron yoke.

(f) (3 points) Estimate the inductance of the magnet.

2 C-M 3-4: Beamline Matrix Identities

The usual representation for Pauli spin matrices of SU(2) is:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

You can see that the symplectic form S is related to $i\sigma_y$.

(a) (5 points) Given M for some beamline, show that $\breve{M} \equiv \sigma_z M^{-1} \sigma_z$ is the transfer matrix for the s-direction mirror image of the beamline.

(b) (5 points) Recall that the Pauli matrices satisfy the cyclic relation $\sigma_j \sigma_k = i\epsilon_{jkl}\sigma_l$. Using this, show that the following relations hold:

$$\begin{aligned} M^{\mathrm{T}} &= \sigma_x \breve{M} \sigma_x \\ M^{-1} &= \sigma_y M^{\mathrm{T}} \sigma_y \end{aligned}$$

3 C-M 3-8: Solenoid Matrix

(a) (5 points) Following the methods of Section 3.7 of the text (and as done on Tuesday Feb 8 in class), and starting from the Hamiltonian (CM:3.70), find the linear equations of motion for a particle in a uniform magnetic field along the s-axis, with the vector potential

$$\vec{A} = (-\frac{B_0}{2}y, \frac{B_0}{2}x, 0) \ \text{ for } 0 < s < l \qquad \vec{A} = 0 \text{ otherwise}$$

Be sure to use a canonical system of coordinates inside the magnet.

(b) (10 points) Find the generator G and use it to obtain the linear transformation matrix

$$\mathbf{M} = \begin{pmatrix} \frac{1+\cos\phi}{2} & r\sin\phi & \frac{\sin\phi}{2} & r(1-\cos\phi) \\ -\frac{\sin\phi}{4r} & \frac{1+\cos\phi}{2} & -\frac{1-\cos\phi}{4r} & \frac{\sin\phi}{2} \\ -\frac{\sin\phi}{2} & -r(1-\cos\phi) & \frac{1+\cos\phi}{2} & r\sin\phi \\ \frac{1-\cos\phi}{4r} & -\frac{\sin\phi}{2} & -\frac{\sin\phi}{4r} & \frac{1+\cos\phi}{2} \end{pmatrix},$$

for the transformation through a solenoid magnet in the hard-edge approximation, with $r = p_s/(qB_0)$, and $\phi = l/r$.