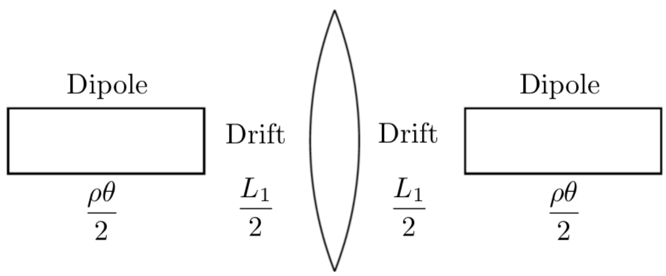
Introduction To Accelerator Physics Homework 4

Due date: Thursday Mar 29, 2018

(Need help? Email `satogata at jlab.org`)

1 Double Bend Achromat (30 points)



Consider the arrangement of magnets shown in Fig. 1, where two sector bend dipole magnets are equally spaced around a single focusing quadrupole of focal length f. This system is known as a **double bend achromat** (DBA), and this problem goes through the steps to calculate the quadrupole focusing strength f required for this system to be an achromat. Each dipole bends by angle $\theta/2$ and has bending radius ρ, and so has length $L = \rho\theta/2$. Each drift has drift length L_1. This system is very similar to the xkcd cartoon shown on page four of the slides, although here both dipoles bend in the same direction.

(a) (5 points) Since we are calculating aspects of a system with dipoles and dispersion, we will need to use 3×3 matrices as in the class notes, where the third coordinate is fractional momentum deviation $\delta \equiv (p - p_0)/p_0$. For a sector dipole of bend angle $\theta/2$ and bend radius ρ, the 3×3 transport matrix is

$$M_{\text{dipole}} = \begin{pmatrix} \cos(\theta/2) & \rho\sin(\theta/2) & \rho[1-\cos(\theta/2)] \\ -\frac{1}{\rho}\sin(\theta/2) & \cos(\theta/2) & \sin(\theta/2) \\ 0 & 0 & 1 \end{pmatrix}$$

Show that for weak dipoles ($\theta \to 0$ with $\rho\theta$ constant), this matrix can be written to first order in θ as

$$M_{\text{dipole}} = \begin{pmatrix} 1 & L & L\theta/4 \\ 0 & 1 & \theta/2 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) (8 points) Write out the five 3×3 matrices for this system and calculate their product. This matrix will have the form

$$M_{\text{DBA}} = \begin{pmatrix} C & S & D \\ C' & S' & D' \\ 0 & 0 & 1 \end{pmatrix}$$

Hints: When the system is left-right symmetric like this, you should end up with $C = S'$. D and D' should be proportional to θ since they should go to zero if the dipoles become drifts ($\theta \to 0$).

(c) (4 points) Using the equations on p. 35 of the Feb 15-22 slides, show that the periodic solution for η' is $\eta' = 0$.

(d) (5 points) Show that the periodic solution for η is 0 if the quadrupole focal length is

$$f = \frac{L + L_1}{4}$$

Under these conditions, the dispersion η and its derivative η' will be zero on both sides of this group of magnets. This is the property for an optical system (or accelerator magnet system) to be **achromatic**.

(e) (8 points) For the case when this system is achromatic, calculate the maximum dispersion $\hat{\eta}$, which will be located at the center of the quadrupole.

2 Cookie Can Resonator Frequencies

(10 points): Using the equations on pages 20-25 of the RF cavities lecture (Mar 20-22 slides), calculate the frequencies of the TM_{010}, TM_{020}, TM_{110}, TE_{011}, and TE_{111} resonant modes of the cavities shown in class:

- Long Piroulline can: $l = 16.4$ cm, $2a = 8.6$ cm
- Short cookie can: $l = 4.4$ cm, $2a = 13.5$ cm

Remember the rule of thumb given in class: CEBAF SRF cavities that are 10 cm long (therefore with 20 cm wavelength) have a frequency close to 1.5 GHz. So your answers should likely be in the 1-10 GHz range.

3 C-M 9.1: Plane Wave Acceleration

(10 points): How strong must the electric field intensity of a *traveling plane wave* be to accelerate electrons with an energy gradient of 10 MeV/m? (Hint: Use the Poynting vector.)

4 C-M 9.2: RF Cavity Power Loss

(10 points): Show that the RF power loss in the conducting walls of a cavity is given by equation (9.16) in the text.