USPAS Graduate Accelerator Physics Homework 3

Due date: Thursday January 28, 2021

1 One-turn matrix properties

(5 points) Consider the one-turn matrix M given in Equation 3.16

a) Show that M is unimodular.

b) Show that the trace of M does not vary as the reference point moves around the lattice from s_1 to s_2. Hint: pre- and post-multiply M by M_{21} and M_{21}^{-1}, where M_{21} represents motion from s_1 to s_2.

2 Floquet transformation

(5 points) Prove that $M = T^{-1} R T$, where M is a 2×2 one-turn matrix, R is a rotation matrix, and T is the Floquet transformation matrix defined in Equation 3.21.

3 Light lens doublet

(5 points) Consider the net effect of axially symmetric convex and concave lenses on a light beam. If two lenses with focal lengths f and $-f$ are placed distance L apart, demonstrate that there is net focussing if $|f| > L$, no matter which way round the lenses are placed.

4 FODO cell matrix properties

(10 points) Consider the FODO cell matrix

$$M = \begin{pmatrix} 1 - 2(qL)^2 & 2L(1 + (qL)) \\ -2q(qL)(1 - qL) & 1 - 2(qL)^2 \end{pmatrix}$$

a) Demonstrate directly that M is unimodular.

b) What are the eigenvalues and eigenvectors of M? Why doesn't it matter that they are complex?

5 `madx` lab: Introduction using a toy FODO cell

(0 points)

This "question" assumes that you have installed `madx` on your computer, and walks you through some easy initial exercises. Extensive – likely even overwhelming – documentation is located at `http://madx.web.cern.ch` under the "Documentation" link in the left sidebar and shortcuts, including some "tutorials".

Download the RHIC FODO lattice `madx` file from `http://www.toddsatogata.net/2021-USPAS/lab/fodo.madx` to your working directory. Open this file in a text editor. Lines that begin with `//` or `!` are comments and *all statements must end with a semicolon.* You will see several sections, including some precursor information; beam, element, and lattice definitions; and actions. madx also lets you input commands from other files with the `call, file='filename';` statement. madx is mostly case-**in**sensitive.

The element and lattice definition sections both have some interesting features. The `K1` in the quadrupole definitions is the linear focusing (the 1 stands for first order) K value, so the focal length of each quadrupole is $f = 1/(K1 \cdot L)$. The quadrupoles are split in half so we can easily calculate parameters at their centers. The drift is also sliced up into sections so the Twiss plots show the beta function curves more clearly rather than linearly interpolating the plots between distant points. The lattice definition also uses a nice shorthand in madx for repeated occurrences of an element.

Type `madx` in the terminal command line, then at the prompt type

```
call, file=fodo.madx;
```

You can hit a carriage return at the various prompts when madx displays lattice functions. You may have to hit "q" in the plot window to move forward to the next plot. When the prompt returns at the end, type

`exit;` or `quit;` or

to exit from the program. You will also see that madx has written the files `madx.ps` (a PostScript file containing both plots), and `fodo.dat` (a text file containing a summary of parameters). The parameters Q1 and Q2 are the two phase advances of this FODO cell, in radians modulo 2π.

Open and print the `madx.ps` lattice function plots. You can fiddle with the initial conditions of $\beta_{x,y}$ in the first plot (such as moving them closer to or further from the periodic conditions found by the second `TWISS` command) and re-run madx to see how the first plot changes. Try figuring out how you adjust their slope by changing $\alpha_{x,y}$ in the first `TWISS` command from the madx documentation. The second plot uses a `TWISS` command that does not specify the Twiss parameters to track, so it automatically calculates the periodic solution *if it exists*. Remember that $\beta_{x,y} > 0$ but $\alpha_{x,y}$ can be positive or negative.

6 `madx` lab: The RHIC FODO cell

Preparation

There are design parameters for the RHIC arc FODO main arc optics in a document at the class website, `http://www.toddsatogata.net/2021-USPAS/lab/rhicdm.pdf`. These are taken from the full RHIC configuration manual located at `http://www.bnl.gov/cad/accelerator/docs/pdf/RHICConfManual.pdf`.

Download the RHIC FODO lattice madx file from `http://www.toddsatogata.net/2021-USPAS/lab/rhicfodo.madx` to your working directory. You will see several sections again, including some precursor information; beam, element, and lattice definitions; and actions.

Type `madx` in the terminal command line, then at the prompt type

```
call, file=rhicfodo.madx;
```

You can hit a carriage return at the various prompts when madx displays lattice functions. When the prompt returns at the end, type

```
exit;
```

to exit from the program. You will also see that madx has written the files `rhicfodo.ps` (containing both plots), `rhicfodo1.twi` (containing ascii twiss parameters from the FODO cell with given input conditions), and `rhicfodo2.twi` (containing raw ascii twiss parameters for the periodic FODO cell).

Open and print the `rhicfodo.ps` lattice function plots, and compare them to the RHIC design optics. (Note that the RHIC design optics plot actually plots $\beta_{x,y}^{1/2}$.) You can fiddle with the initial conditions of the beta functions in the first plot and re-run madx to see how the first plot changes. The second plot is the periodic solution found by the second `TWISS` command. Remember that $\beta_{x,y} > 0$!

Questions

a) (2 points) Why is $\beta_y(\max)$ smaller than $\beta_x(\max)$ in the matched cell? How would you change magnet strengths to make them equal?

b) (2 points) Find the phase advance per cell from numbers in the `rhicfodo2.twi` matched twiss file and compare to the design values of $2\pi\times 0.224$ horizontal and $2\pi\times 0.237$ vertical. Is this lattice overfocused or underfocused compared to design?

c) (3 points) Given the FODO lattice parameters, do the $\beta_x(\min)$, $\beta_x(\max)$, $\eta_x(\min)$, and $\eta_x(\max)$ agree with equations (3.50) and (4.18)?

d) (3 points) For IBS suppression, we have decided to change the phase advance per cell to 120 degrees in each plane. How much stronger do we need to make the quadrupoles? What happens to the periodic beta functions and dispersion? The reduced periodic dispersion is what limits the coupling between longitudinal and transverse motion and makes this an "IBS suppression" lattice.