USPAS Graduate Accelerator Physics Homework 5

Due date: Tuesday January 29, 2019

1 Projection map Hamiltonian H_p

Define the projection map P as linear motion R from a reference point to the location of a nonlinear magnet, where

$$R = \begin{pmatrix} c_x & s_x \\ -s_x & c_x \end{pmatrix} \qquad c_x = \cos(\phi_x) \quad s = \sin(\phi_x)$$

followed by the nonlinear kick $\Delta x' = gx^n$, finally followed by inverse linear motion R^{-1} back to the reference point. Show that the discrete projection Hamiltonian representing P is given by

$$H_p = -\frac{g}{n+1}(c_x x + s_x x')^{n+1} \tag{1.1}$$

2 Hénon triangle near $Q = 1/3$

Consider the equilateral triangle in (x, x') normalised phase space predicted by Equations 9.27 and 9.28.

(a) What is the radius of the largest circle that can be inscribed inside the triangle?

(b) What is the orientation of the triangle?

(c) What happens to the area and the orientation of the triangle as the tune Q is (slowly) swept through the value of 1/3?

3 Hénon dynamic aperture simulation

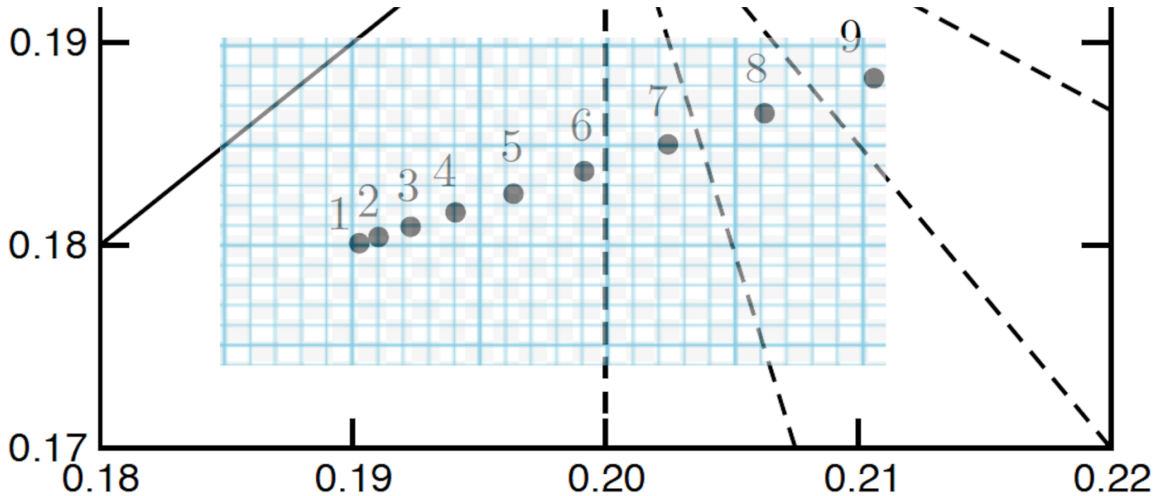
Use the simulation at `http://www.toddsatogata.net/2019-USPAS/lab/Henon.html` to investigate motion under the Hénon map by adjusting the two control parameters: tune Q and the number of turns tracked T. Launch multiple trajectories at many initial locations in phase space. Consider the plot of Hénon dynamic aperture (DA) versus tune shown in Figure 9.4.

(a) How small must $|Q - 1/3|$ be, for the triangular predictions of Exercise 9.4 (question 2 in this homework) to be reasonably valid?

(b) Devise and define a convenient quantitative measure of the size of the stable region – the DA. (There are many ways to do this.)

(c) How does the DA in the range $0.5 < Q < 1.0$ relate to the DA in the range below $Q = 0.5$? Why?

4 Nonlinear Tune Tracking Data



(Modified from Peggs/Satogata problem 10.1) You have simulated the RHIC accelerator with a set of nine particles launched with design momentum ($\delta = 0$), $x' = 0$, and initial x offsets of 1, 2, … 9 mm at a location with horizontal beta function $\beta_x = 40$ m. You "measure" the fractional tunes of these particles from the plot shown above to be:

x [mm]	Q_x	Q_y
1	0.1903	0.1800
2	0.1910	0.1802
3	0.1923	0.1809
4	0.1941	0.1816
5	0.1963	0.1825
6	0.1991	0.1837
7	0.2024	0.1951
8	0.2061	0.1866
9	0.2105	0.1884

(a) plot Q_x and Q_y vs. J_x from the above table.

(b) What is the simplest fit to the tune vs. action data?

(c) What is the simplest and most likely dominant nonlinearity?

5 Dodecapole Detuning

(Modified from Peggs/Satogata problem 10.2) If a single dodecapole (12-pole) magnet delivers an angular kick of

$$\Delta x' = -g_{12} x^5 \tag{5.1}$$

and causes normalized phase space detuning

$$Q_x = Q_{0x} + A g_{12} a^B \tag{5.2}$$

what are the numerical values of coefficients A and B? Note that here you must only calculate the effect of the detuning to *first order* in dodecapole strength.