

# USPAS Accelerator Physics 2019

## Northern Illinois University and UT-Batelle

### Quantum Excitation, Low Emittance Lattices and Synchrotron Light Sources

(with particular thanks to Andy Wolski and Pantaleo Raimondi)  
(extra slides on Compton Sources from Geoff Krafft)

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<http://www.toddsatogata.net/2019-USPAS>

Happy birthday to Anton Chekhov, W.C. Fields, Oprah, and Abdus Salam!

Happy Freethinkers Day, Puzzle Day, National Corn Chip Day, and Curmudgeons Day!

# Electron Emittances

- Steve introduced classical synchrotron radiation (Chapter 11) this morning
  - Bending high energy electrons radiate power
  - Average total particle energy is restored by RF acceleration
- Question:
  - Why don't electron transverse emittances shrink to zero?
- Answer:
  - Transverse noise “thermal equilibrium”
  - Limiting factor is NOT RF cavity alignment or other general accelerator system noise
  - Synchrotron radiated power dominates. It is **quantized**, so power radiated is a **stochastic lumpy noisy** process rather than smooth and continuous

# Quantum Mechanics (No grease monkey, I, but a quantum mechanic)

- Characteristic energy of emitted photons

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho} \quad u_c = \hbar\omega_c = \frac{3}{2} \frac{\hbar c\gamma^3}{\rho} \quad \hbar = 6.582 \times 10^{-16} \text{ [eV} \cdot \text{s]}$$

- We can simplify using our old friends  $\frac{p}{c} = B\rho$   $p = \beta\gamma mc \approx E/c$  to find

$$u_c \text{ [keV]} \approx 0.665 E^2 \text{ [GeV}^2\text{]} B \text{ [T]}$$

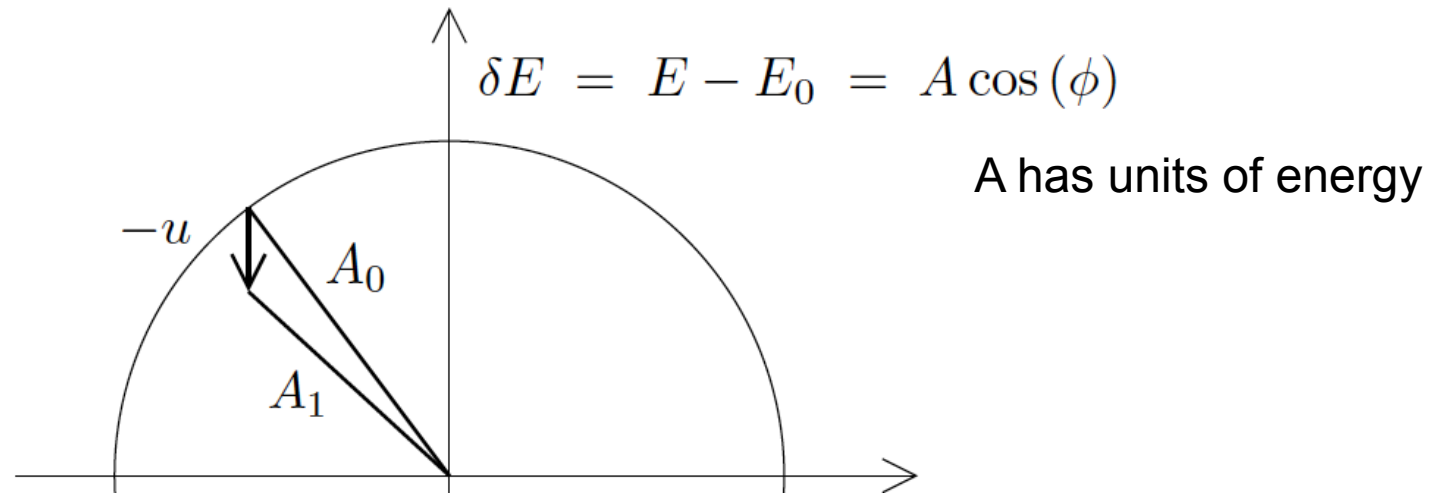
- How many photons? We integrate over a quantum spectrum to find N (photons/sec) and apply random walk

$$N = \int n(u) du \quad \mathcal{P}(\omega) = u n(u) \frac{du}{d\omega} = \hbar u n(u)$$

$$N \langle u^2 \rangle = \int n(u) u^2 du = \frac{55}{24\sqrt{3}} r_0 \hbar m c^4 \frac{\gamma^7}{\rho^3} \approx 8.5 \times 10^{-8} \frac{\gamma^7}{\rho^3 [\text{m}^3]} \left[ \frac{\text{eV}^2}{\text{s}} \right]$$

# Longitudinal Emittance Growth and Equilibrium

- Consider single particle change in energy  $-u$  producing change in longitudinal phase space amplitude  $A$



- Use law of cosines, average over phase  $\phi$  and photon distribution to find the average square amplitude growth over time; at equilibrium this is equal to the damping rate

$$\frac{d\langle A^2 \rangle}{dt} = N \langle u^2 \rangle = \frac{\langle A^2 \rangle}{\tau_s/2} \quad (11.32)$$

# Longitudinal Equilibrium Energy Spread

- We can average around the ring as well as the distribution and longitudinal phases to connect  $A$  and the “natural” energy spread:

$$\sigma_E^2 = \frac{1}{2} \langle A^2 \rangle = \frac{\tau_s}{4} \langle N \langle u^2 \rangle \rangle_s$$

- Then we can finally find the equilibrium energy spread

$$\left( \frac{\sigma_E}{E_0} \right)^2 = \frac{C_q}{J_s} \gamma^2 \frac{\langle 1/\rho^3(s) \rangle_s}{\langle 1/\rho^2(s) \rangle_s}$$

$J_s$  : partition function

Averages: “radiation integrals”

$$C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc}$$

$$= 3.84 \times 10^{-13} \text{ [m]} \quad \text{electrons}$$

$$= 2.09 \times 10^{-16} \text{ [m]} \quad \text{protons}$$

Only geometry: live with it!  
o(few e-4) for most light sources

# Transverse Equilibrium Emittance

- The equilibrium emittance, balanced between synchrotron radiation damping and quantum excitation effects, is

$$\epsilon_x = \frac{C_q}{J_x} \gamma^2 \frac{\langle \mathcal{H}(s) / \rho^3(s) \rangle_s}{\langle 1 / \rho^2 \rangle_s}$$

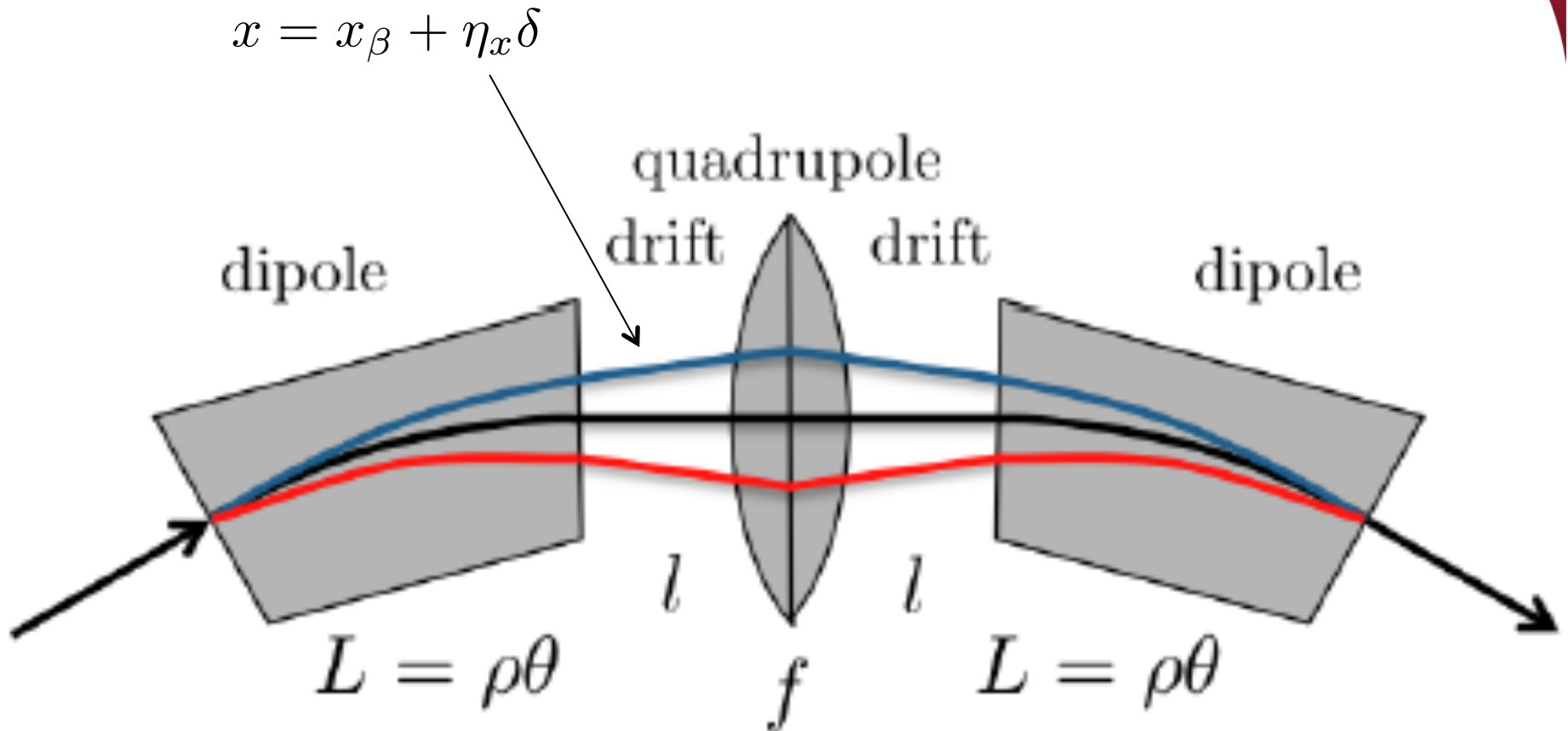
- This depends on details of the lattice, in particular dispersion located in the dipoles (at locations of synchrotron radiation emission that couple dimensions)

$$\mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x(s) \eta_x \eta_x' + \gamma_x \eta_x^2$$

(“Curly – H function”)

- Vertical is like this but dispersion and curly-H are zero!

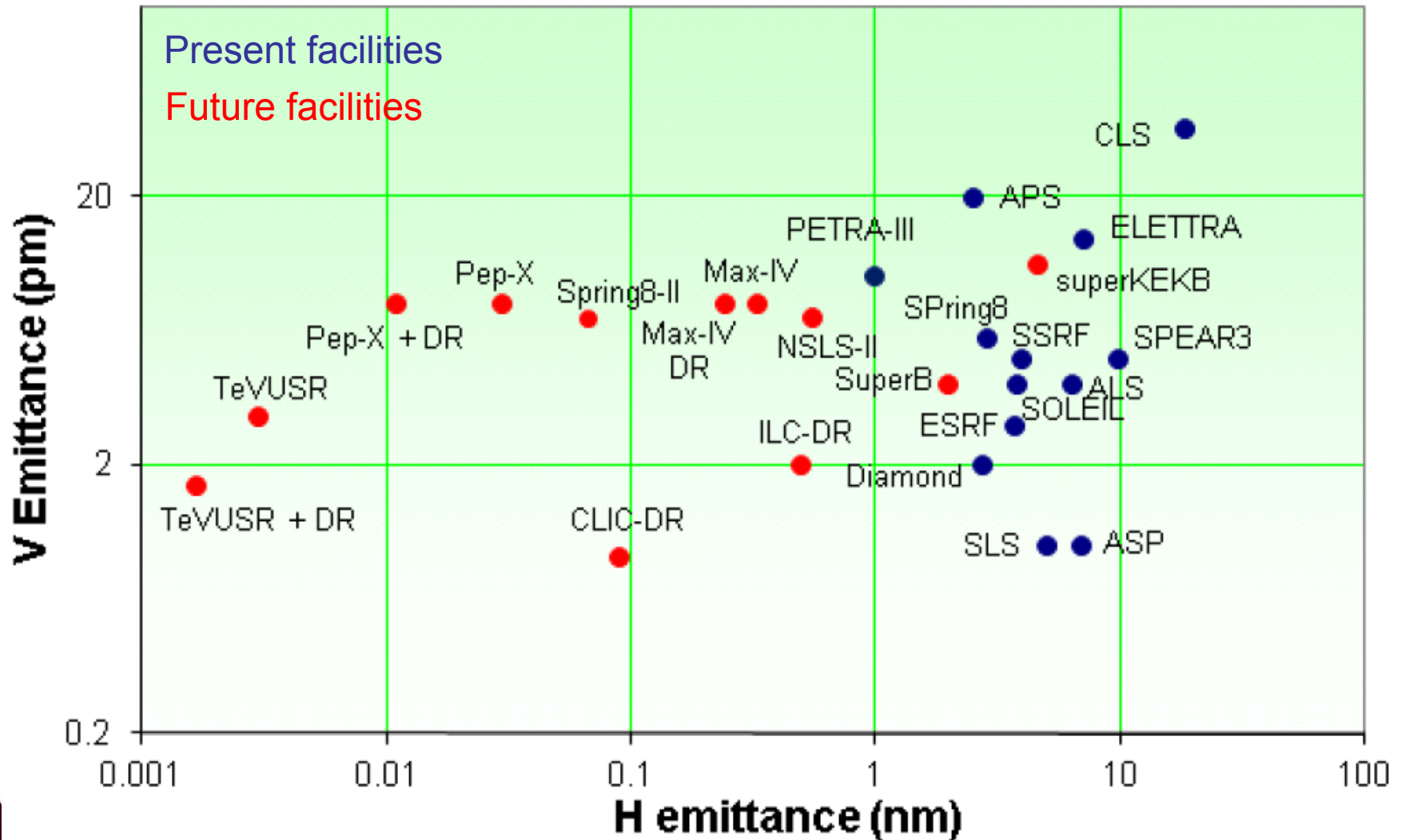
# Review: Double Bend Achromat



- DBA is also known as a Chasman-Green lattice
  - Used in early third-generation light sources (e.g. NSLS at BNL)
  - But now we know about synchrotron radiation,  $\mathcal{H}$  functions

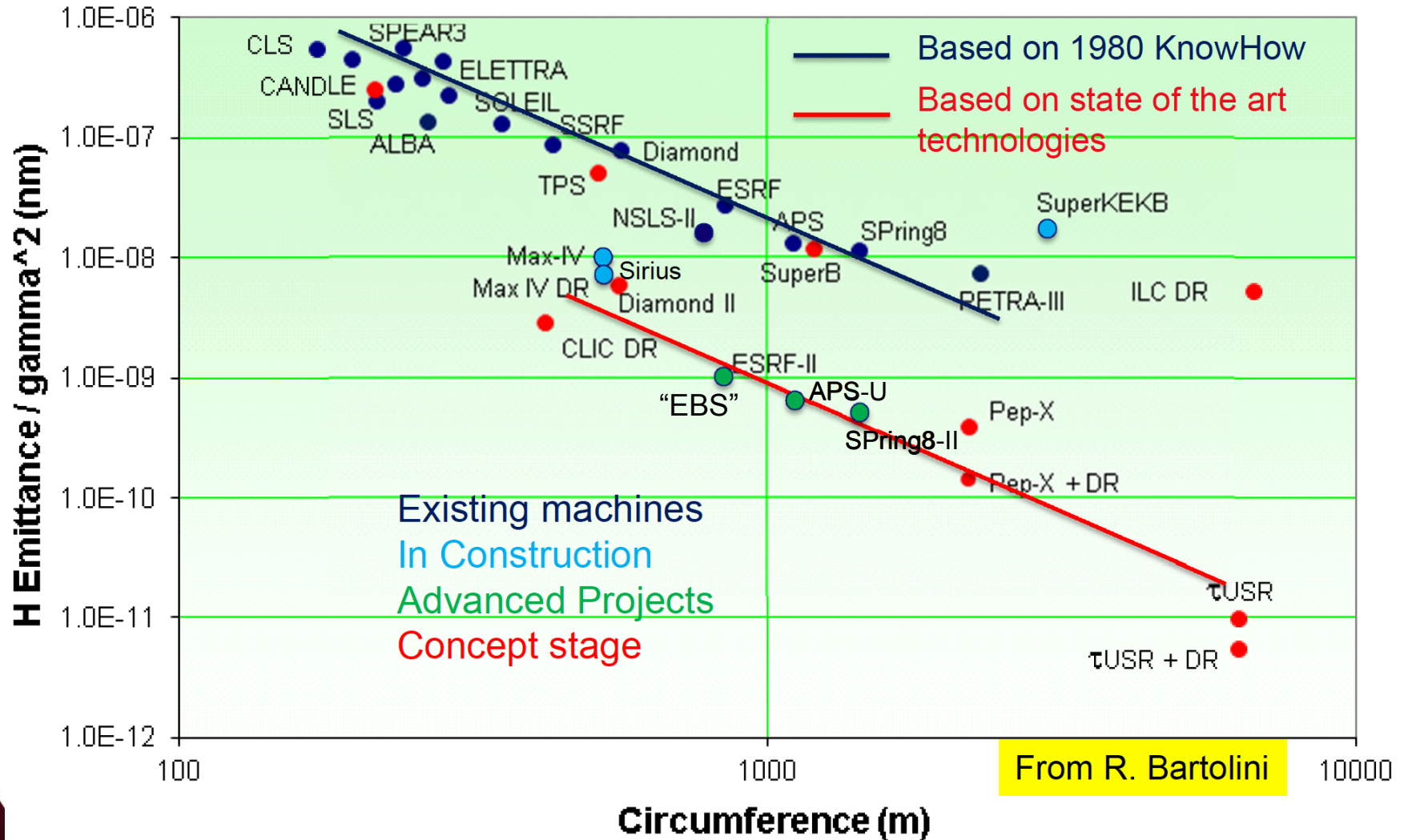
# One View of Light Source Emittances

Note the logarithmic emittance scales and relatively constant vertical emittance!





# Another View of Light Source Emittances



# Radiation Integrals

- There are several other radiation integrals that come into play in evaluation of effects of radiation on dynamics of ultra-relativistic particles in a storage ring or beamline, including one that depends on curly-H.

$$I_1 \equiv \oint \frac{\eta_x(s)}{\rho(s)} ds \quad \text{momentum compaction } \alpha_p = \frac{I_1}{L}$$

$$I_2 \equiv \oint \frac{ds}{\rho^2(s)} \quad I_4 \equiv \oint \frac{\eta_x(s)}{\rho(s)} \left( \frac{1}{\rho^2(s)} + 2k_1(s) \right) ds$$

$$I_3 \equiv \oint \frac{ds}{|\rho(s)|^3} \quad I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds$$

These integrals only depend on the lattice design

# Relation of Integrals to Partition Numbers

- For a horizontal planar ring we can write relationships between the radiation integrals and partition numbers:

Partition numbers

$$J_x = 1 - \frac{I_4}{I_2} \quad J_y = 1 \quad J_s = 2 + \frac{I_4}{I_2}$$

- See Handbook of Accelerator Physics, Chao and Tigner, p. 210

# Equilibrium Horizontal Emittance

- The evolution of horizontal emittance, including both damping and quantum excitation, is

$$\frac{d\epsilon_x}{dt} = -\frac{2}{\tau_x}\epsilon_x + \frac{2}{J_x\tau_x}C_q\gamma^2\frac{I_5}{I_2} \quad J_x = 1 - \frac{I_4}{I_2}$$

damping

Quantum excitation

“quantum constant”  $C_q = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} \approx 3.83 \times 10^{-13} \text{ m}$

- This is at an equilibrium for the “natural” emittance

$$\epsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2}$$

- This only depends on beam energy and radiation integrals!

# Equilibrium Energy Spread

- We can average the quantum excitation effects on beam momentum offset to find the evolution of energy spread:

$$\frac{d\sigma_\delta^2}{dt} = C_q \gamma^2 \frac{2}{J_u \tau_u} \frac{I_3}{I_2} - \frac{2}{\tau_u} \sigma_\delta^2 \quad J_u = 2 + \frac{I_4}{I_2}$$

Quantum excitation

damping

- We can also find the equilibrium energy spread and bunch length

$$\sigma_{\delta 0}^2 = C_q \gamma^2 \frac{I_3}{J_u I_2}$$

bunch length :  $\sigma_{z0} = \frac{\alpha_p c}{Q_s} \sigma_{\delta 0}$

- Note the lack of RF parameters! This equilibrium distribution is again determined only by the lattice (and collective effects). We can shorten bunch length by raising RF voltage

# Evaluating Radiation Integrals

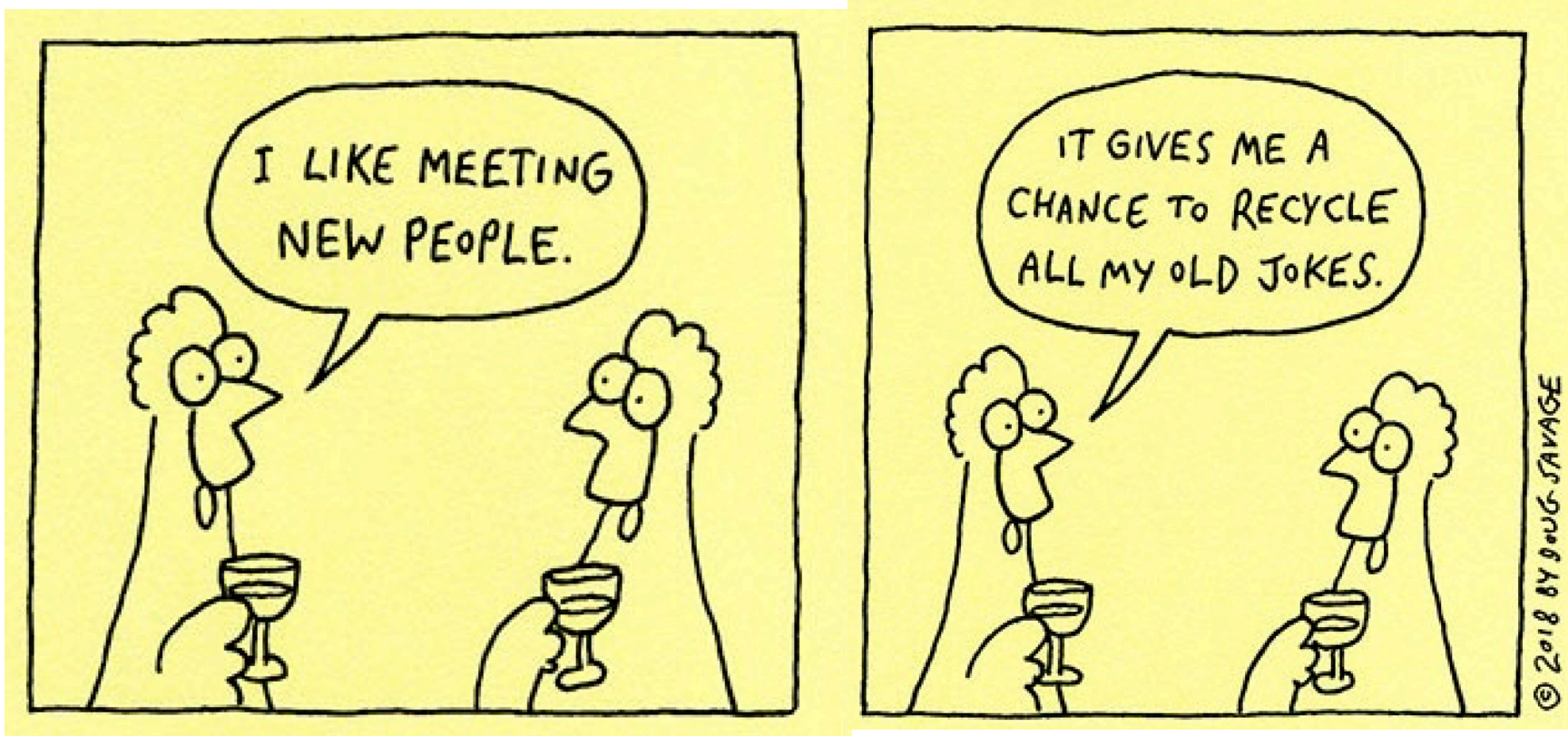
- If bends have no quadrupole component (a modern separated function synchrotron),  $J_x \approx 1$  and  $J_u \approx 2$
- To find the equilibrium emittance, we then need to evaluate two synchrotron radiation integrals
- $I_2$  depends on only detailed knowledge of dipole magnets
  - e.g. for all dipole magnets being the same, total bend  $2\pi$

$$I_2 = \oint \frac{ds}{\rho^2(s)} = \frac{2\pi B}{(B\rho)} \approx \frac{2\pi cB}{U/e}$$

- Evaluating  $I_5$  depends on detailed knowledge of optics

$$I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \quad \mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

## (Take a breath)



## FODO Lattice I<sub>5</sub>

- Just like our excursions into the FODO lattice before, we had calculated our optical functions in terms of
  - Thin quadrupole focal length  $f$
  - Dipole bending radius  $\rho$  (for dispersion contributions)
  - Dipole lengths  $L = \rho\theta$  (full space between quadrupoles)
- These calculations are usually done with computer programs that find the optical functions and integrate  $\mathcal{H}$  for us.
  - But Wolski (see below) writes out some of the logic to progress through a FODO lattice and evaluate some reasonably realistic approximations

$$\theta \ll 1 \quad \Rightarrow \quad \rho \gg 2f \quad \Rightarrow \quad 4f \gg L$$



## FODO Lattice $I_5$

- Similar to the dogleg, the analysis is most easily done in an expansion of small dipole bend angle  $\theta$

$$\begin{aligned}\frac{I_5}{I_2} &= \left(4 + \frac{\rho^2}{f^2}\right)^{-\frac{3}{2}} \left[8 - \frac{\rho^2}{2f^2}\theta^2 + O(\theta^4)\right] \\ &\approx \left(1 - \frac{\rho^2}{16f^2}\theta^2\right) \left(1 + \frac{\rho^2}{4f^2}\right)^{-\frac{3}{2}} \quad \sin \frac{\mu}{2} = \frac{\rho\theta}{2f}\end{aligned}$$

$$\rho \gg 2f \quad \Rightarrow \quad \frac{I_5}{I_2} \approx \left(1 - \frac{L^2}{16f^2}\right) \frac{8f^3}{\rho^3}$$

$$4f \gg L \quad \Rightarrow \quad \frac{I_5}{I_2} \approx \frac{8f^3}{\rho^3}$$

# Approximate Natural Emittance of FODO Lattice

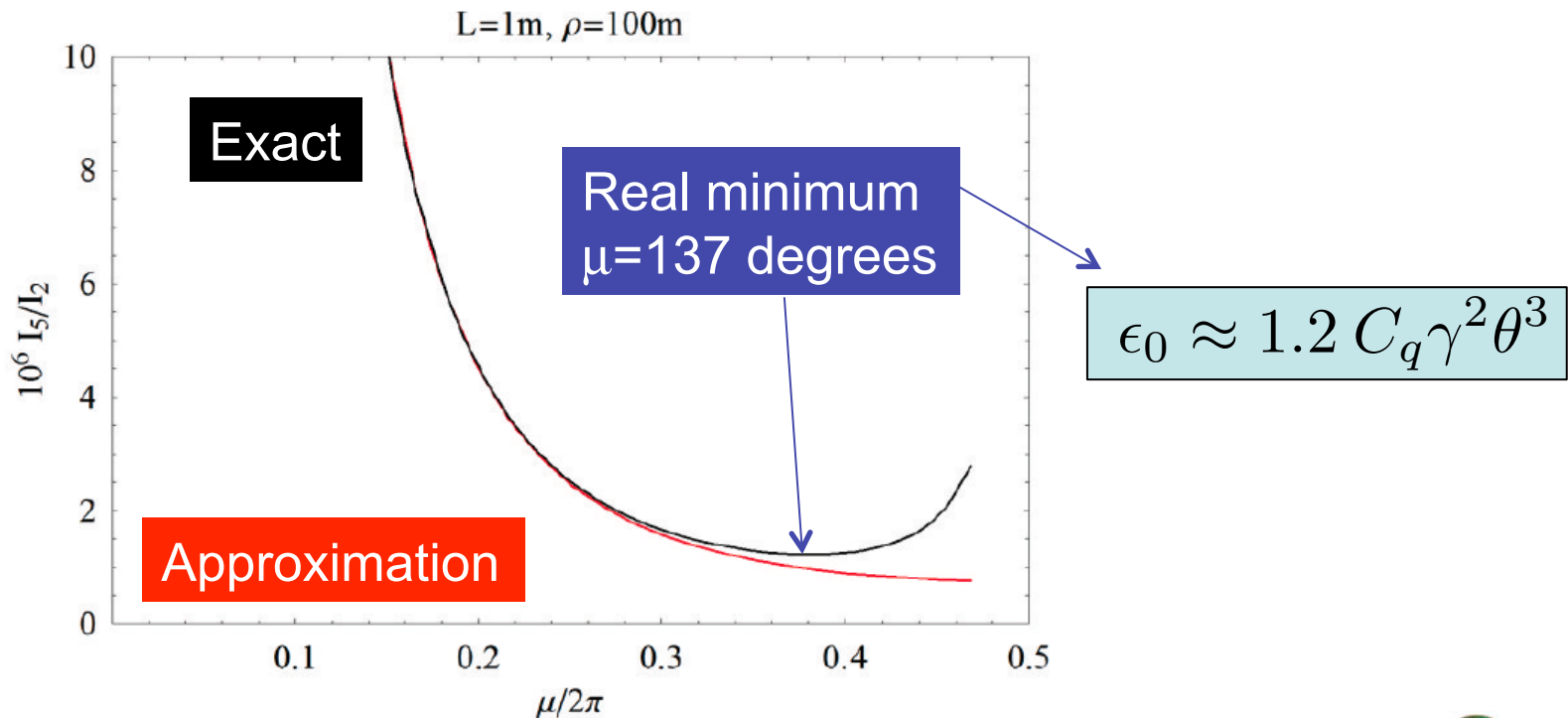
- We can then write the approximate natural horizontal emittance of the FODO lattice, again with  $J_x \approx 1$

$$\epsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2} \approx C_q \gamma^2 \left( \frac{2f}{L} \right)^3 \theta^3$$

- Proportional to square of beam energy  $\gamma$
- Proportional to cube of bending angle per dipole
  - Increase number of cells to reduce bending angle per dipole and thus reduce FODO emittance.
- Proportional to cube of quadrupole focal length
  - Stronger quads gives stronger focusing, lower natural emittance
- Inversely proportional to cube of the cell (or dipole) length
  - Longer cells also reduce overall natural emittance

# Minimum Emittance of FODO Lattice?

- The stability criterion for FODO lattices with these parameters is  $f \geq L/2$  with a minimum of  $f/L = 1/2$ 
  - Estimated FODO lattice minimum emittance
$$\epsilon_0 \approx C_q \gamma^2 \theta^3$$
  - But approximations start to break down for strong focusing



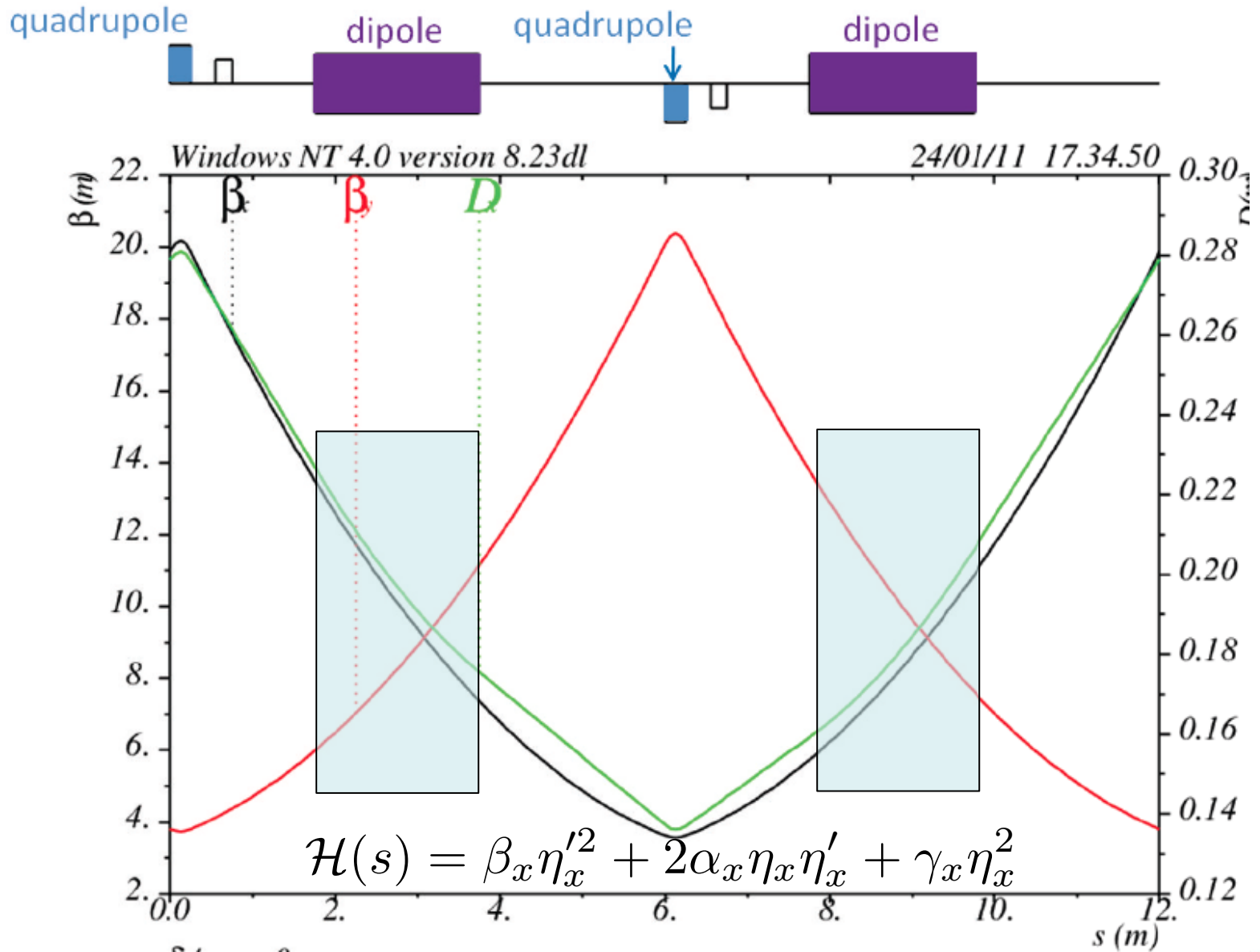
# We Can Do Better!

- It turns out that this emittance isn't usually good enough for modern third-generation light source requirements
  - 1-2 orders of magnitude too big
- How do we fix this?
  - Beam energy determines some properties of the sync light
  - So the remaining handle we have is the optics

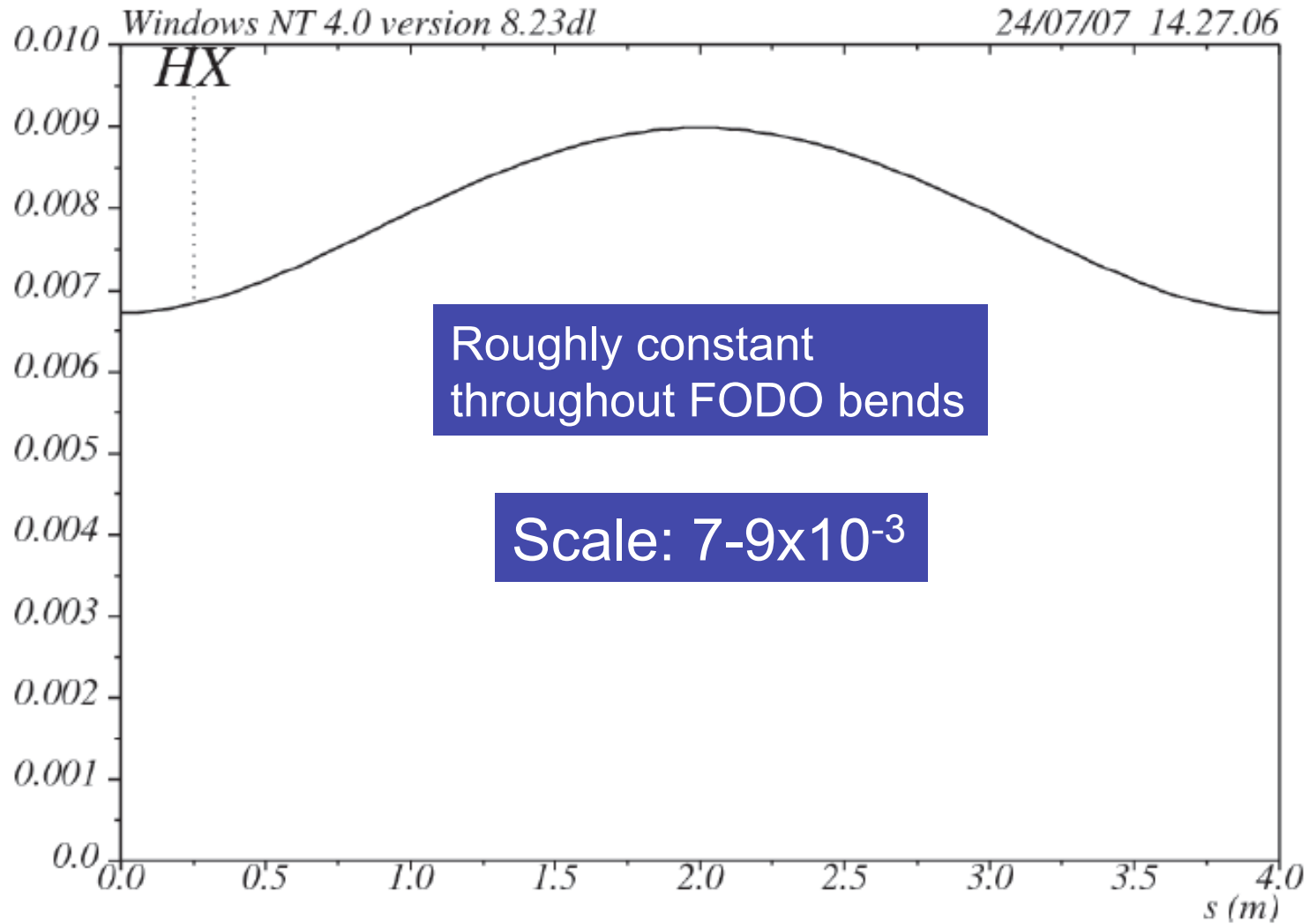
$$I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \quad \mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

- Minimizing  $\eta$  and  $\eta'$  in the dipoles will minimize the overall integral of  $\mathcal{H}$  and thus  $I_5$
- How do the dispersion functions look though FODO dipoles?

# FODO Cell Optics

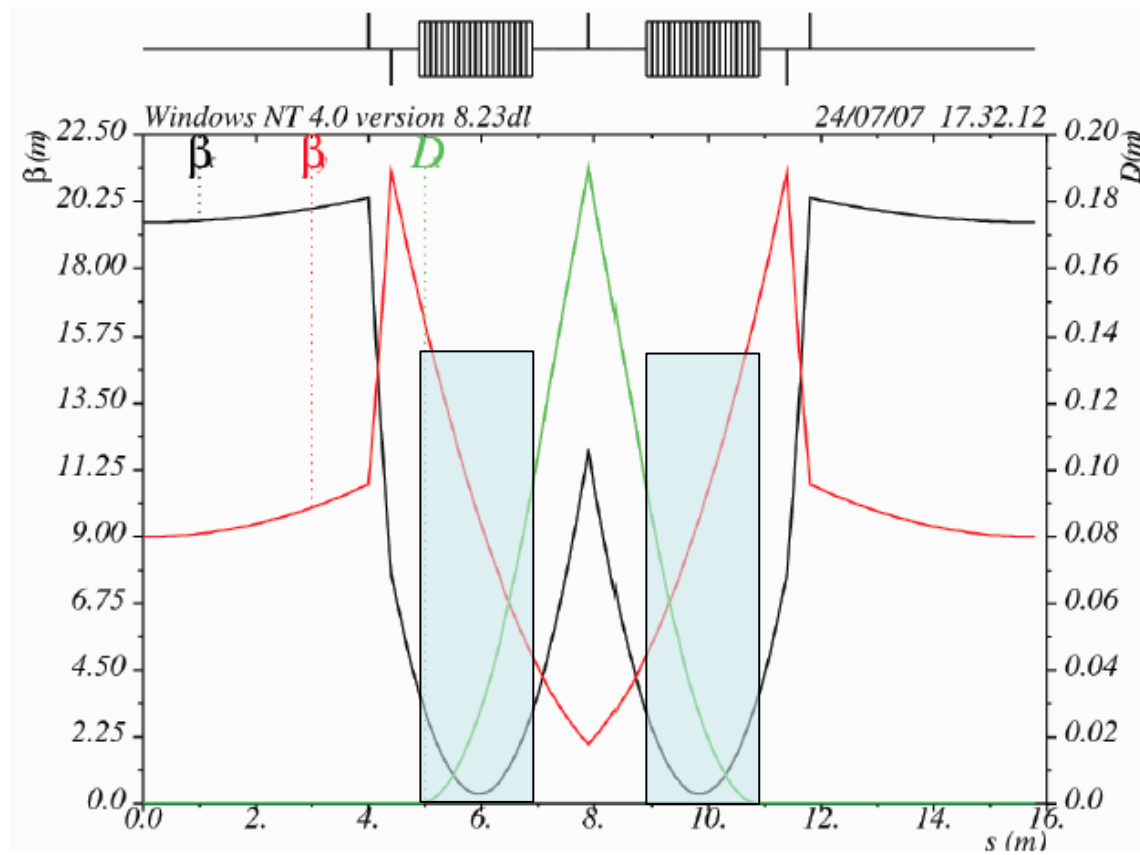


# FODO Dipole $\mathcal{H}$

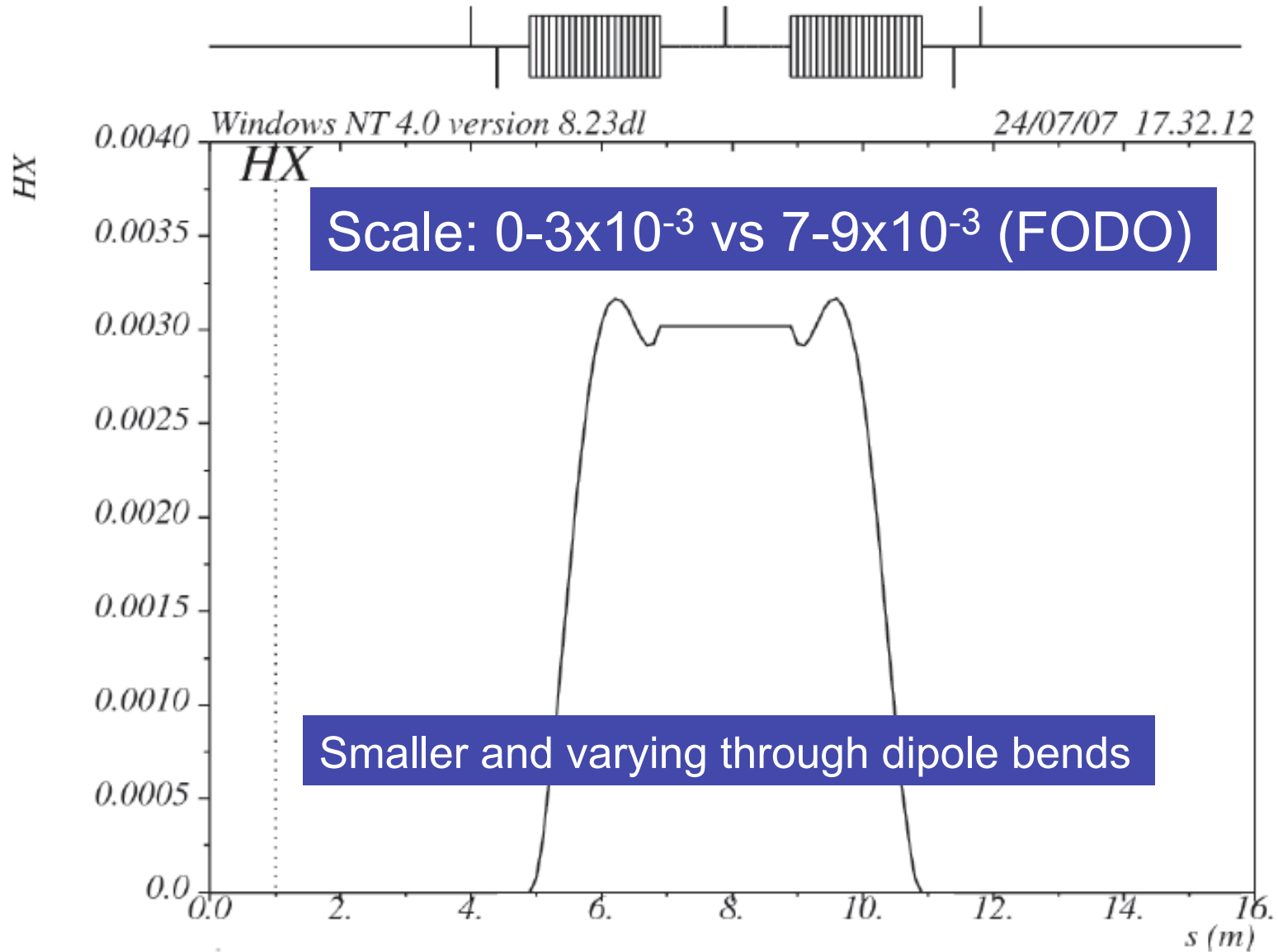


# Double Bend Achromats

- If only we had a lattice that had dipoles that had zero  $\eta$  and  $\eta'$  somewhere near their ends
- We do, the double bend achromat!
  - Add extra focusing at ends for periodic matching with  $\alpha_{x,y} \approx 0$



# DBA Dipole $\mathcal{H}$





## DBA Radiation Integrals

- We can optimize the beta functions and matching vs dipole length to produce a best (minimum) integral of  $I_5$

$$I_{5,\min} = \frac{1}{4\sqrt{15}} \frac{\theta^4}{\rho} + o(\theta^6)$$

$$I_2 = \int \frac{ds}{\rho^2} = \frac{\theta}{\rho}$$

dipole ends

$$\beta_x \approx L\sqrt{12/5}$$

$$\alpha_x \approx \sqrt{15}$$

$$\epsilon_{0,\text{DBA},\min} = C_q \gamma^2 \frac{I_{5,\min}}{J_x I_2} \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$$

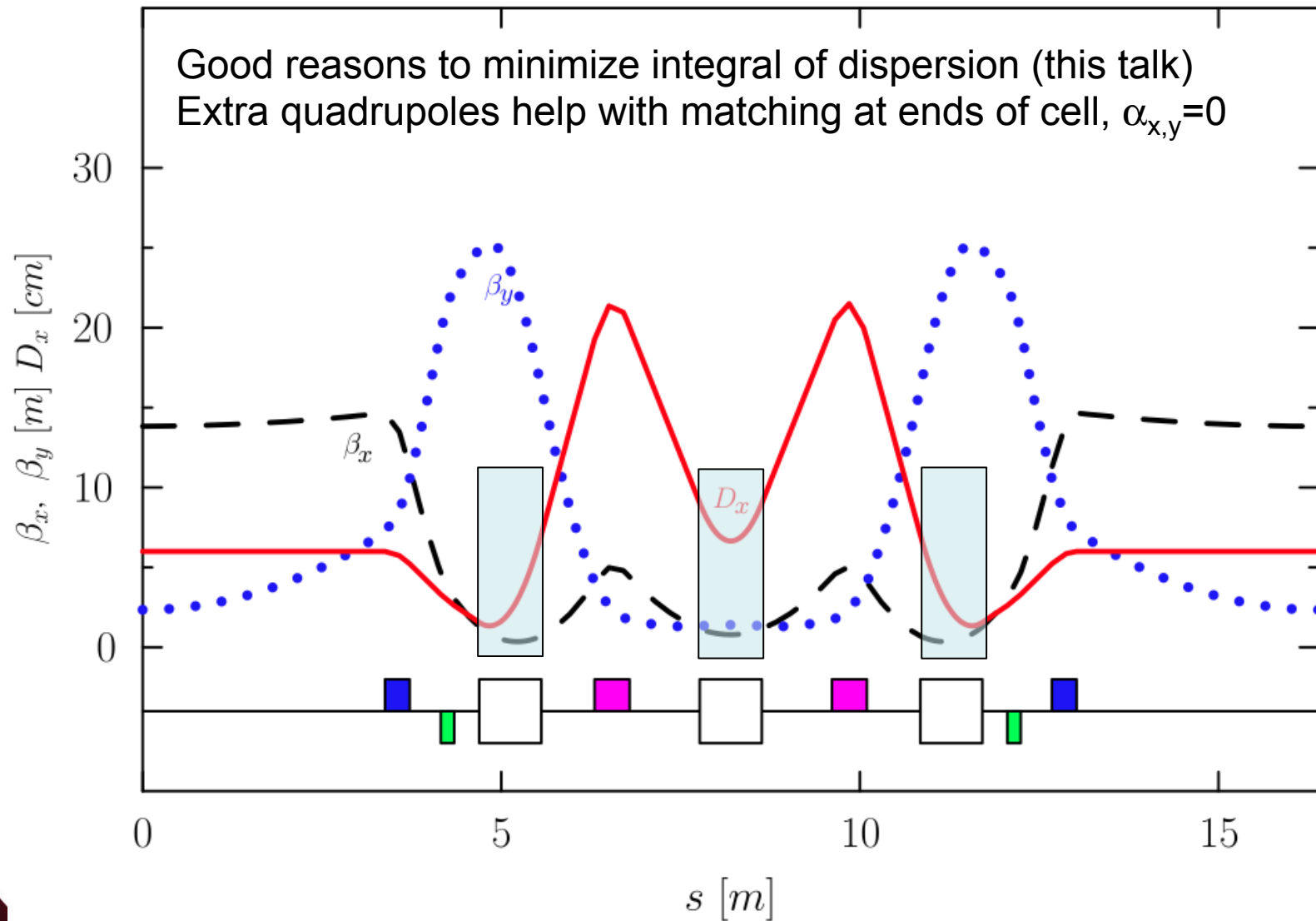
- This is about 13 times smaller (!) than the FODO lattice minimum emittance!

$$\epsilon_0 \approx 1.2 C_q \gamma^2 \theta^3$$

## But We Can Still Do Better

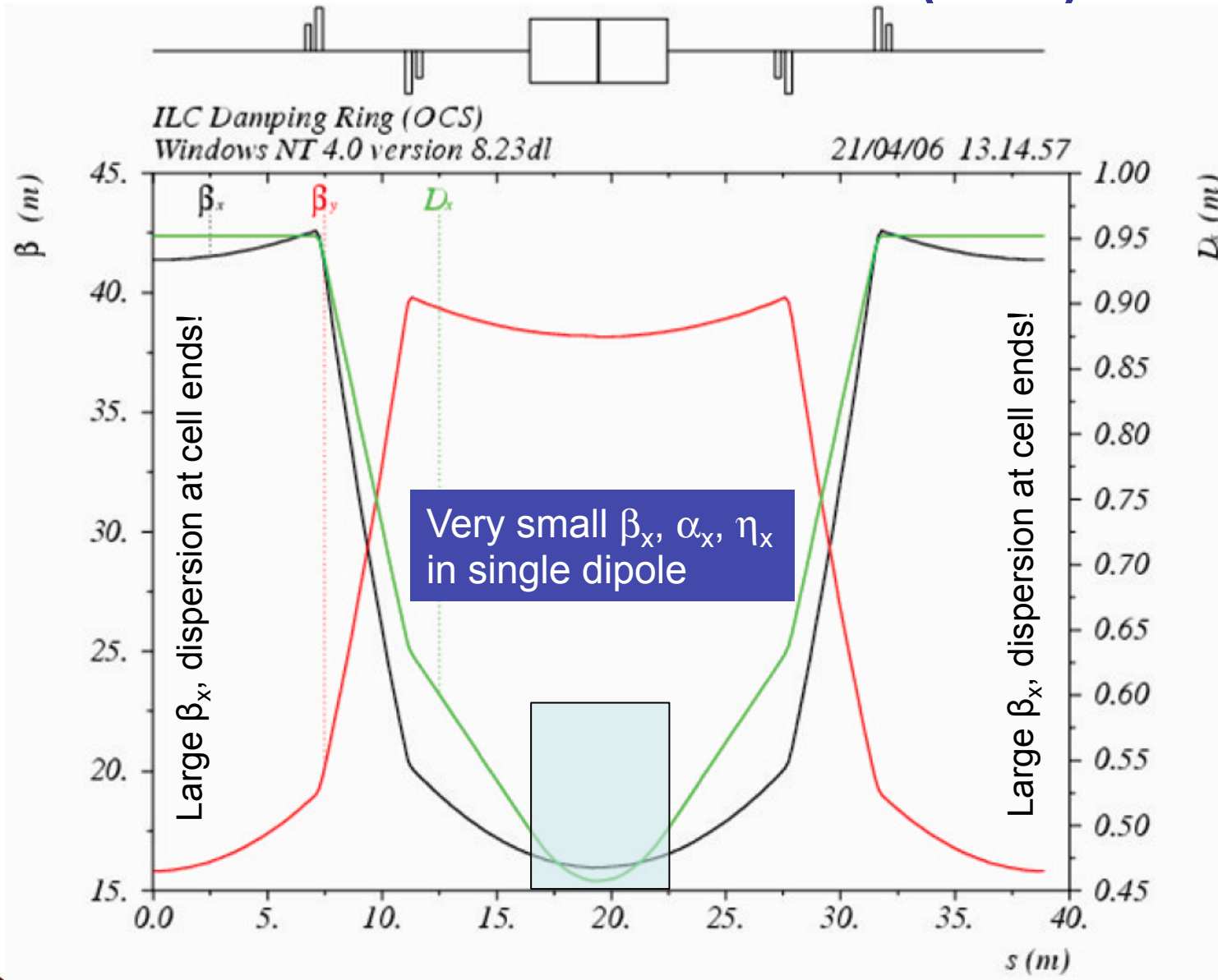
- The double bend achromat was a huge step forward
  - Made NSLS into a very successful light source
  - But we can still further optimize  $I_5$
- One way to do this is the triple bend achromat shown (very briefly) earlier
  - e.g. the ALS, BESSY-II, SLS (Swiss Light Source, PSI)
  - This can place local minima at the dipoles
  - One tradeoff: more complicated lattice, more expensive...
  - More focusing also provides stronger chromatic effects
    - Correction with sextupoles requires nonlinear optimization
- Another solution: minimize  $I_5$  wrt all lattice parameters
  - So-called TME (theoretical minimum emittance) lattices
  - Tend to not be very locally robust solutions
  - But they sure get close to minimizing the natural emittance

# Triple Bend Achromat Cell (ALS at LBL)



L. Yang et al, Global Optimization of an Accelerator Lattice Using Multiobjective Genetic Algorithms, 2009

# “Theoretical Minimum Emittance” (TME) Lattice



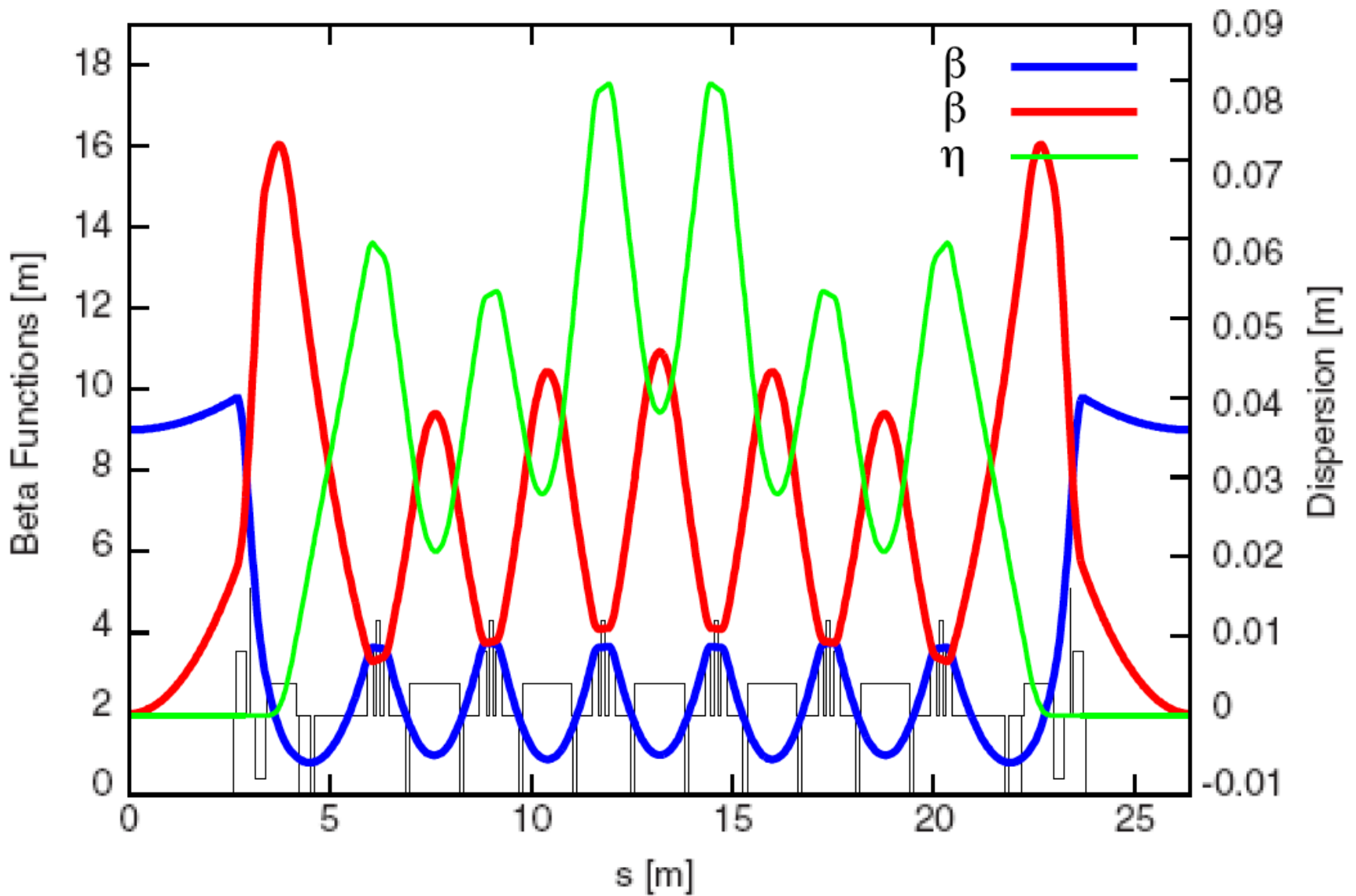
## Summary of Some Minimum Emittance Lattices

Lattice style	Minimum emittance	Conditions/comments
90° FODO	$\varepsilon_0 \approx 2\sqrt{2}C_q\gamma^2\theta^3$	$\frac{f}{L} = \frac{1}{\sqrt{2}}$
	/2.36	
137° FODO	$\varepsilon_0 \approx 1.2C_q\gamma^2\theta^3$	minimum emittance FODO
	/18.6	
DBA	$\varepsilon_0 \approx \frac{1}{4\sqrt{15}}C_q\gamma^2\theta^3$	$\eta_{x,0} = \eta_{px,0} = 0$ $\beta_{x,0} \approx \sqrt{12/5}L \quad \alpha_{x,0} \approx \sqrt{15}$
	/3	
TME	$\varepsilon_0 \approx \frac{1}{12\sqrt{15}}C_q\gamma^2\theta^3$	$\eta_{x,\min} \approx \frac{L\theta}{24} \quad \beta_{x,\min} \approx \frac{L}{2\sqrt{15}}$

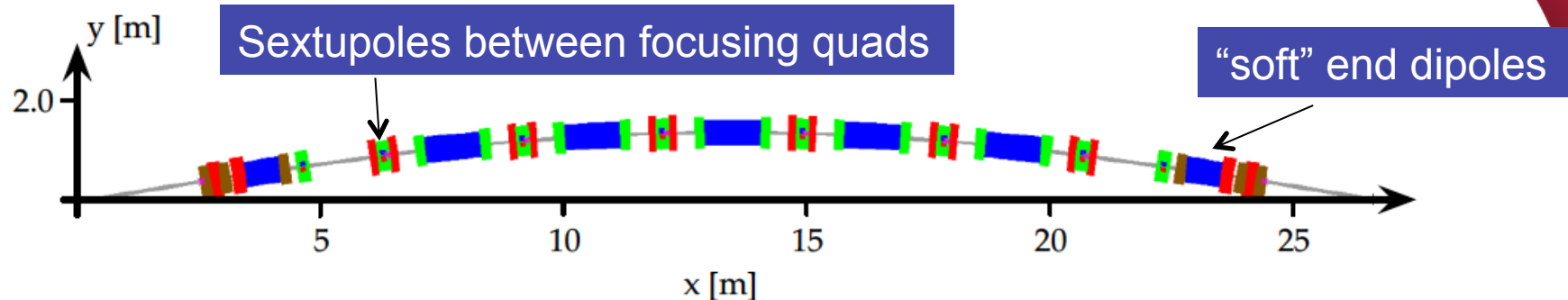
# Why Not TME All The Time?

- Optimizing one parameter (beam emittance) does not necessarily optimize the facility performance!
  - TME lattices are considered by many to be over-optimized
  - High chromaticities give very sensitive sextupole distributions
    - These in turn give very sensitive nonlinear beam dynamics
    - Momentum aperture, dynamic aperture, ...
    - More tomorrow and Thursday
- Usually best to back off TME to work on other optimization
  - Another alternative is to move towards machines with many dipoles
    - Reduces bending angle per dipole and brings emittance down
    - MAX-IV: 7-bend achromat; SPRING-8 6- and 10-bend achromats

# MAX-IV 7-Bend Achromat



# MAX-IV Nonlinear Optics



**Figure 1:** Schematic of one of the 20 achromats of the MAX IV 3 GeV storage ring. Magnets indicated are gradient dipoles (blue), focusing quadrupoles (red), sextupoles (green), and octupoles (brown).

- MAX-IV represents an interesting case in optics design
  - Soft end dipoles minimize synchrotron radiation on SC IDs
  - All dipoles have vertical gradient
  - Strong focusing -> large chromaticities
  - Low dispersion -> very strong chromaticity sextupoles
  - Three sextupole families optimize higher-order chromaticity and driving terms
  - Additional octupoles also correct tune change vs amplitude



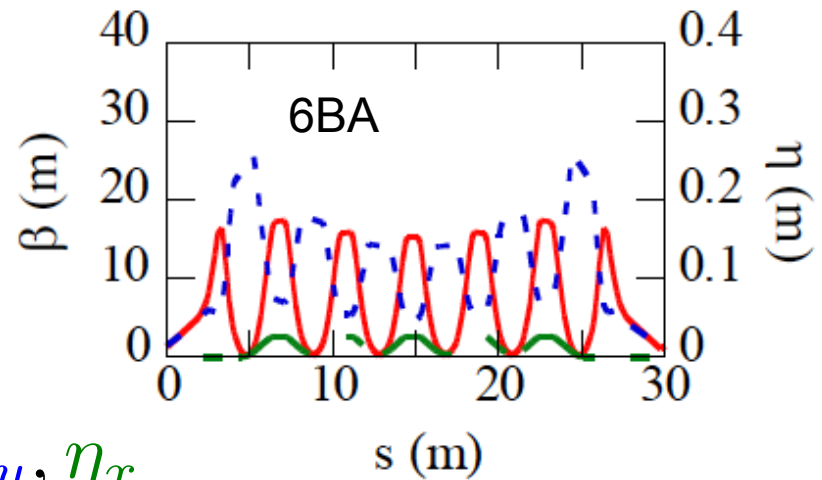
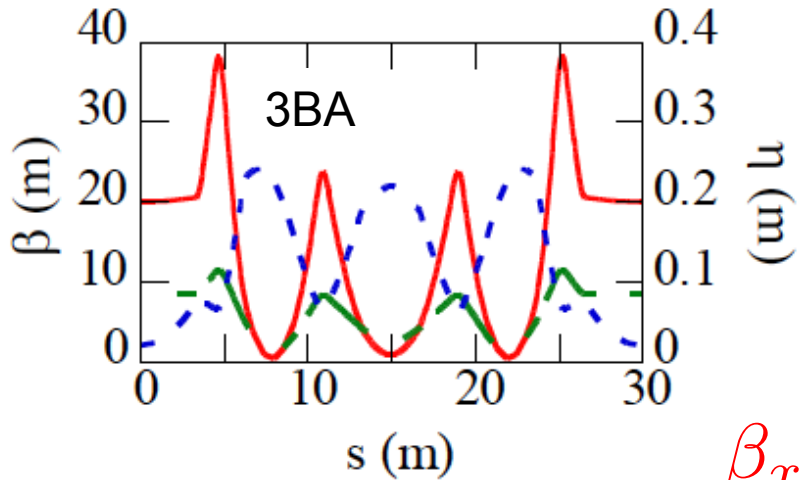
## MAX-IV Parameters

<i>Parameter</i>	<i>Unit</i>	<i>Value</i>
Energy	GeV	3.0
Main radio frequency	MHz	99.931
Circulating current	mA	500
Circumference	m	528
Number of achromats	...	20
Number of long straights available for IDs	...	19
Betatron tunes (H/V)	...	42.20 / 16.28
Natural chromaticities (H/V)	...	-50.0 / -50.2
Corrected chromaticities (H/V)	...	+1.0 / +1.0
Momentum compaction factor	...	$3.07 \times 10^{-4}$
Horizontal damping partition	...	1.85
Horizontal emittance (bare lattice)	nm·rad	0.326
Radiation losses per turn (bare lattice)	keV	360.0
Natural energy spread	...	0.077%
Required momentum acceptance	...	4.5%

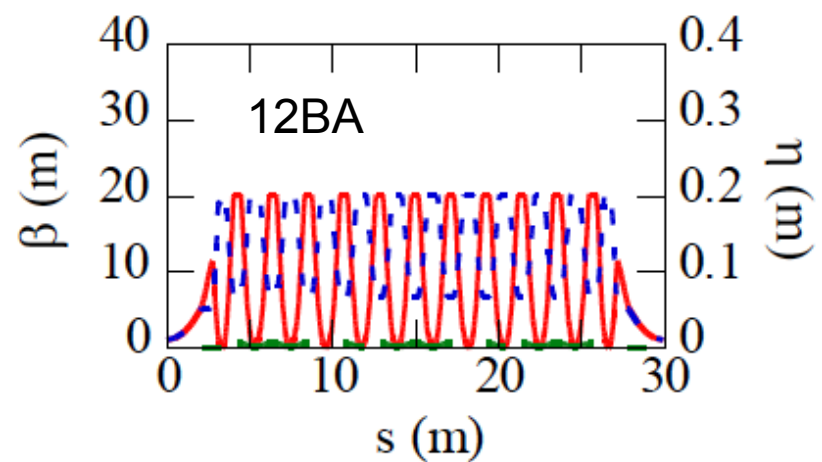
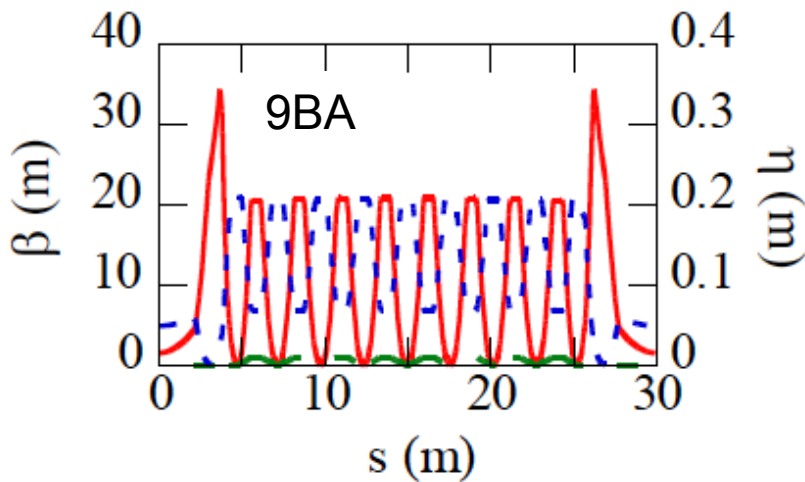
Lattice	Type	$E$ [GeV]	$\epsilon_x$ [nm·rad]	$\epsilon_x^*$ [nm·rad]	$J_x$	$\langle \mathcal{H}_x \rangle$ [ $\times 10^{-3}$ ]	$F_{rel}$	$\xi_x/v_x$	$S$
<a href="#">Spring-8</a>	11×DB-4	8	3.4	3.7	1.0	1.42	4.6	2.2	58
<a href="#">ESRF</a>	DB-32	6	3.8		1.0	1.68	3.5	3.6	89
<a href="#">APS</a>	DB-40	7	2.5	3.1	1.0	1.35	3.3	2.5	69
<a href="#">PETRA III</a>	Mod. FODO	6	1		1.0	3.62	39.8	1.2	20
<a href="#">SPEAR3</a>	DB-18	3	11.2		1.0	5.73	7.4	5.5	73
<a href="#">ALS</a>	TB-12	1.9	6.3	6.4	1.0	4.99	10.4	1.7	24
BESSY II	TBA-10	1.9	6.1		1.0	4.83	2.9	2.8	40
SLS	TBA-12	2.4	5		1.0	3.38	2.6	3.2	56
DIAMOND	DB-24	3	2.7		1.0	1.46	4.2	2.9	76
ASP	DB-14	3	7		1.4	5.60	3.0	2.1	28
<a href="#">ALBA</a>	DB-16	3	4.3		1.3	2.96	2.6	2.1	39
<a href="#">SOLEIL</a>	DB-16	2.75	3.7	5.5	1.0	1.79	2.0	2.8	67
CLS	DBA-12	2.9	18.3		1.6	16.79	2.0	1.3	10
ELETTRA	DBA-12	2	7.4		1.3	9.12	1.4	3.0	31
TPS	DB-24	3	1.7		1.0	1.08	2.7	2.9	87
NSLS-II	DBA-30	3	2		1.0	3.78	2.0	3.1	50
MAX-IV	7BA-20	3	0.33		1.9	0.40	18.1	1.2	59
<a href="#">PEP-X (TME)</a>	4×8TME-6	4.5	0.095		1.0	0.34	3.3	1.7	90
PEP-X (USR)	8×7BA-6	4.5	0.029		1.0	0.10	5.3	1.4	145
<a href="#">TeVUSR</a>	30×7BA-6	11	0.0031		2.4	0.02	12.0	1.4	360
TeVUSR	30×7BA-6	9	0.0029		2.7	0.02	18.4	1.4	281

J. Bengtsson, 2012, Nonlinear Dynamics Optimization in Low Emittance Rings, ICFA Beam Dynamics Newsletter 57, April 2012

# Multiple Bend Achromats (BAs)



$\beta_x, \beta_y, \eta_x$



Y. Shimosaki, 2012, Nonlinear Dynamics Optimization in Low Emittance Rings, ICFA Beam Dynamics Newsletter 57, April 2012

# Ultimate Storage Ring Concept: PEP-X

$E = 4.5 \text{ GeV}$

$I = 1.5 \text{ A}$

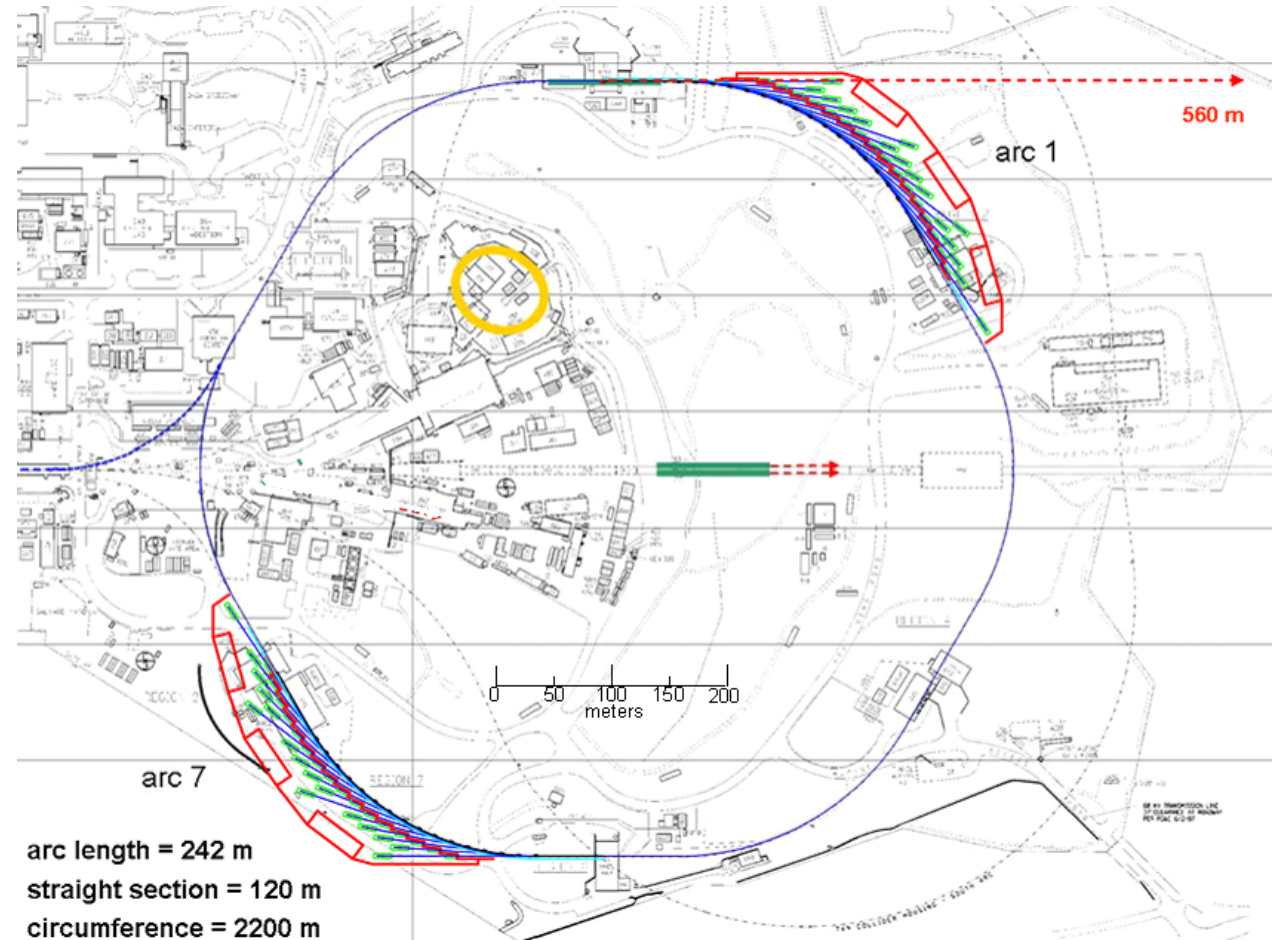
$e_x = 150 \text{ pm-rad}$   
( $\sim 0.06 \text{ nm-rad}$  w/o IBS)

$e_y = 8 \text{ pm-rad}$

$\sigma_s = 3/6 \text{ mm}$   
(without/with 3rd harm rf)

$t = \sim 1 \text{ h}$

top-up injection  
every few seconds  
( $\sim 7 \text{ nC}$ , multiple bunches)



arc length = 242 m  
straight section = 120 m  
circumference = 2200 m

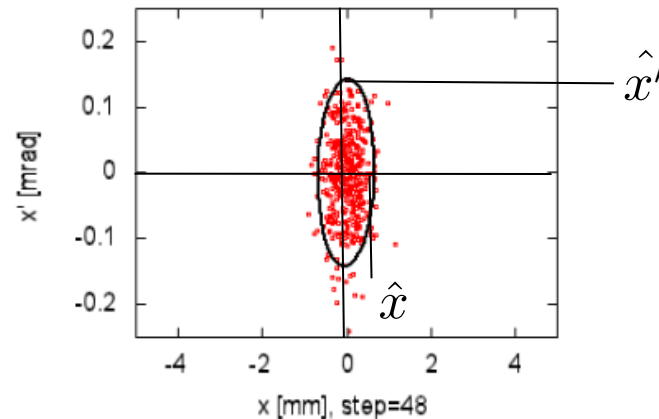
- 2 arcs of DBA cells with 32 ID beam lines (4.3-m straights)
- 4 arcs of TME cells
- $\sim 90 \text{ m}$  damping wigglers
- 6 ea 120-m straights for injection, RF, damping wigglers long IDs, etc.

B. Hettel (SLAC) Future Light Sources 2012 Workshop

# Small Emittance Drawbacks: Touschek Scattering

- Electrons within the electron bunches in a synchrotron light storage ring do sometimes interact with each other
  - They're all charged particles, after all
- Fortunately most of these interactions are negligible for high energy, ultrarelativistic electron beams
  - $\gamma \gg 1$  so, e.g., time dilation reduces effect of space charge  $\propto \gamma^{-2}$
  - But these are long-distance Coulomb repulsions
  - High angle scattering can lead to sudden large momentum changes for individual electrons
  - Low emittance and high brilliance enhances this effect
    - Tighter distributions of particles => more likelihood of interactions
  - Large momentum changes can move electrons out of the stable RF bucket => particle loss

## Rough Order of Magnitude



$$\hat{x} = \sqrt{\mathcal{W}\beta(s)}$$

$$\hat{x}' = \sqrt{\mathcal{W}/\beta(s)}$$

$$\sigma_{\text{RMS}} = \sqrt{\epsilon\beta}$$

$$\sigma'_{\text{RMS}} = \sqrt{\epsilon/\beta}$$

$$\epsilon = \pi\mathcal{W}$$

- For a given particle,  $\hat{x} = \beta\hat{x}' = \frac{\beta\hat{p}_x}{p_0}$      $\hat{p}_x = \frac{p_0\hat{x}}{\beta}$
- If **all** transverse momentum is transferred into  $\delta$  then

$$\Delta p = \gamma p_x = \gamma \frac{p_0\hat{x}}{\beta}$$

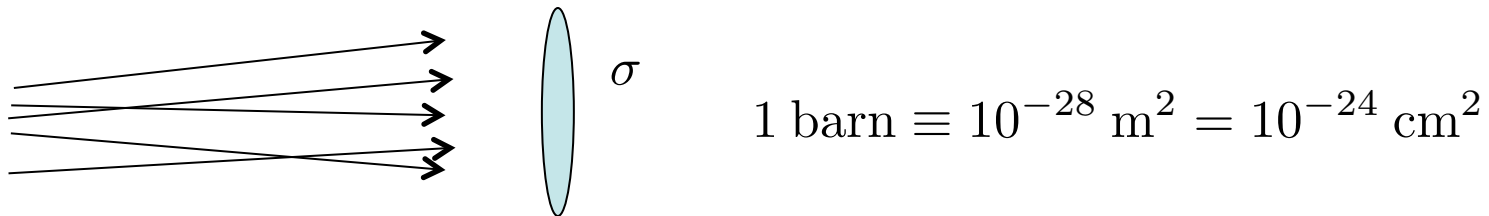
- For realistic numbers of 2 GeV beam ( $\gamma \sim 4000$ ),  $\beta_x = 10\text{m}$ , and  $100\mu\text{m}$  beam displacement, we find

$$\Delta p \approx 80 \text{ MeV}/c \approx 0.04 p_0$$

- This scattering mechanism can create electron loss
  - Even worse for particles out in Gaussian tails

# Cross Section

- **Cross section** is used in high energy physics to express the probability of scattering processes: units of area



- Often expressed as a **differential cross section**, probably of interaction in a given set of conditions (like interaction angle or momentum transfer):  $d\sigma/d\Omega$
- In particle colliders, **luminosity** is defined as the rate of observed interactions of a particular type divided by the cross section  $\mathcal{L} \equiv \frac{\text{event rate}}{\sigma}$  units  $[\text{s}^{-1} \text{ cm}^{-2}]$

Integrating this over time gives an expected number of events in a given time period to calculate experiment statistics

# Touschek Scattering Calculations

- Touschek Scattering calculations use the Moller electron elastic interaction cross section in the rest frame of the electrons
  - Then relativistically boost back into the lab frame
  - This is all too involved for this lecture!
    - Really 2<sup>nd</sup> year graduate level scattering theory calculation
  - See Carlo Bocchetta's talk at CERN Accelerator School
    - <http://cas.web.cern.ch/cas/BRUNNEN/Presentations/PDF/Bocchetta/Touschek.pdf>
  - As usual we'll just quote the result
  - Touschek loss exponential decay lifetime

$$\tau = \frac{\gamma^3 V_{\text{bunch}} \sigma'_{x,\text{RMS}} \delta_{\text{acceptance}}^2}{c r_0^2 N_{\text{bunch}} (\ln(2) \sqrt{\pi})} \frac{1}{C(\epsilon)}$$

$$\delta_{\text{acceptance}}: \frac{\Delta p}{p_0} \text{ at which particles are lost}$$

$$V_{\text{bunch}} = 8\pi \sigma_x \sigma_y \sigma_z$$

$$C(\epsilon) \approx -[\ln(1.732\epsilon) + 1.5]$$

$$\epsilon \equiv \left( \frac{\delta_{\text{acceptance}}}{\gamma \sigma'_{x,\text{RMS}}} \right)^2$$

$$r_0 \approx 2.818 \times 10^{-13} \text{ cm}$$



# Touschek Scaling

$$\tau = \frac{\gamma^3 V_{\text{bunch}} \sigma'_{x,\text{RMS}} \delta_{\text{acceptance}}^2}{c r_0^2 N_{\text{bunch}} (\ln(2) \sqrt{\pi})} \frac{1}{C(\epsilon)}$$

$\delta_{\text{acceptance}}$ :  $\frac{\Delta p}{p_0}$  at which particles are lost

$$V_{\text{bunch}} = 8\pi \sigma_x \sigma_y \sigma_z$$

$$C(\epsilon) \approx -[\ln(1.732\epsilon) + 1.5]$$

$$\epsilon \equiv \left( \frac{\delta_{\text{acceptance}}}{\gamma \sigma'_{x,\text{RMS}}} \right)^2$$

$$r_0 \approx 2.818 \times 10^{-13} \text{ cm}$$

- High lifetime is good, low lifetime is bad
  - Higher particle phase space density  $N_{\text{bunch}}/V_{\text{bunch}}$  makes loss faster
    - But we want this for higher brilliance!
  - Smaller momentum acceptance makes loss faster
    - But tighter focusing requires sextupoles to correct chromaticity
    - Sextupoles and other nonlinearities reduce  $\delta_{\text{acceptance}}$
  - Higher beam energy  $\gamma_r$  makes loss slower
    - Well at least we win somewhere!

# Touschek Lifetime Calculations

- Generally one must do some simulation of Touschek losses

## Touschek Lifetime Calculations for NSLS-II

B. Nash, S. Kramer, Brookhaven National Laboratory, Upton, NY 11973, USA

### Abstract

The Touschek effect limits the lifetime for NSLS-II. The basic mechanism is Coulomb scattering resulting in a longitudinal momentum outside the momentum aperture. The momentum aperture results from a combination of the initial betatron oscillations after the scatter and the non-linear properties determining the resultant stability. We find that higher order multipole errors may reduce the momentum aperture, particularly for scattered particles with energy loss. The resultant drop in Touschek lifetime is minimized, however, due to less scattering in the dispersive regions. We describe these mechanisms, and present calculations for NSLS-II using a realistic lattice model including damping wigglers and engineering tolerances.<sup>1</sup>

### LINEAR AND NON-LINEAR DYNAMICS MODELING

NSLS-II has a 15-fold periodic DBA lattice. The lattice functions for NSLS-II are shown in Figure 1. The linear lattice results in the equilibrium beam sizes around the ring that enter into Eqn. (1). Non-linear dynamics enter through the parameter  $\delta_{acc}(s)$ . This is the maximum momentum change that a scattered particle can endure before it is lost. There are two elements to this stability question. The first is the amplitude of the initial orbit which comes from the off-momentum closed orbit (dispersion) and beta functions. These are shown in Figures 2 and 3. The amplitude of the induced betatron oscillation following a scatter with relative energy change  $\delta = \frac{\Delta E}{E_0}$  is given by

$$x_2 = (\eta^{(1)}(s_2) + \sqrt{\mathcal{H}(s_1)\beta_x(s_2)})\delta + \eta^{(2)}(s_2)\delta^2 \quad (3)$$

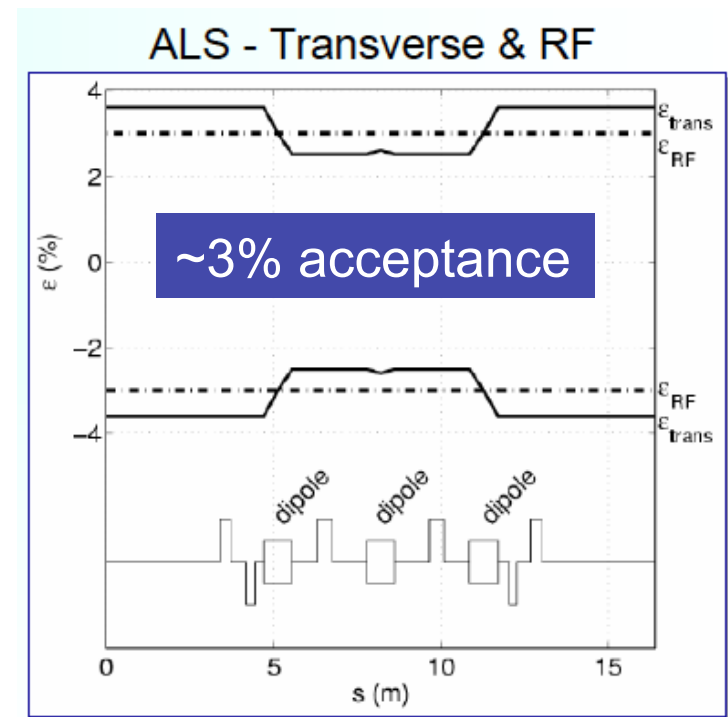
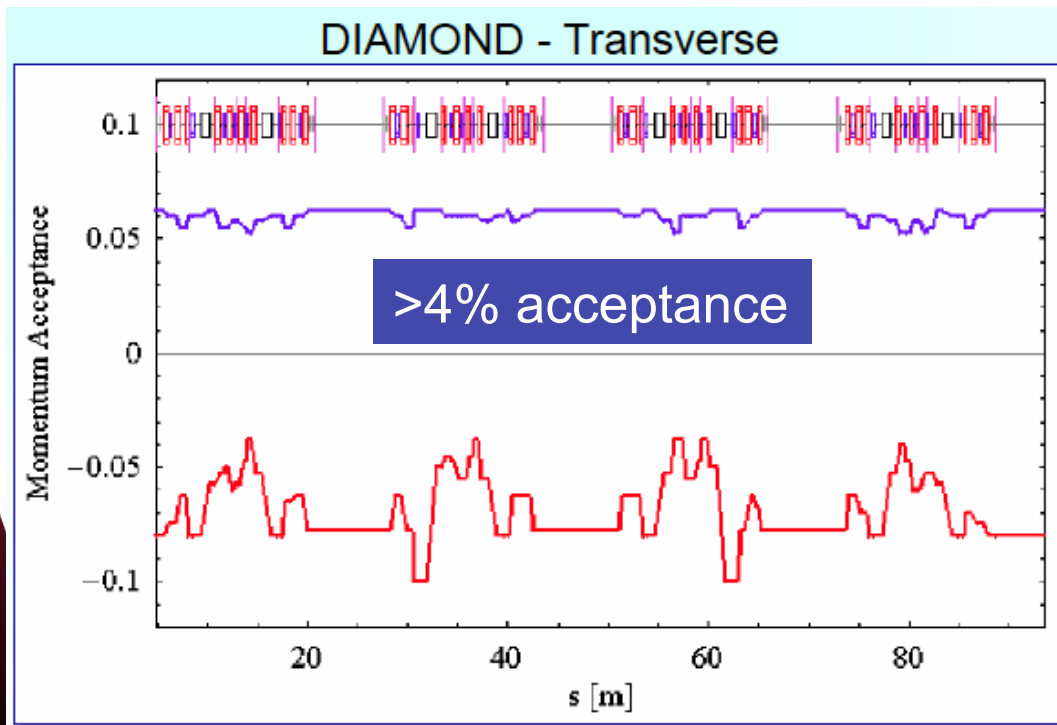
where  $\mathcal{H} = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta_x'^2$  is the dispersion in-

### INTRODUCTION

PAC' 09 Conference: <http://www.bnl.gov/isd/documents/70446.pdf>

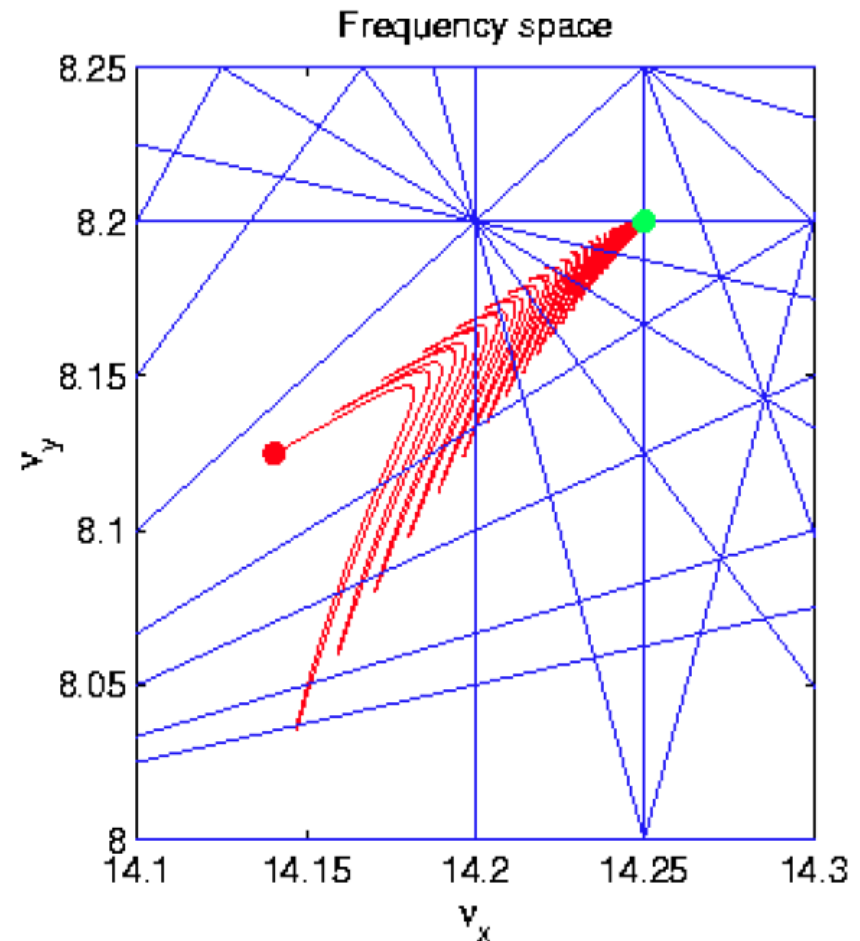
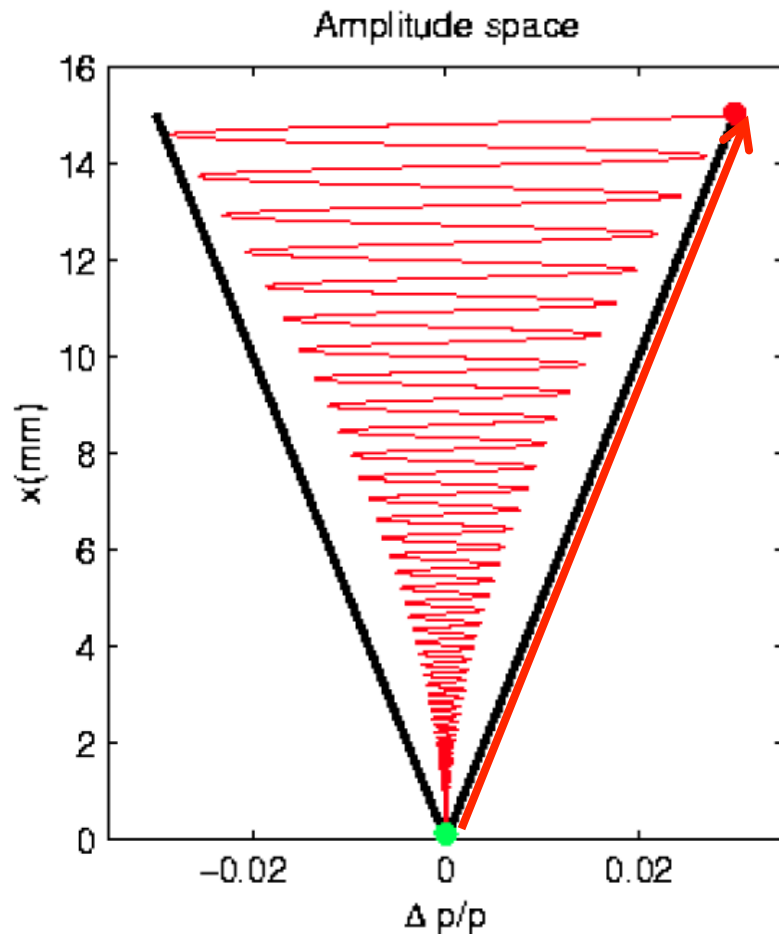
# Momentum Aperture and Touschek

- Most third generation storage rings have limiting transverse acceptance
  - Much work to optimize transverse momentum aperture
  - Particularly modern machines (e.g. DIAMOND, SOLEIL)
  - Detailed nonlinear dynamics measurements required



Carlo Bocchetta's talk at CERN Accelerator School

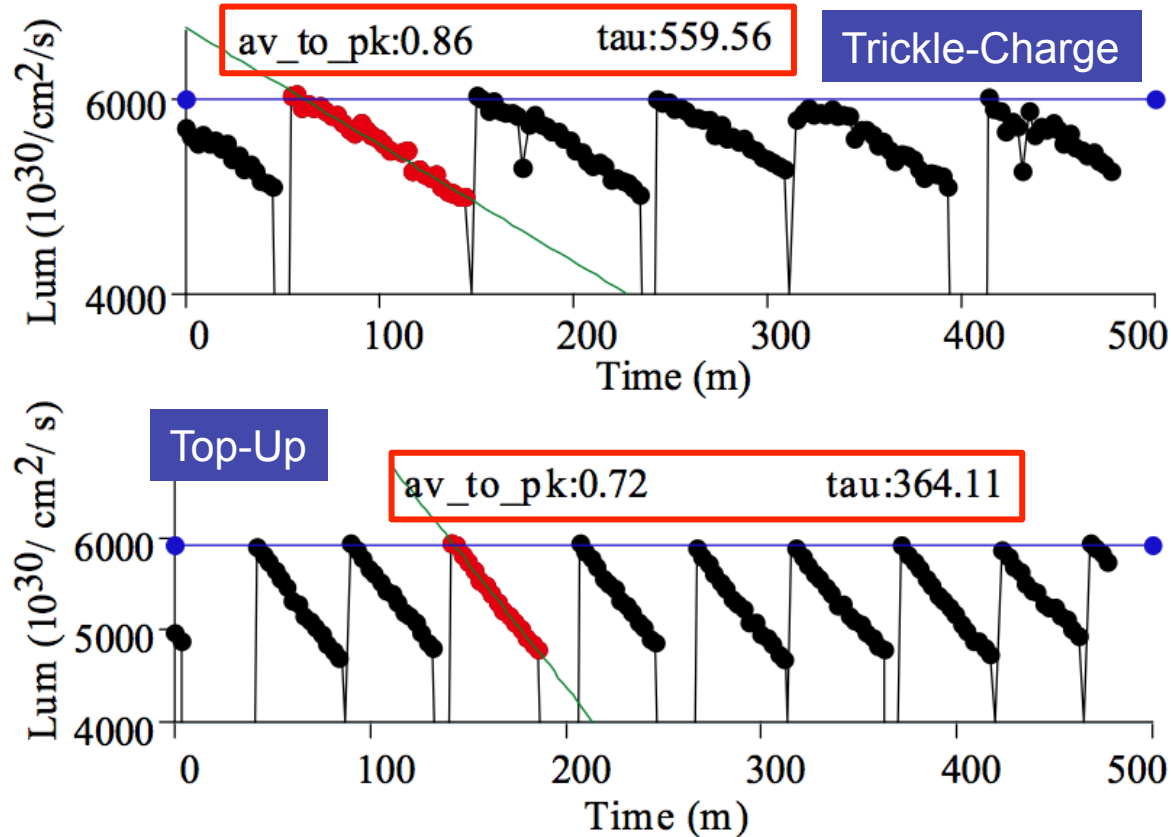
# Kicked Electron Damping



- After a Touschek kick, electrons damp again
  - But they move through tunes and amplitudes in complicated way
  - Will see more of “tune space” and resonances tomorrow

D. Robin, ALS

# Top-Up and Trickle-Charge



- Top-up: add beam at discrete times to “top-up” beam current
  - Turn off detectors during top-up, dominated by beam lifetime
- Trickle-charge: add small trickle of beam continuously
  - Dominated by injection jitter detector trips, other injector stability

J. L. Turner et al, “Trickle-Charge: A New Operational Mode for PEP-II”, SLAC-PUB-11175



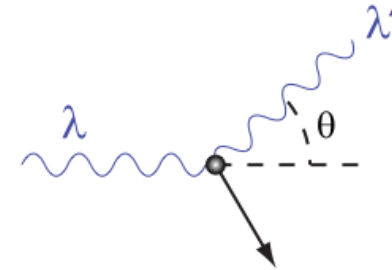
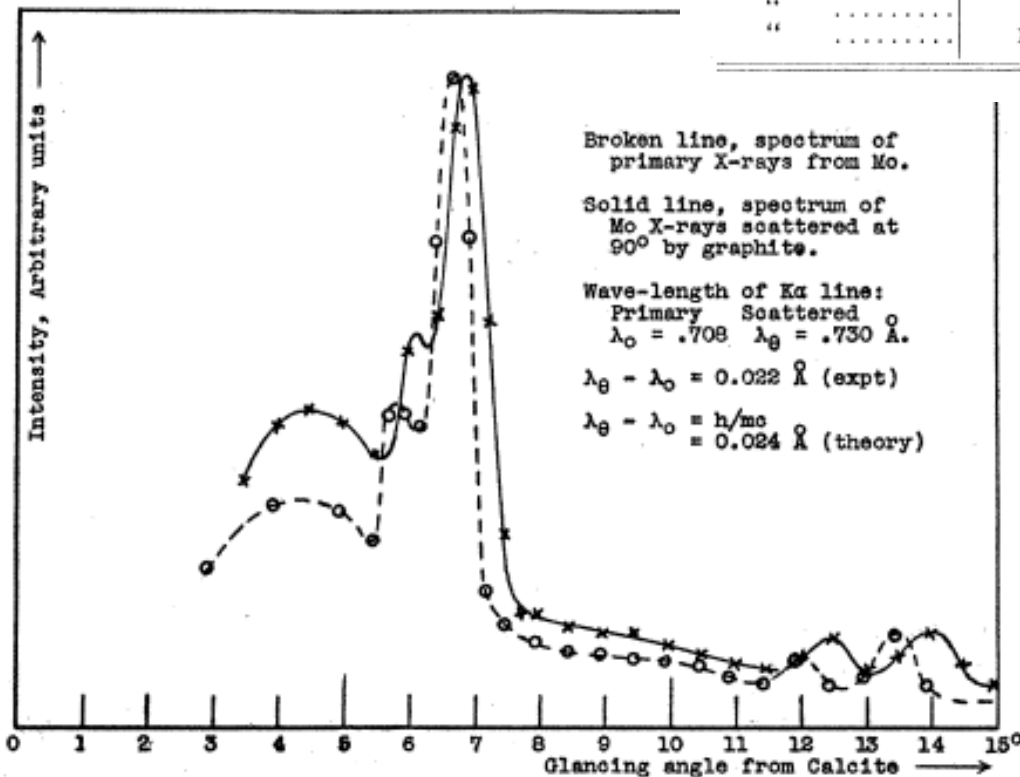
# Compton Effect and Inverse Compton Sources

(Slides from G. Krafft)

TABLE I

Wave-length of Primary and Scattered  $\gamma$ -rays

	Angle	$\mu/\rho$	$\tau/\rho$	$\lambda$ obs.	$\lambda$ calc.
Primary .....	$0^\circ$	.076	.017	0.022 Å	(0.022 Å)
Scattered .....	$45^\circ$	.10	.042	.030	0.029
" .....	$90^\circ$	.21	.123	.043	0.047
" .....	$135^\circ$	.59	.502	.068	0.063

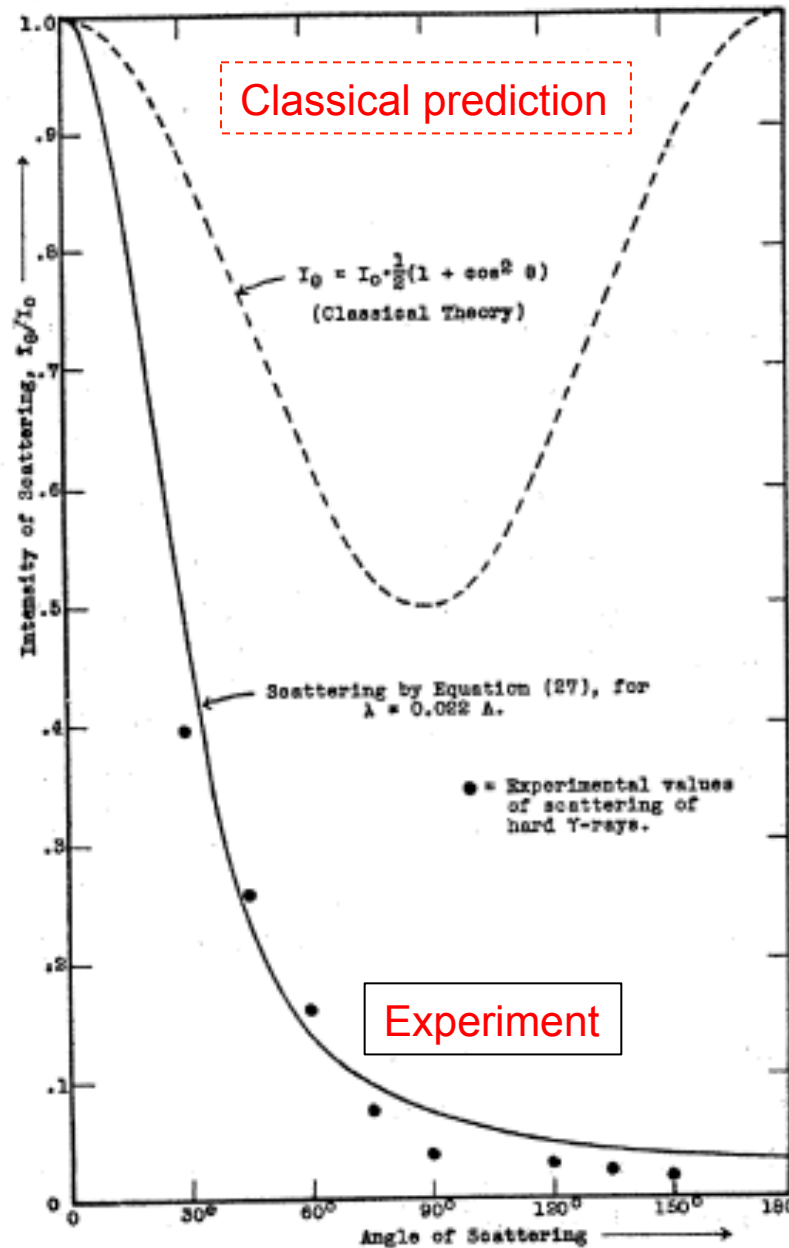


A. H. Compton, Phys. Rev., 21, 483 (1923)

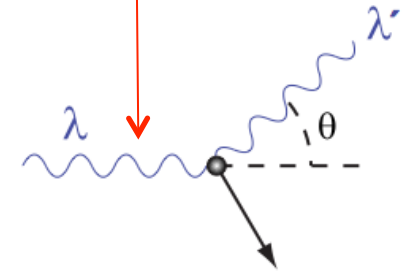
Proved light had particle-like properties when observed with low intensity light

Nobel prize in Physics, 1927!!

Fig. 4. Spectrum of molybdenum X-rays scattered by graphite, compared with the spectrum of the primary X-rays, showing an increase in wave-length on scattering.



Unpolarized Incident X-ray Beam



$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

Planar scattering  
Quantum effect!



# Undulators/Wigglers vs Compton

- Undulators and wigglers get small wavelength light from high-energy (expensive, multi-GeV) electrons

$$\lambda = \frac{\lambda_{\text{undulator}}}{2\gamma^2} \left( 1 + \frac{\kappa^2}{2} \right) \quad \kappa = \frac{eB\lambda_{\text{undulator}}}{2\pi m_e c} \quad \text{Deflection parameter}$$

- Synchrotron light sources:

$$\gamma \approx \text{thousands} \quad \kappa \approx \sqrt{2} \text{ (undulators), } \approx \text{tens (wigglers)}$$

- Compton sources use a high-powered laser to generate EM fields instead of wigglers or undulators

- Scattered photons from laser are relativistically upshifted into X-ray

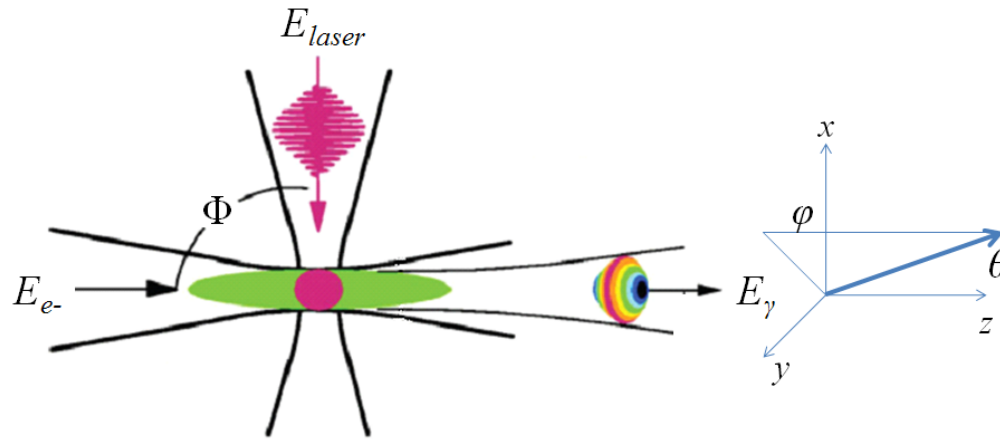
$$\lambda = \frac{\lambda_{\text{laser}}}{4\gamma^2} \left( 1 + \frac{\kappa^2}{2} \right)$$

$$\lambda_{\text{laser}} \approx 10^{-4} \lambda_{\text{undulator}} \quad \Rightarrow \quad \text{lower } \gamma \text{ by } \approx 10^2$$

Big deal!! Only low energy electrons (10s of MeV) needed!

# Energy

- Layout



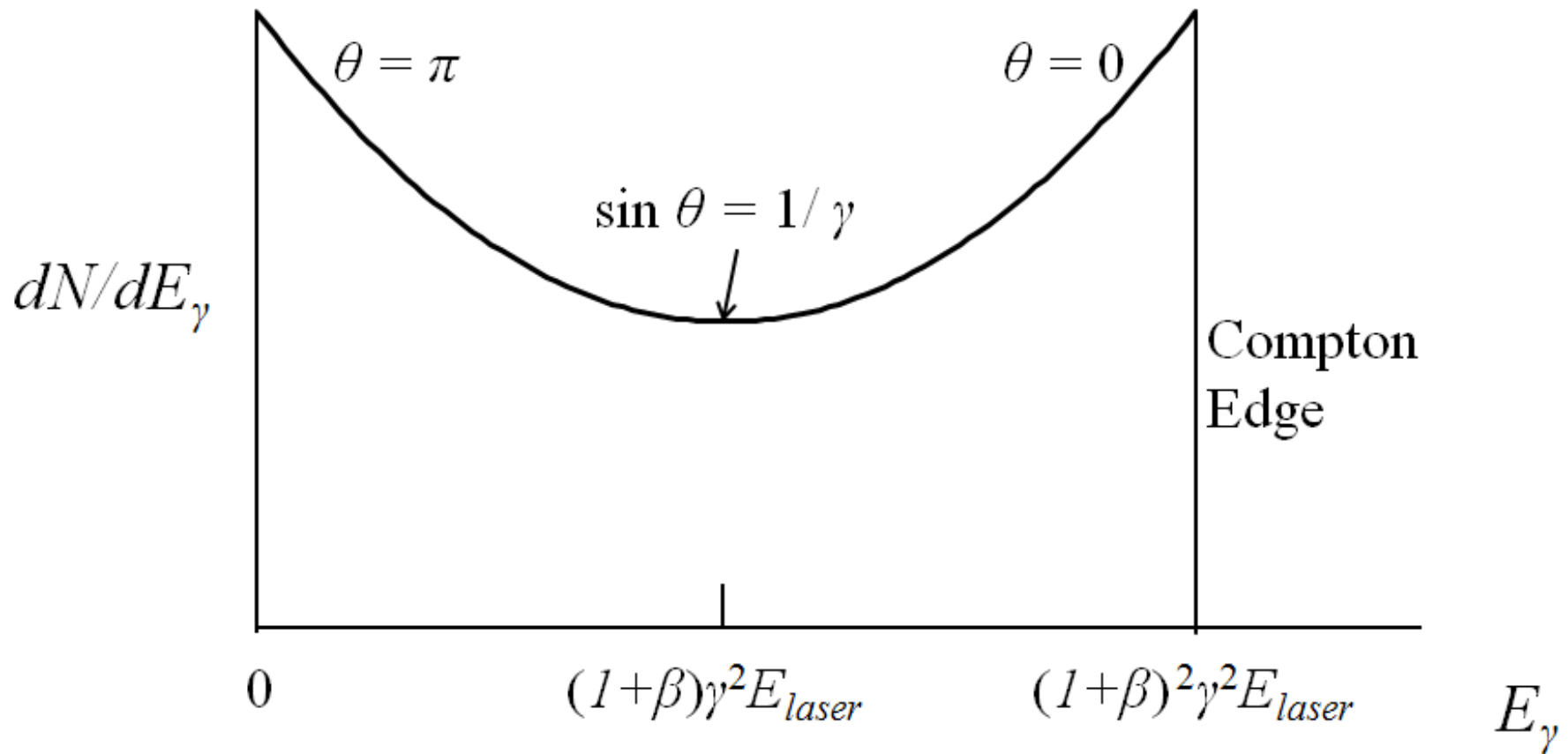
- Energy

$$E_\gamma(\theta, \phi) = \frac{E_{laser} (1 - \beta \cos \Phi)}{1 - \beta \cos \theta + E_{laser} (1 - \cos \Delta\Theta) / E_{e^-}}$$

- Thomson limit

$$E'_{laser} \ll mc^2, \quad E_\gamma(\theta, \phi) \approx E_{laser} \frac{1 - \beta \cos \Phi}{1 - \beta \cos \theta}$$

# Number Distribution of Photons



# Flux

- Percentage in 0.1% bandwidth ( $\theta = 0$ )

$$N_{0.1\%} = 1.5 \times 10^{-3} N_{\gamma}$$

- Flux into 0.1% bandwidth

$$\mathcal{F} = 1.5 \times 10^{-3} \dot{N}_{\gamma}$$

- Flux for high rep rate source

$$\mathcal{F} = 1.5 \times 10^{-3} f N_{\gamma}$$

# Energy Spread

## Sources of Energy Spread in the Scattered Pulse

Source Term	Estimate	Comment
Beam energy spread	$2\sigma_{E_{e^-}} / E_{e^-}$	From FEL resonance
Laser pulse width	$\sigma_{\omega} / \omega$	Doppler Freq Independent
Finite $\theta$ acceptance (full width)	$\gamma^2 \Delta\theta^2$	$\theta = 0$ for experiments
Finite beam emittance	$2\gamma^2 \varepsilon / \beta_{e^-}$	Beta-function

# Spectral Brilliance

- In general

$$\mathcal{B} = \frac{\mathcal{F}}{4\pi^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}}$$

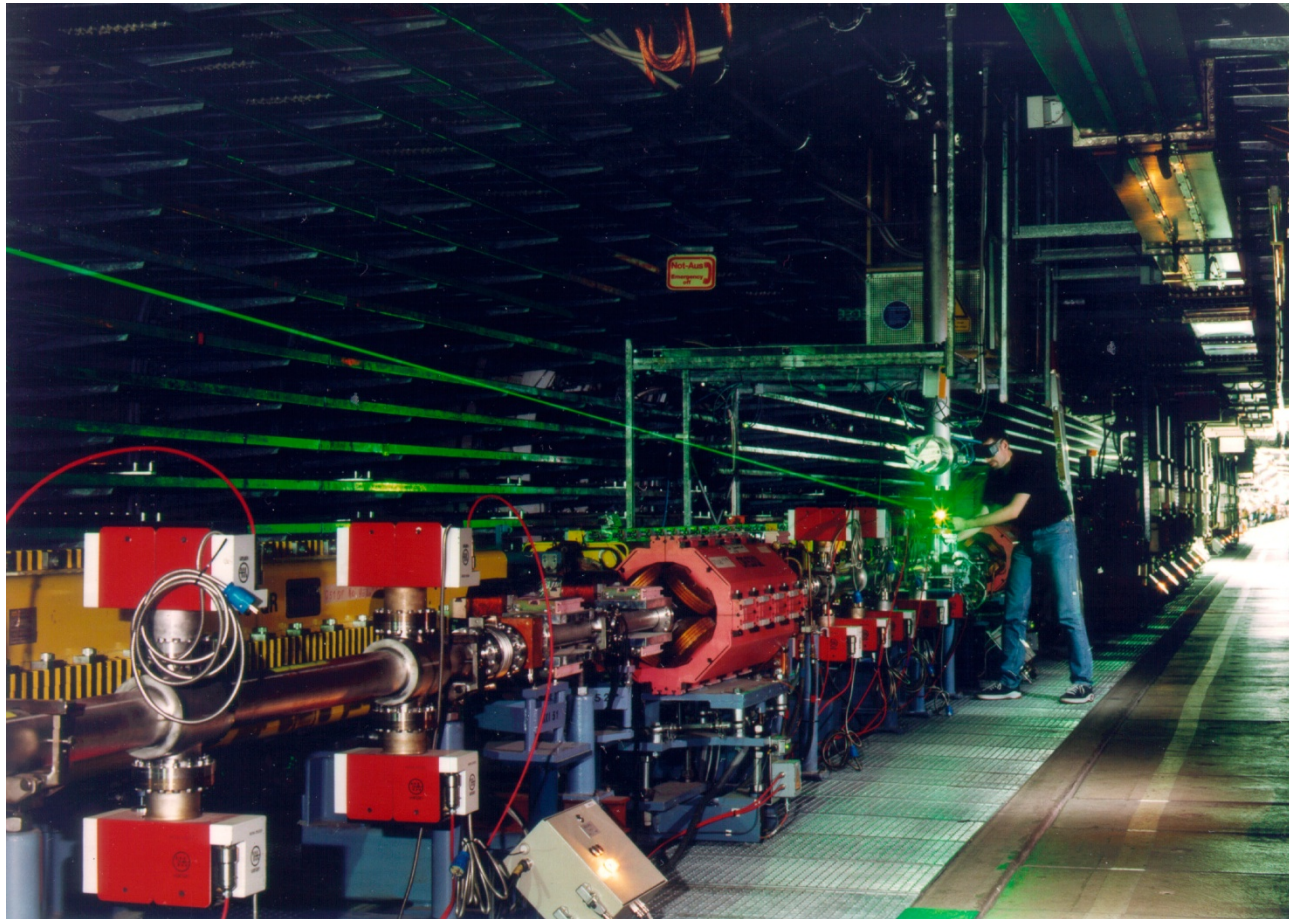
$$\approx \frac{\mathcal{F}}{4\pi^2 \sqrt{\beta_x \varepsilon_x} \sqrt{\varepsilon_x / \beta_x + \lambda / 2L} \sqrt{\beta_y \varepsilon_y} \sqrt{\varepsilon_y / \beta_y + \lambda / 2L}}$$

- For Compton scattering from a low energy beam emittances dominate

$$\mathcal{B} = \frac{\mathcal{F}}{4\pi^2 \varepsilon_x \varepsilon_y}$$

# Compton Polarimetry

- At high photon energy (in beam frame), scattering rate couples to the polarization variables



# High Field Thomson Backscatter

For a flat incident laser pulse the main results are very similar to those from undulators with the following correspondences

	Undulator	Thomson Backscatter
Field Strength	$K$	$a$
Forward Frequency	$\lambda \approx \frac{\lambda_0}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$	$\lambda \approx \frac{\lambda_0}{4\gamma^2} \left( 1 + \frac{a^2}{2} \right)$
Transverse Pattern	$\beta^*_z + \cos \theta'$	$1 + \cos \theta'$

NB, be careful with the radiation pattern, it is the same at small angles, but quite a bit different at large angles



# Source Illumination Method

- Direct illumination by laser
  - Earliest method
  - Deployed on storage rings
- Optical cavities
  - Self-excited
  - Externally excited
  - Deployed on rings, linacs, and energy recovered linacs
- High power single pulses
  - Deployed on linacs

# Early Gamma Ray Sources

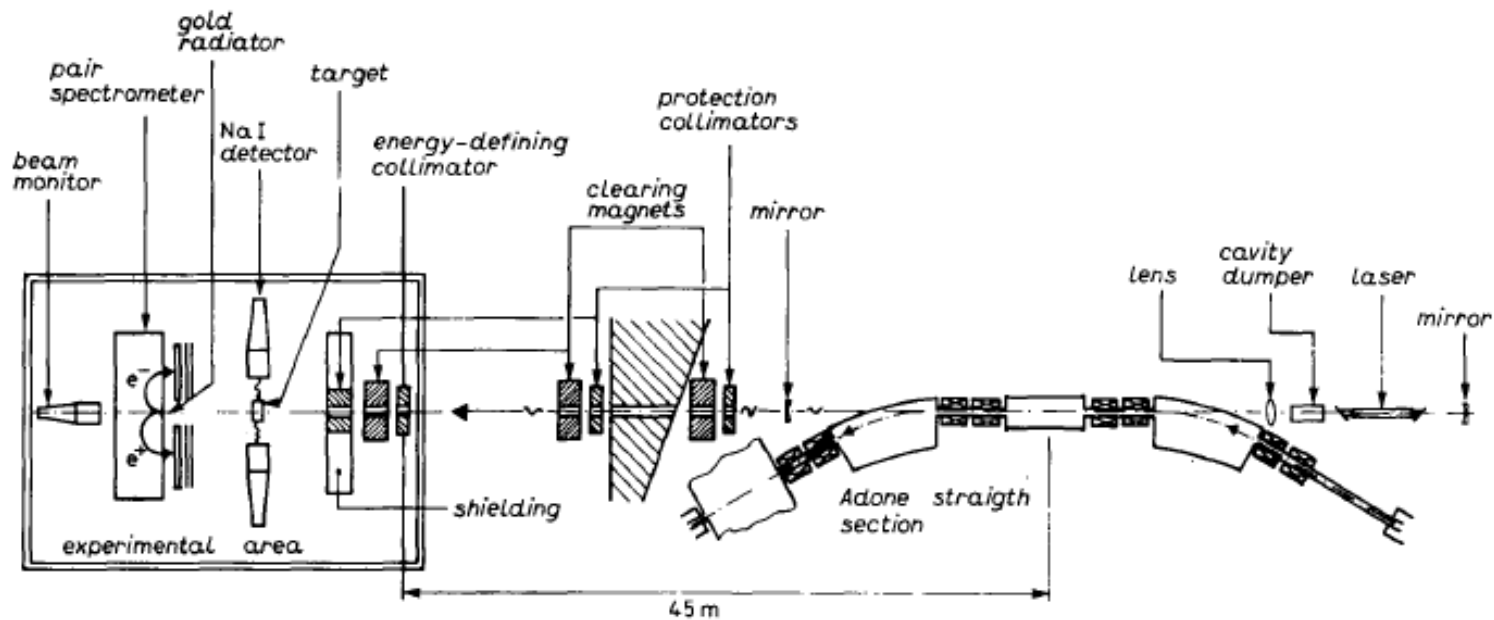
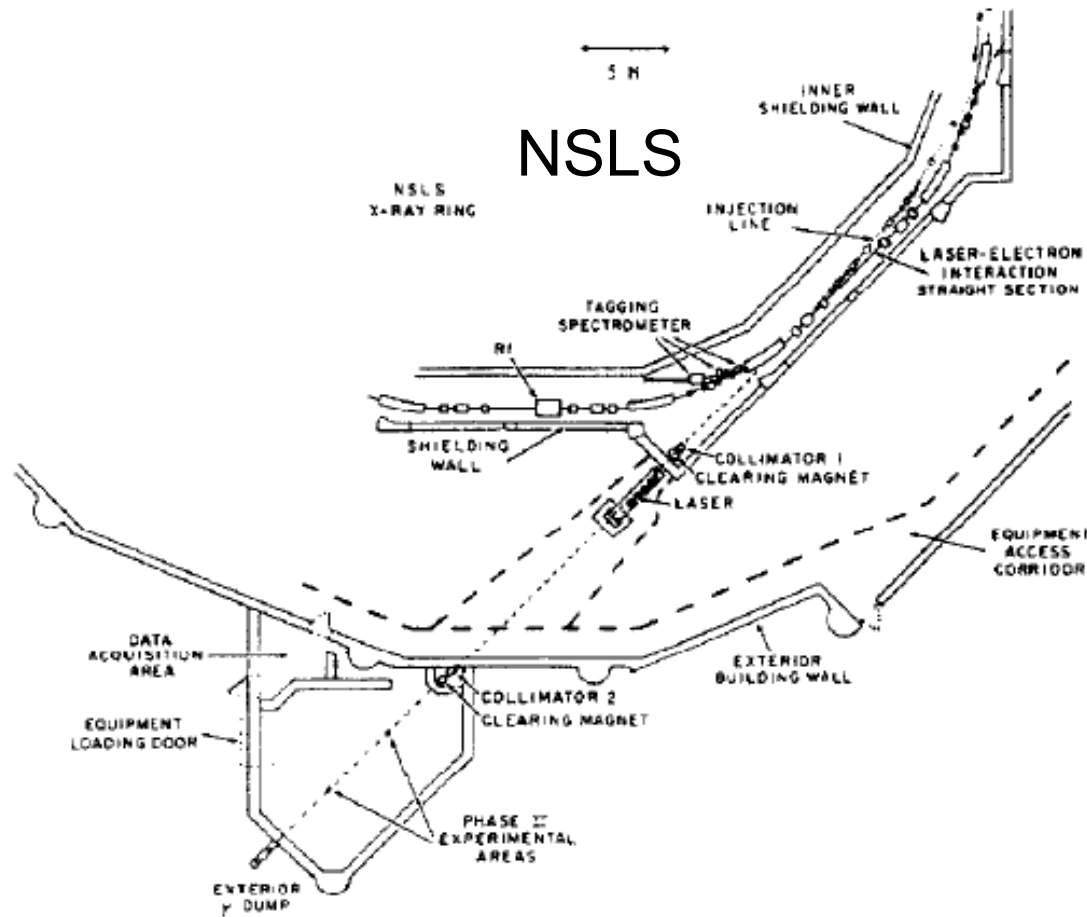


Fig. 1. – Overall view of the experimental set-up.

Compton Edge  
78 MeV

Federici, *et al.*  
Nouvo. Cim. B 59, 247 (1980)

# LEGS



Compton Edge  
270 MeV

Sandorfi, *et al.*  
PAC83, 3083 (1983)

Fig. 4 A plan of the LEGS facility at BNL.

# Electrotechnical Laboratory (Japan)

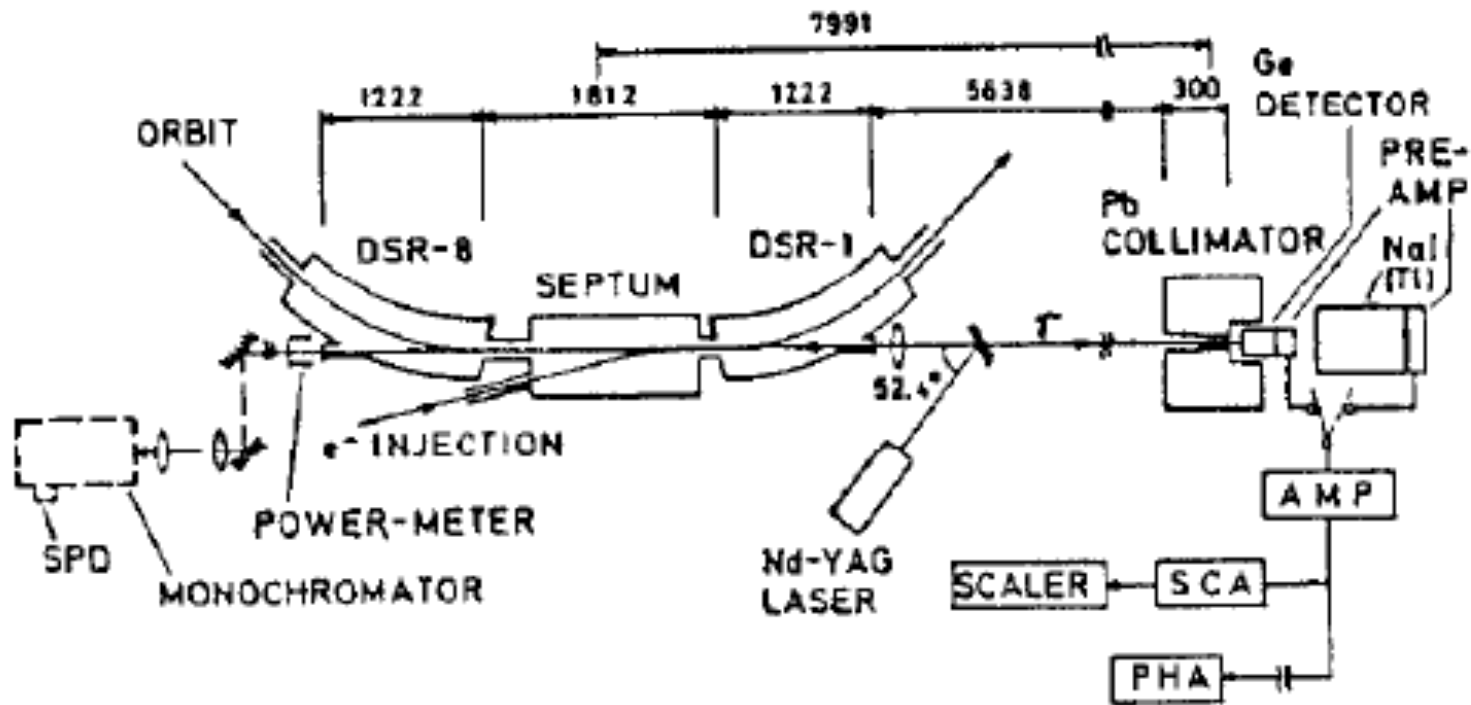
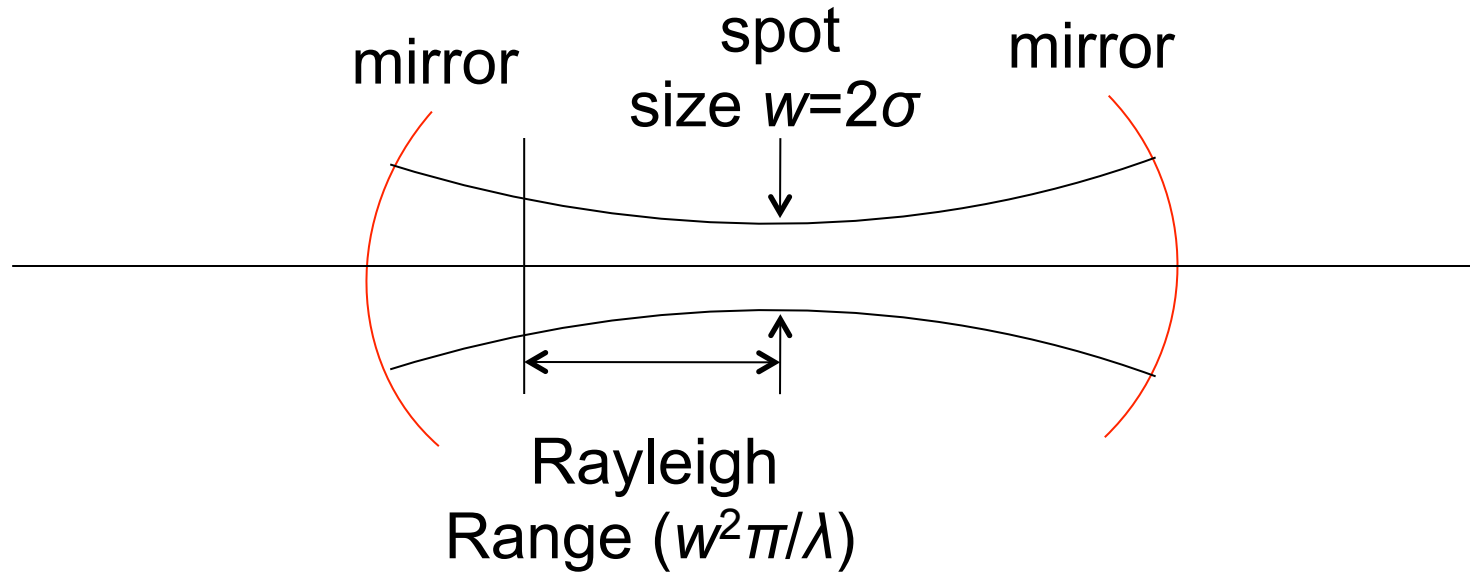


Fig. 2. Experimental arrangement.

Compton Edge 6.5 MeV

Yamazaki, *et al.*  
PAC85, 3406 (1985)

# Optical Cavities



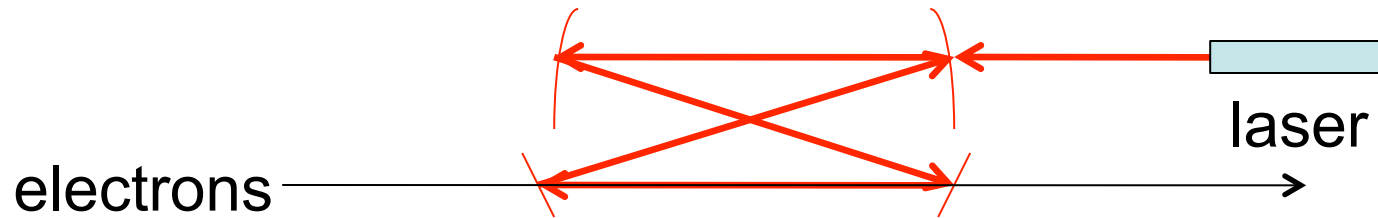
Quantity	Dimensions
Wavelength	200 nm-10 microns
Circulating Power	0.1-200 kW
Spot Size	50-500 microns
Rayleigh Range	40 cm-5 m

# Self Excited



Location	Wavelength	Circulating Power	Spot Size	Rayleigh Range
Orsay	5 microns	100 W	mm	0.7 m
UVSOR	466 nm	20 W	250 microns	0.4 m
Duke Univ.	545 nm	1.6 kW	930 microns	5 m
Super-ACO	300 nm	190 W	440 microns	2 m
Jefferson Lab FEL	1 micron	100 kW	150 microns	1 m

# Externally Excited



Location	Wavelength	Input Power	Circulating Power	Spot Size (rms)
Jefferson Lab Polarimeter	1064 nm	0.3 W	1.5 kW	120 microns
TERAS	1064 nm	0.5 W	7.5 W	900 microns
Lyncean	1064 nm	7 W	25 kW	60 microns
HERA Polarimeter	1064 nm	0.7 W	2 kW	200 microns
LAL	532 nm	1.0 W	10 kW	40 microns

# Modern Ring Based Systems

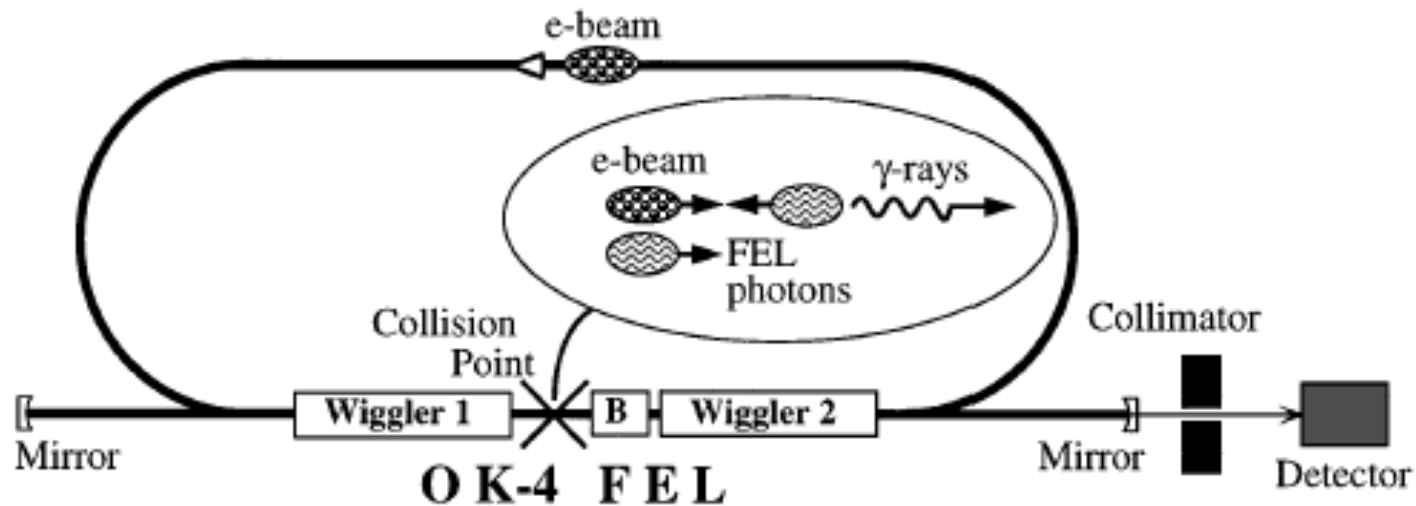
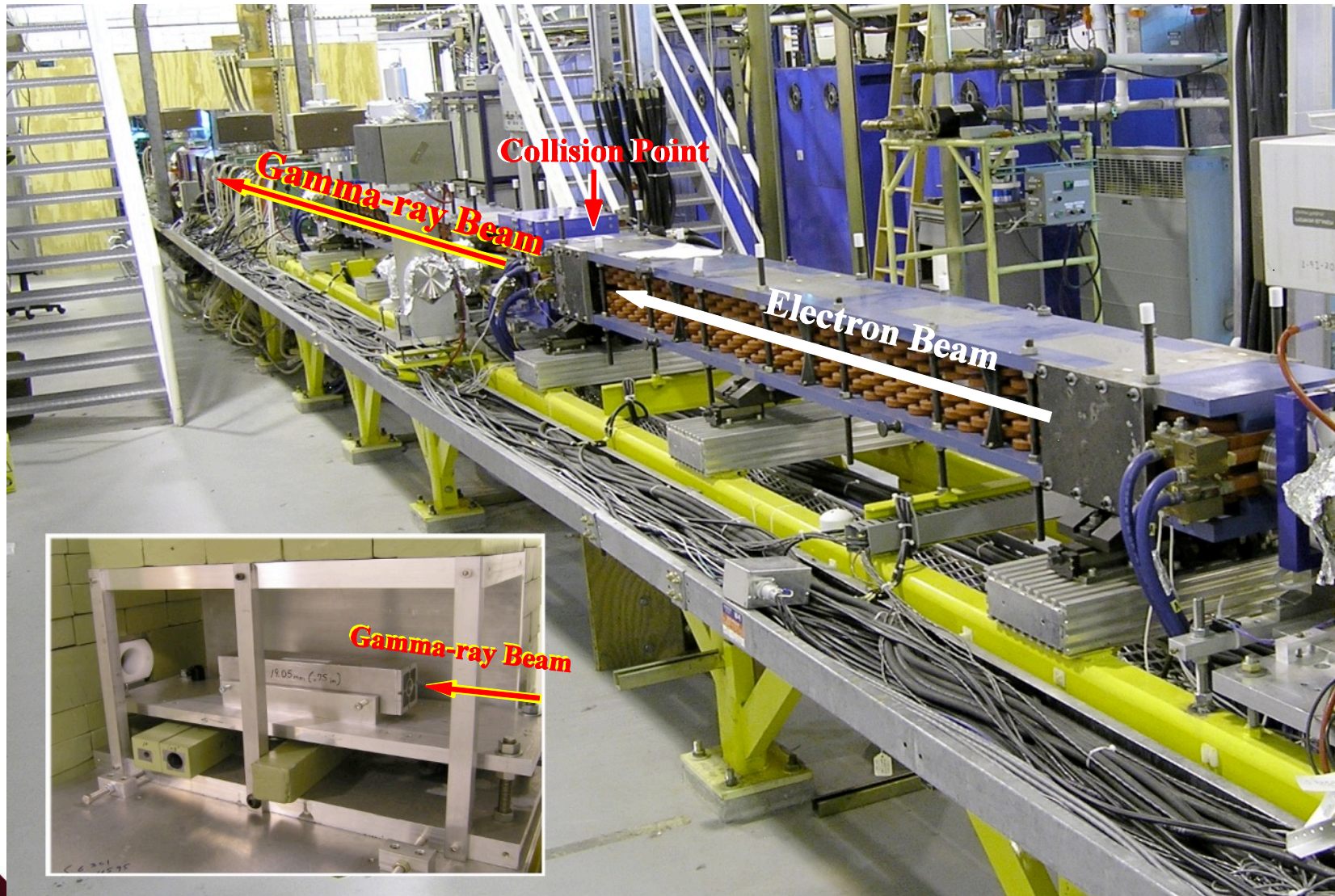


FIG. 1. Schematic of the OK-4/Duke storage ring FEL and  $\gamma$ -ray source. Two electron bunches spatially separated by one-half the circumference of the ring participate both in lasing and  $\gamma$ -ray production via Compton scattering of intracavity photons. A collimator installed downstream selects a narrow cone of quasimonoenergetic  $\gamma$  rays.

Litvinenko, *et al.*, *Phys. Rev. Lett.*, **78**, 4569 (1997)



# Duke HIGS Facility



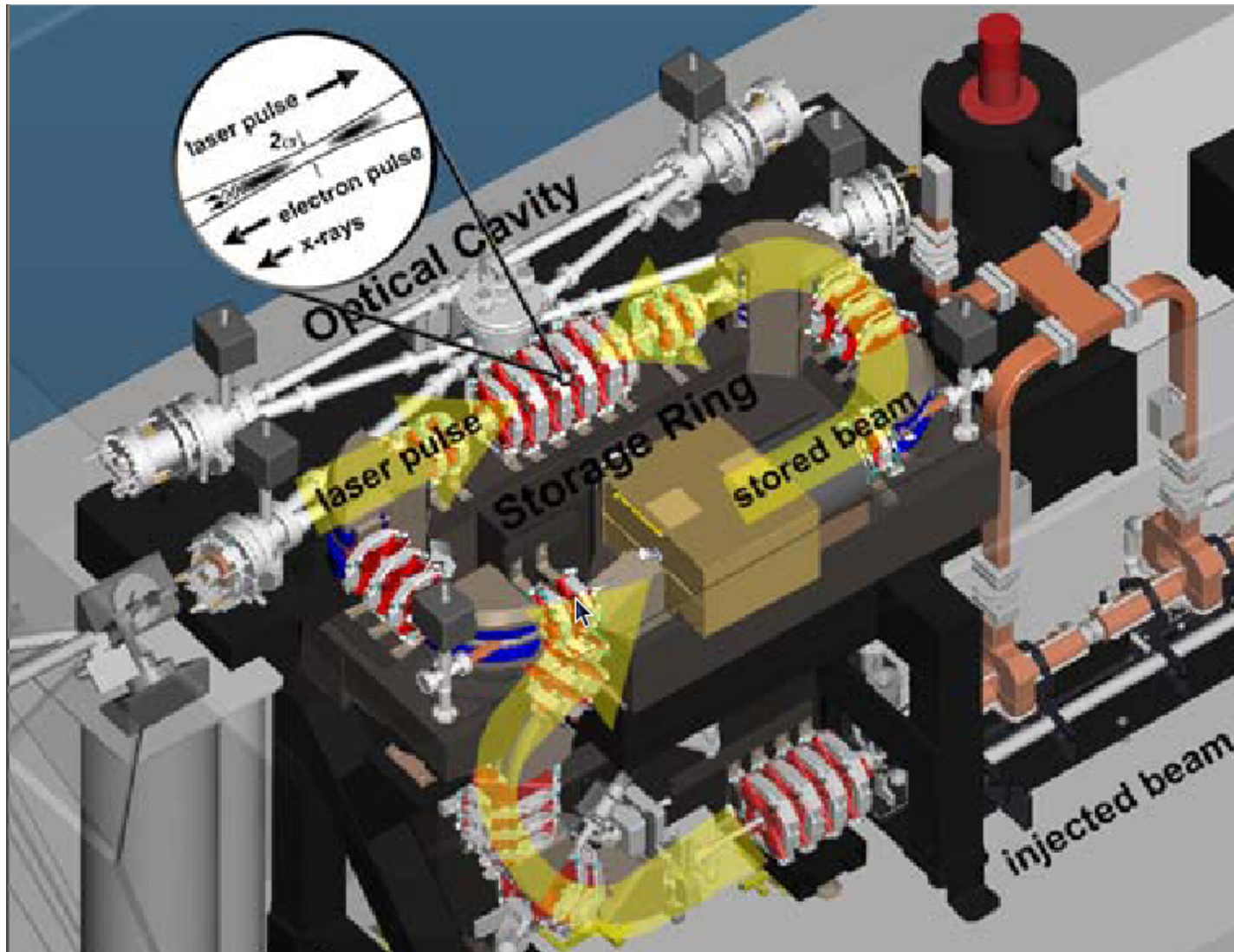
## Some Modern Parameters

Parameter	Value	Unit
Photon Energy	100	MeV
Production Rate	$10^{10}$	photons/sec@9 MeV
Laser Wavelength	545	nm
Circulating Power	1.6	kW
Polarization	100%	

Topoff allows larger circulating power now!

H. R. Weller, *et al.*, *Prog. Part. Nucl. Phys.*, **62**, 4569 (2009)

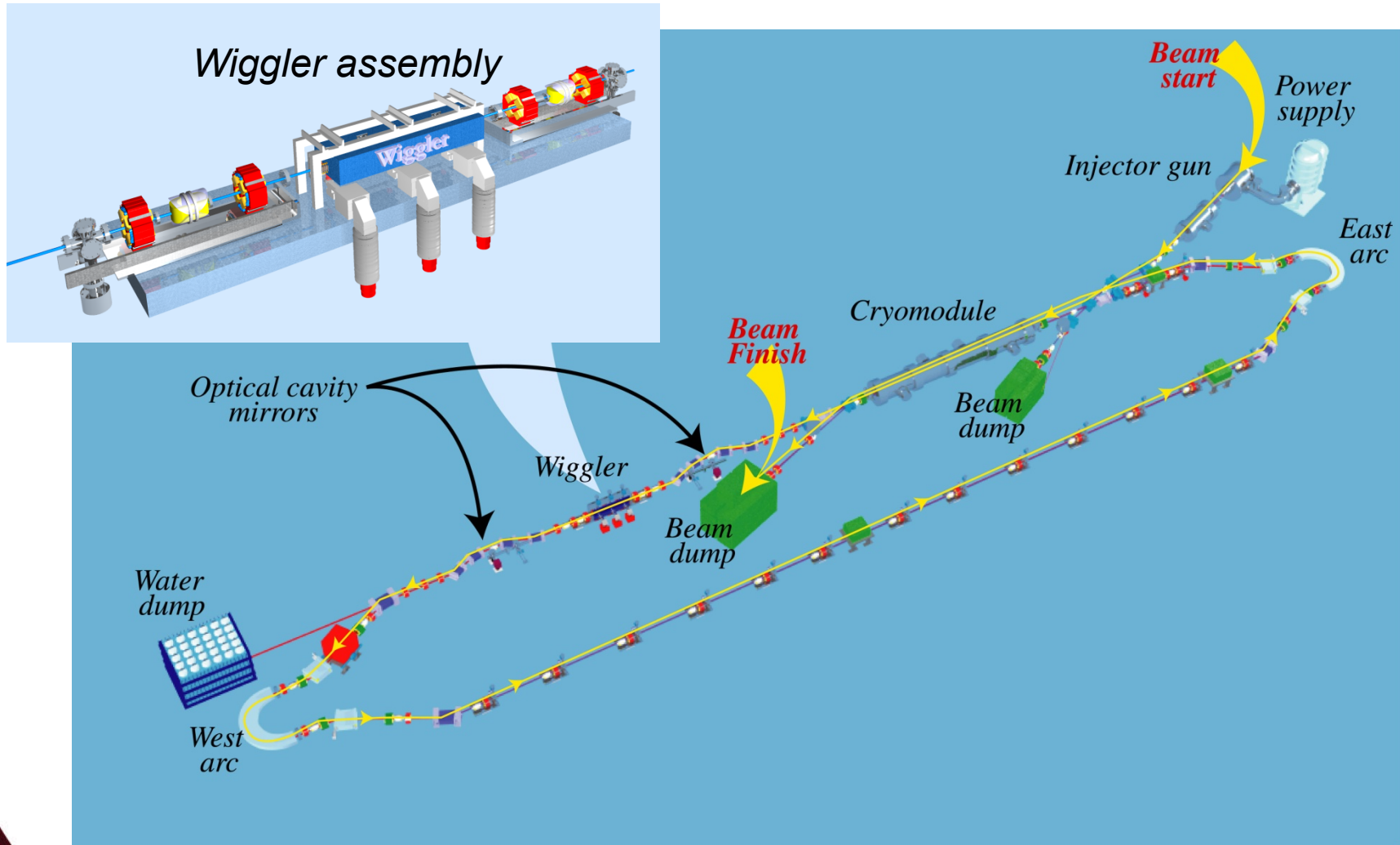
# Lyncean Compact X-ray Source



# Lyncean Source Performance

Parameter	Value	Unit
Photon Energy	10-20	keV
Production Rate	$10^{11}$	photons/sec
Laser Wavelength	1064	nm
Circulating Power	25	kW
Polarization	100%	
Ultimate Brilliance	$5 \times 10^{11}$	p/(sec mm <sup>2</sup> mrad <sup>2</sup> 0.1%)

# The Jefferson Lab IR FEL



Neil, G. R., et. al, *Physical Review Letters*, **84**, 622 (2000)