Introduction To Accelerator Physics Homework 5

Due date: Thursday April 12, 2018

(Need help? Email `satogata at jlab.org`)

1 Chicane Bunch Compressor

Figure 1: A simple four-dipole chicane, with no quadrupoles.

Figure 1 sketches the layout of a simple four-dipole chicane in which the bend angle θ is small, the dipoles are assumed to be short, and a is the distance between the first and second dipoles. Longitudinal motion can be described by

$$\begin{pmatrix} z_2 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} 1 & M_{56} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ \delta_1 \end{pmatrix}$$

where z is the longitudinal position relative to the design particle, and $\delta \equiv \Delta p/p_0$ is the fractional momentum deviation from the design momentum p_0. These types of arrangements of dipoles are used produce M_{56} which looks like drift length in the longitudinal direction, even for highly relativistic electrons.

(a) (10 points) For the four-dipole chicane *with small* θ, use geometry to show

$$M_{56} \equiv \frac{dz_2}{d\delta_1} = -2a\theta^2$$

(b) (5 points) The nonlinear dependence of can be expanded in a Taylor series:

$$z_2 - z_1 = M_{56}\delta_1 + T_{566}\delta_1^2 + U_{5666}\delta_1^3 + \ldots$$

For the same four-dipole chicane, show that

$$T_{566} = -\frac{3}{2}M_{56}$$

and

$$U_{5666} = 2M_{56}$$

2 LHC Synchrotron Radiation

Protons radiate negligible synchrotron radiation at very low energies, but even this cannot be neglected at very high energies such as at the LHC. Consider protons of total energy $E = 7$ TeV in one ring of the LHC, where the circumference of the ring is 26.7 km and the main dipole field is 8.33 T.

(a) (10 points) Calculate the energy loss per turn per proton at total proton energies $E = 500$ GeV and $E = 7$ TeV. What percentage of the proton total energy is this?

(b) (5 points) Calculate the total power radiated by synchrotron radiation for a beam of protons at this energy with an average current of 0.56 A.

(c) (5 points) Calculate the energy loss per turn per *electron* at total *electron* energies $E = 500$ GeV and $E = 7$ TeV. (This is why very high energy electron synchrotrons are impractical.)

3 Synchrotron Radiation and Beam Energy

(10 points) Consider a beam of high energy ultrarelativistic electrons circulating in a synchrotron, where the particles are longitudinally focused by RF electric fields as discussed in previous lectures. The electrons radiate 10^{-3} of their total energy in synchrotron radiation per turn.

Draw the sinusoidal RF voltage vs time as shown in class, and indicate where (in phase) in this curve the design trajectory must be for the electrons to gain back the energy they lose to synchrotron radiation, and remain longitudinally focused. Explain your reasoning.