USPAS Graduate Accelerator Physics Homework 7

Due date: Thursday January 31, 2019

1 ESS RF cavity cell count

Use the synchronism factor calculator at `http://www.toddsatogata.net/2019-USPAS/lab/Synchro.html` to help with the following problem.

The high energy end of an ESS-like proton linac is being designed using either 2 or 3 families of elliptical cell cavities to accelerate from a kinetic energy of 200 MeV to 2.0 GeV. The cavities in each family $i = 1, 2, (3)$ all have the same geometric beta, $\beta_{i,G}$. Family i accelerates from $\beta_{i,min}$ to $\beta_{i,max}$, so that $\beta_{1,max} = \beta_{2,min}$, et cetera. The synchronism factor $S(\beta/\beta_G, N)$ measures the efficiency with which each cavity accelerates, where N is the number of cells per cavity, with a maximum value of $S(1, N) = 1$.

(a) What are the values of γ and β at 200 MeV and 2.0 GeV?

(b) Assuming that $S(x, N) \approx S(1/x, N)$ for any N, what are the optimum β_{tran} values at which to transition from one family to the next, for both 2 and 3 families?

(c) What are the optimum β_G values for those 2 or 3 families?

(d) Using those β_{tran} and β_G values, what are the minimum values of the synchronism factor for 2 or 3 families, with $N = 5, 7$ or 9? How reasonable is the assumption that $S(x, N) \approx S(1/x, N)$?

(e) How would you decide whether to use 2 or 3 families? What are the competing cost and performance drivers?

2 Round Beam-Beam Phase Space

Investigate motion under a single round Gaussian 1-D interaction by working with the simulation code located at `http://www.toddsatogata.net/2019-USPAS/lab/RoundBeamBeam.html` . You can adjust the tune Q and the beam-beam parameter ξ, and launch trajectories at any initial location in phase space on the plot. Consider the $(\phi/2\pi, a/\sigma)$ normalized space shown in Figure 15.4, in which almost-flat lines correspond to regular resonancefree motion, with detuning.

(a) Set $Q = 0.331$ and $\xi = 0.006$, and observe the resonance islands that appear at an amplitude where the beam-beam tune shift moves particles across the $Q = 2/6$ resonance. Compare the amplitude of the resonance island centres to the theoretical prediction.

(b) Why does only every second resonance island appear?

(c) Motion near $Q = 1/3$ becomes unbounded at modest amplitudes when significant sextupoles are present, but here the motion is regular for even the largest amplitudes. Why?

(d) Set the tune to $Q = 0.305$ and print (or save) phase space diagrams for four or five values of ξ in the range from 0.05 to 0.2. What happens as ξ increases?