USPAS Graduate Accelerator Physics Homework 1

Due date: Tuesday January 26, 2021

1 RHIC frequency/field

The RHIC collider collides fully stripped gold ions (A=197, Z=79) at a total energy of E_{coll}=100 GeV/nucleon per beam. The circumference of each ring is 3834 m. Assume the mass of a gold ion is 197×0.93113 GeV/c^2.

(a) (5 points) Calculate the revolution frequency of a particle at the injection energy of E_{inj}=10.5 GeV/nucleon, and at the storage energy of E_{coll}=100 GeV/nucleon. What is the change in revolution frequency for particles accelerated from E_{inj} to E_{coll}?

(b) (5 points) If we assume that there are 192 identical dipoles per ring, each of length $L = 10$ m, what is the required dipole field in each at the collision energy of E_{coll}?

2 Pion decay kinematics

Consider a charged pion decaying into a muon plus an antineutrino:

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$
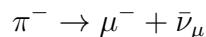
Use $m_{\pi^\pm} = 140$ MeV/c^2, $m_\mu = 106$ MeV/c^2, and $m_{\bar{\nu}} \approx 0$.

(a) (3 points) In the rest system of the pion, what are the energies and momenta of the muon and antineutrino?

(b) (3 points) Since neutrinos have now been discovered to have mass, how high must a pion beam energy be to produce some neutrinos at rest during their decays? Assume a rest mass of 0.01 eV/c^2 for muon neutrinos (and antineutrinos); you do not need to recalculate results from part (a).

(c) (4 points) For a moving pion with total energy $U_\pi = \gamma m_\pi c^2$ find an expression for the direction, θ_μ of the muon relative to the pion in the lab in terms of the angle θ_μ^* in the in the pion's rest system.

Pion at rest   Pion in lab
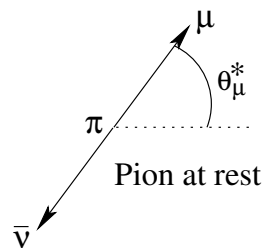
(flip the page...)
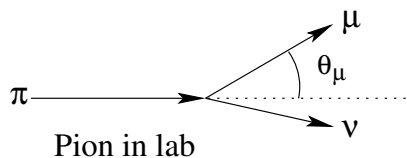
3   Fractional Relativistic Quantities

(5 points) Show that the incremental increase in energy dE is related ot the incremental increase in momentum dp through the dimensionless relationship

$$\frac{dE}{E} = \beta^2 \, \frac{dp}{p} \qquad\qquad (3.1)$$