USPAS Graduate Accelerator Physics Homework 4

Due date: Friday January 25, 2019

1 Mode TM_{010}

Why is the TM_{010} mode usually preferred in an RF cavity?

2 Kilpatrick criterion

The Kilpatrick criterion

$$f \;=\; 1.64\;E_k^2\;\exp(-8.5/E_k). \tag{2.1}$$

is an empirical equation from the 1950s that predicts the relation between frequency f (in MHz) and electrical field E_k (in MV/m) on a room-temperature copper surface at the limit of electrical breakdown. Higher frequencies support higher gradients. Contemporary vacuum systems allow the Kilpatrick limit E_k to be exceeded by bravery factors as large as 2.

If the maximum surface field on the walls of a single-cell pill box cavity is $1.8E_k$, then how many cavities are required to accelerate beam at 5 MeV per turn when the frequency is 200 MHz, 400 MHz, and 800 MHz?

3 Tune plane resonances

Consider a unit square in the tune plane (Q_x, Q_y) with corners at (n, n), $(n+1, n)$, $(n, n+1)$, and $(n+1, n+1)$.

(a) On graph paper or with a computer program, draw the lines representing all sum resonances $p = q\,Q_x + r\,Q_y$ through fourth order – for positive integer values of q and r, with $q + r \leq 4$.

(b) Plot all difference resonances $p = q\,Q_x - r\,Q_y$ through fourth order.

(c) Where are the largest areas of tune space that are resonance-free?

4 Closed three-bumps

The trigonometric law of sines states that

$$\frac{a}{\sin A} \;=\; \frac{b}{\sin B} \;=\; \frac{c}{\sin C} \tag{4.1}$$

where A, B, and C are the angles of a triangle, while a, b, and c are the lengths of the opposing sides.

(a) Use the law of sines to show that Equation 8.17 guarantees the localisation of a three-bump.

(b) What are the ratios of corrector strengths that close the three-bump if the phase advance between neighbouring correctors is 60 degrees, or 90 degrees?

(c) What phase advance conditions make three-bump localisation difficult in practice? Why?

5 Interaction region quadrupole errors

The interaction region quadrupole Q2 in RHIC has a focal length of about 3.0 m, at a location where the β-function is about 1400 m in collision optics with $\beta^* = 1$ m.

(a) How accurately must the strength of this magnet be known and set, if the strength error must be guaranteed to generate a β-wave amplitude of less than 1%?

(b) What tune shift is generated at this level of error?