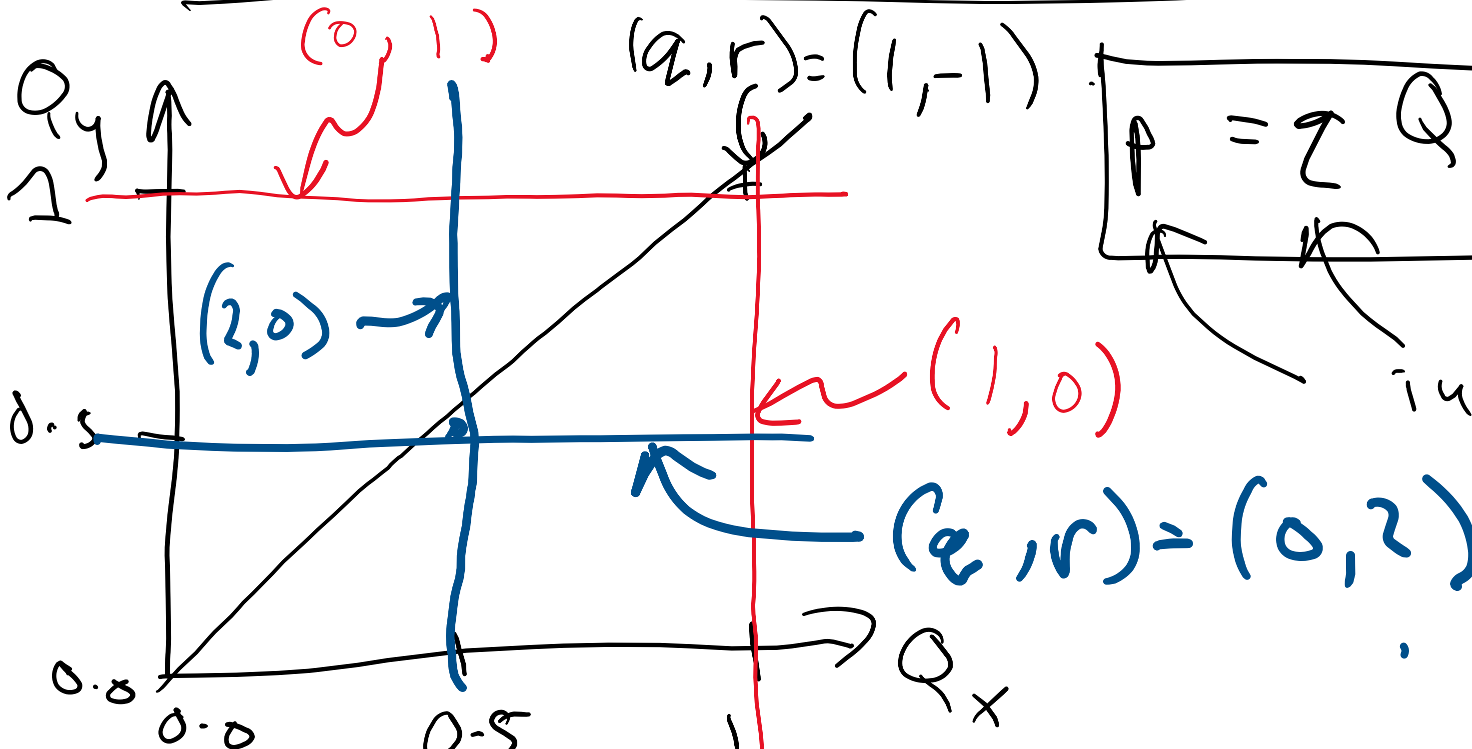
# #8: LINEAR ERRORS & THEIR CORRECTION 1/24/19

## OR: INTRODUCTION TO RESONANCES




$$\boxed{p = q\,Q_x + r\,Q_y}$$

integers

- $p$ can be very large .... but (usually) just ignore the integer parts of $Q_x$ & $Q_y$

# MAGNET DISPLACEMENTS → TRAJECTORIES + CLOSED ORBITS

Take a multipole magnet

$$B_y + iB_x = C_n (x+iy)^n$$

| | | |
|---|---|---|
| $C_1$ | quad | $x^1$ |
| $C_2$ | sext | $x^2$ |
| $C_n$ | ... | $x^n$ |

DISPLACE it : $x \to x + \Delta x$ , $y \to y + \Delta y$

$$\boxed{B_y + iB_x = C_n\left[(x+iy)^n \underbrace{- n(x+iy)^{n-1}(\Delta x + i\Delta y)}_{\text{FEED DOWN}} + \dots\right]}$$

QUADRUPOLE

$$\Delta x' = -\frac{x}{f} + \frac{\Delta x}{f}$$

CONSTANT TERM: DIPOLE !!

E.G. $f = 10$ m
$\Delta x = 1$ mm

$$\boxed{\Delta x / f = 100\ \mu\text{rad}}$$

# DIPOLE ROLL

- Dipole displacements do nothing!
- But a roll of a dipole with bend angle $\theta$, by $\alpha$ about a longitudinal axis

$$\Delta y' = \alpha\theta \quad [\text{CONSTANT}]$$

E.G. $\theta = 40$ mrad, $\alpha = 1$ mrad.

$$\boxed{\Delta y' = 40\ \mu\text{rad}}$$

So a SINGLE misaligned quad or dipole generates angular kicks $\sim 100\ \mu$rad, V + H

Q: How bad is this?

Q: Free wave or closed orbit?

Q: How to correct it?

FREE-WAVE

In a transfer line (or linac)

$$\boxed{\frac{x}{\sqrt{\beta(s)}} = \Delta x' \sqrt{\beta_0} \sin\left(\psi(s) - \psi_0\right)} \qquad s > s_0$$

EG $\Delta x' = 10^{-4}$ rad, $\beta_0 = \beta = 50$ m

$\langle x^2 \rangle^{\frac{1}{2}} \simeq 5$ mm

Can't take many of these RMS errors before hitting the beam pipe!

Q: WHAT happens in a storage?

WHAT do PBC do?

# CLOSED ORBIT ERRORS

Apply PBC with 1-turn matrix $M$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{co} + \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = (I - M)^{-1} \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix}}$$

$I$ Identity matrix

– Watch out if $\det(I-M) \to 0$ !!

$$\boxed{(I-M)^{-1} = \frac{1}{2(1-C)} \begin{pmatrix} 1-C & \beta S \\ -\frac{S}{\beta} & 1-C \end{pmatrix}}$$

$C = \cos(2\pi Q_x)$

$S = \sin(2\pi Q_x)$

BEWARE

watch out for $C \triangleq 1$ $(Q_x = 0, 1, 2 \ldots)$ !!

AT KICK LOCATION

$$\boxed{\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = \frac{\Delta x'}{2\sin(2\pi Q_x)} \begin{pmatrix} \beta_0 \cos(\pi Q_x) \\ \sin(\pi Q_x) - \alpha_0 \cos(\pi Q_x) \end{pmatrix}}$$

for any $\alpha_0$, includes angular perturbation

$$\boxed{\frac{x_{co}}{\sqrt{\beta}} = \frac{\Delta x' \sqrt{\beta_0}}{2\sin(\pi Q_x)} \cdot \cos\left( |\psi - \psi_0| - \pi Q_x \right)}$$

AT ANY LOCATION

**RESONANCE EXAMPLE 1** (LINEAR)

Tunes with $p \approx Q_x$ are dangerous.

Beware $(q,r) = (1,0)$ and $(0,1)$ !!

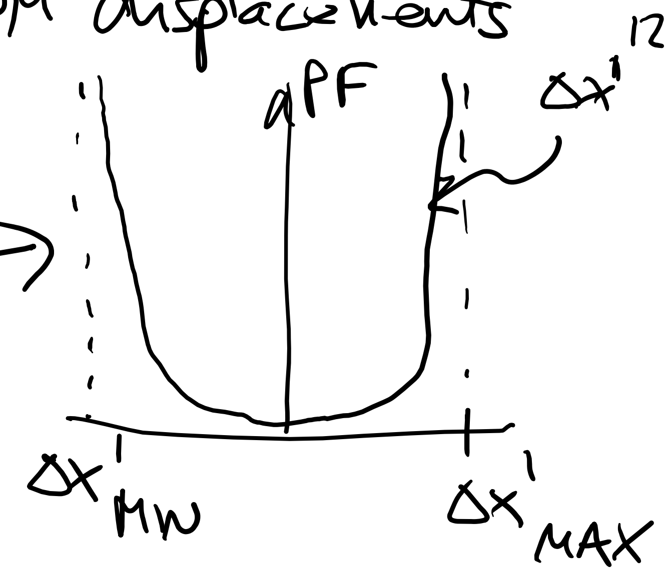
# CLOSED ORBIT CORRECTION

Measure BPM's
Adjust dipole corrector strengths

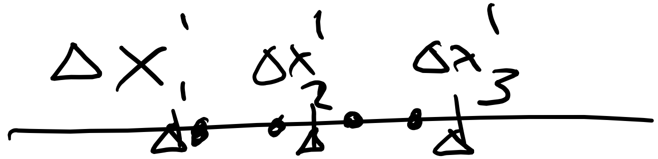
Q: What is PENALTY FUNCTION?

eg. SUM of squares of BPM displacements

Q: What about DIPOLE CORRECTOR LIMITS




There are many ways, but "SLIDING 3-BUMP" method allows ANY P.F.




Downstream of 3 dipole correctors — FREE WAVE

$$\boxed{\frac{x(\psi)}{\sqrt{\beta}} = \sum_{i=1}^{3} \Delta x'_i \sqrt{\beta_i} \cdot \sin(\psi - \psi_i)}$$

IF this sum is ZERO, no need to apply PBC!!

$\Rightarrow$ closed orbit corrections are localised $\psi_1 < \psi < \psi_3$

$$\boxed{\frac{\Delta x'_i \sqrt{\beta_i}}{\sin(\psi_j - \psi_k)} = \text{constant } K}$$

for $(i,j,k)$ acyclic combination of $(1,2,3)$

SLIDING 3-BUMP: Adjust bump strength $K$ to minimise local PF for

$\underbrace{(1,2,3)}_{2}, \underbrace{(2,3,4)}_{3}, \ldots\ldots (N,1,2),(123)\ldots\ldots$

REPEAT until the GLOBAL PF is minimised.

$p = 2, 3$

Repetition is often required in REAL WORLD, as well as in the calculation/algorithm.

# LINEAR COUPLING – ROLLED QUADS

Roll ANY magnet with 4x4 matrix $M$ by angle $\alpha$

$$\boxed{M_{ROLLED} = R(-\alpha)\, M\, R(\alpha)}$$

where

$$R = \begin{pmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{pmatrix}$$

$C \equiv \cos(\alpha) \simeq 1$

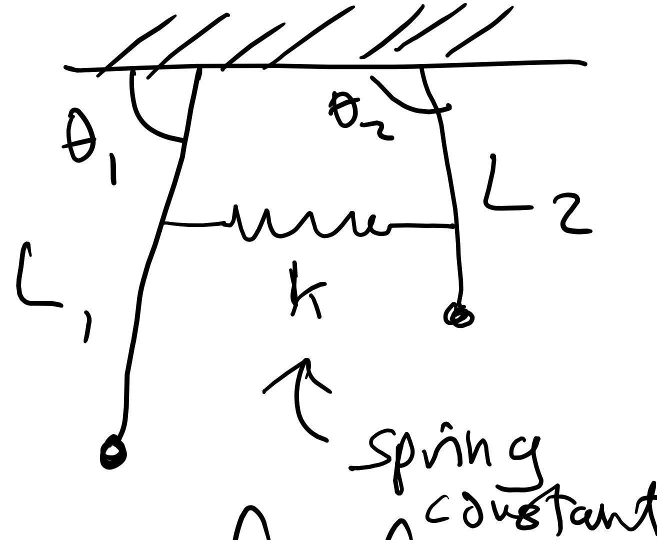
$S \equiv \sin(\alpha) \simeq \alpha$

so a Rolled Thin Quad

$$M_{RTQ} \simeq M_{TQ} + \alpha \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & -2q & 0 \\ \hline 0 & 0 & 0 & 0 \\ -2q & 0 & 0 & 0 \end{array}\right) \qquad q = \frac{1}{f}$$

Couples H to V!!

# ANALOGY: COUPLED PENDULA




With NO coupling $(k=0)$

$$f_1 \sim \frac{1}{\sqrt{L_1}}, \quad f_2 \sim \frac{1}{\sqrt{L_2}}$$

WITH COUPLING, still 2 eigenfrequencies $f_A, f_B$

$$\theta_1 = a_{1A}\cos(f_A t + \phi_{1A}) + a_{1B}\cos(f_B t + \phi_{1B})$$

$$\theta_2 = a_{2A}\cos(f_A t + \phi_{2A}) + a_{2B}\cos(f_B t + \phi_{2B})$$

Q: What happens when $L_1$ & $L_2$ are very different?

Q: What happens when $\frac{1}{\sqrt{L_1}}$ increases across (constant) $\frac{1}{\sqrt{L_2}}$?

- $f_A$ is ALWAYS $> f_B$ : association with $f_1$ & $f_2$ swaps
- There is a closest approach of $|f_A - f_B|$

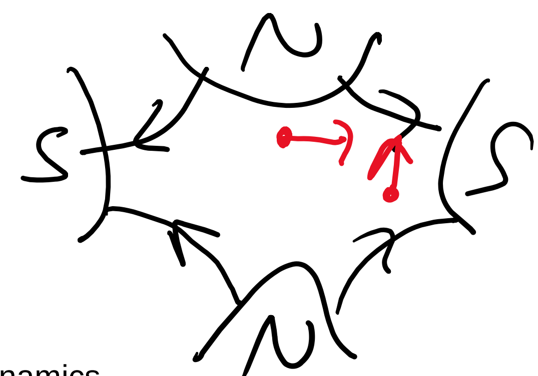
SIMILARLY, H & V accelerator motion is coupled & confused if

$$\boxed{|p + Q_x - Q_y| \lesssim Q_{MIN}}$$

DESIGN TUNES

BADLY TUNED: 0.1
WELL TUNED: .001

Correct with skew quads



## QUAD STRENGTH ERRORS : TUNE SHIFTS & BETA WAVES

A quad at $s_0$ has a strength error $\Delta q = \Delta\left(\frac{1}{f}\right)$

A) Is the motion still linearly stable? If yes...

B) How much do $Q_x$ & $Q_y$ change?

C) What happens to $\beta$-functions everywhere?

One turn matrix

$$\tilde{M} = M \begin{pmatrix} 1 & 0 \\ -\Delta q & 1 \end{pmatrix} \quad \text{(thin quad)}$$

Perturbed tune is found by solving

$$\boxed{\mathrm{Tr}(\tilde{M}) = 2\cos(2\pi \tilde{Q})}$$

$$\boxed{\cos(2\pi\tilde{Q}) = \cos(2\pi Q) - \frac{\beta\,\Delta q}{2}\sin(2\pi Q)}$$

EXACT

A) Motion is still stable if $\left|\mathrm{Tr}(\tilde{M})\right| \leq 2$

B) If $\Delta q$ is small, then

$$\boxed{\Delta Q = \tilde{Q} - Q = \frac{\beta_0\,\Delta q}{4\pi}}$$

- $\Delta q$ has different signs in H & V : also $\Delta Q_x$ & $\Delta Q_y$
- EG 1% error of $f = 10\,m$ quad $(\beta_x, \beta_y) = (50, 10)\,m$ gives $(\Delta Q_x, \Delta Q_y) = (.0040, -.0008)$
- Tune shifts are easily corrected if (as common) there are families of F & D quads on single power supplies
- $\beta$-functions are TRICKIER !!

## β- WAVES : FREE

JUST DOWNSTREAM of error quad

$$\begin{pmatrix} \tilde{\beta} \\ \tilde{\alpha} \\ \tilde{\gamma} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \Delta q & 1 & 0 \\ (\Delta q)^2 & 2\Delta q & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

launching a free wave:

$$\boxed{\frac{\Delta\beta}{\beta} = -\Delta q\, \beta_0 \sin\left(2\left(\psi - \psi_0\right)\right)} \qquad \psi > \psi_0$$

advancing at TWICE the speed of a betatron oscillation

E.G. $\Delta q = 10^{-2} \times \left(\frac{1}{10}\right)$, $\beta_0 = 50$ $\Rightarrow$ 5% error wave

Dangerous if many such random errors! Control at the $10^{-4}$ level if possible.

# $\beta$- WAVES : PBC

In a circlar accelerator with PBC

$$\boxed{\frac{\Delta\beta}{\beta} = \frac{-\Delta q\, \beta_0}{2\sin(2\pi Q)} \cos\left(2\left|\psi-\psi_0\right|-2\pi Q\right)}$$

resonance denominator

twice as fast

## RESONANCE EXAMPLE 3

Optics are especially vulnerable to small errors if

$$p \simeq 2Q_x \quad \text{or} \quad r \simeq 2Q_y$$

thats, $(q,r) = (2,0)$ or $(0,2)$

Q: WHY also include NONLINEAR correctors)