USPAS Graduate Accelerator Physics Homework 1

Due date: Tuesday January 22, 2019

1 RHIC frequency/field

The RHIC collider collides fully stripped gold ions (A=197, Z=79) at a total energy of $E_{\rm coll}$=100 GeV/nucleon per beam. The circumference of each ring is 3834 m. Assume the mass of a gold ion is 197×0.93113 GeV/c^2.

(a) (5 points) Calculate the revolution frequency of a particle at the injection energy of $E_{\rm inj}$=10.5 GeV/nucleon, and at the storage energy of $E_{\rm coll}$=100 GeV/nucleon. What is the change in revolution frequency for particles accelerated from $E_{\rm inj}$ to $E_{\rm coll}$?

(b) (5 points) If we assume that there are 192 identical dipoles per ring, each of length $L = 10$ m, what is the required dipole field in each at the collision energy of $E_{\rm coll}$?

2 Pion decay kinematics

Consider a charged pion decaying into a muon plus an antineutrino:

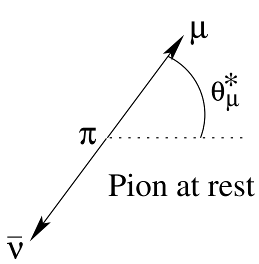
$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

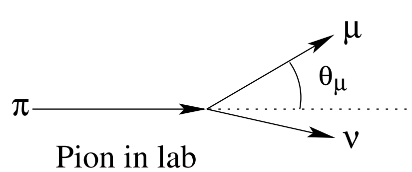
Use $m_{\pi^\pm} = 140$ MeV/c^2, $m_\mu = 106$ MeV/c^2, and $m_{\bar{\nu}} \approx 0$.

(a) (3 points) In the rest system of the pion, what are the energies and momenta of the muon and antineutrino?

(b) (3 points) Since neutrinos have now been discovered to have mass, how high must a pion beam energy be to produce some neutrinos at rest during their decays? Assume a rest mass of 0.01 eV/c^2 for muon neutrinos (and antineutrinos); you do not need to recalculate results from part (a).

(c) (4 points) For a moving pion with total energy $U_\pi = \gamma m_\pi c^2$ find an expression for the direction, θ_μ of the muon relative to the pion in the lab in terms of the angle θ^*_μ in the in the pion's rest system.





(flip the page...)

3 Lithium lens (yes, you can do it)

(10 points) A lithium lens of length l and radius a has a constant total current I flowing through its end caps with uniform current density as pictured in Fig. 1. (So in this figure, the current is flowing from either left to right *or* right to left.) This current creates a magnetic induction $\vec{B}(r)$ within the lithium lens.

Consider a beam of antiprotons with momentum p that are passing left to right through this lithium lens. (Yes, the antiprotons actually pass through the lithium material fairly easily.)

(a) What is the magnetic induction $\vec{B}(r)$ in the lithium lens?

(b) What is the focal length of this lens for the antiprotons? (Recall that the focal length for a focusing lens is defined as the distance at which incoming parallel rays converge on the center axis.)

(c) Does the lithium lens current need to flow from left to right or right to left for the lens to focus this antiproton beam?

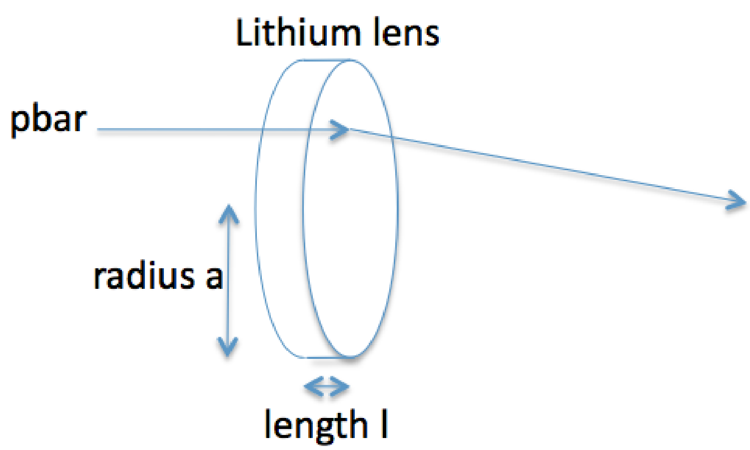


Figure 1: Lithium lens diagram. A uniform total current I is applied through the end caps (that is, pointing left to right or right to left) of the lithium cylinder to create a focusing lens.