# USPAS Graduate Accelerator Physics Homework 2

Due date: Wednesday January 27, 2021

## 1 What charge?

(5 points) Is the particle shown in Figure 2.5 positively or negatively charged? And in Figure 2.6? Explain your reasoning.

#### 2 Parametric expression for rigidity

(5 points) Derive Equation 2.13 for a singly-charged particle, from Equation 2.12. What is the magnetic rigidity of a fully stripped ion with atomic number Z and atomic weight A, when the momentum is measured in units of (GeV/u)/c?

### 3 Thick and thin quadrupole matrices

(10 points) Derive Equation 2.27, and show that it reduces to Equation 2.26 in the limit that  $L \to 0$  while KL is held constant.

### 4 Thin and thick quadrupole parameters

(10 points) You inherit a set of quadrupoles L = 0.5 m long from a defunct accelerator. The quadrupoles have a 70 mm bore radius, and a maximum pole tip field of 1.1 T.

- a) What is the minimum focal length f, for a 15 GeV/c beam?
- b) Is this quadrupole thin, meaning  $L \ll |f|$ ?
- c) How long can a quadrupole with the same cross-section extend before it becomes thick, as a function of beam energy?

## 5 Lithium lens (yes, you can do it)

(10 points) A lithium lens of length l and radius a has a constant total current I flowing through its end caps with uniform current density as pictured in Fig. 1. (So in this figure, the current is flowing from either left to right *or* right to left.) This current creates a magnetic induction  $\vec{B}(r)$  within the lithium lens.

Consider a beam of antiprotons with momentum p that are passing left to right through this lithium lens. (Yes, the antiprotons actually pass through the lithium material fairly easily.)

a) What is the magnetic induction  $\vec{B}(r)$  in the lithium lens?

- b) What is the focal length of this lens for the antiprotons? (Recall that the focal length for a focusing lens is defined as the distance at which incoming parallel rays converge on the center axis.)
- c) Does the lithium lens current need to flow from left to right or right to left for the lens to focus this antiproton beam?



Figure 1: Lithium lens diagram. A uniform total current I is applied through the end caps (that is, pointing left to right or right to left) of the lithium cylinder to create a focusing lens.