USPAS Graduate Accelerator Physics Homework 5

Due date: Monday, February 1, 2021

1 Courant-Snyder Invariant

(5 points) Derive Equation 5.4.

2 Gaussian Distribution

(5 points) If the horizontal projection of the beam distribution is Gaussian at a point with zero dispersion $\rho(x) \sim e^{-x^2/2\sigma^2}$, what is the shape of $\rho(J)$, the action distribution?

3 FODO Cell and Acceptance

(See also question 4 in homework 2.) You inherit a set of identical quadrupoles 0.5 m long from a defunct accelerator. The quadrupoles have a 70 mm bore radius, and a maximum pole tip field of 1.1 T.

a) (5 points) You build a FODO line to transport this beam with a phase advance of 60 degrees per cell in each plane. What is the closest apart that you can place consecutive F and D quadrupoles?

b) (5 points) What are the minimum and maximum β-functions?

c) (5 points) What is the largest RMS emittance that can be accepted, if $\pm 4\sigma$ of the beam must be transmitted without scraping?

4 `madx` lab: Phase Space Tracking

Preparation

This continues the `madx` example from earlier in the week, making use of the same RHIC FODO cell that you examined in Wednesday's homework/lab.

Download the RHIC FODO lattice madx file from `http://www.toddsatogata.net/2021-USPAS/lab/rhicFodoTrack.madx` to your working madx directory. You will see several sections again, including some precursor information; beam, element, and lattice definitions; and actions. Read through the comments to familiarize yourself with the commands and structure. Note that there is a variable, nturns, defined at the beginning; those variables can be used in arithmetic expressions within madx, which is one of its benefits.

The ptc section comments also define ptc (polymorphic tracking code), and provide links to some of the madx documentation for the options for ptc commands used here. You do

NOT have to understand these options! Note that the particles are created in a while() loop that contains a simple example of madx performing math with variables.

This file performs two ptc tracking runs. One is with the periodic RHIC FODO cell you used in the last class. Here the cell is defined to start and end in the middle of a (split) focusing half-quadrupole (QFH). Recall that $\alpha = 0$ in the middle of a quadrupole so you expect the phase space ellipse to be "upright" rather than tilted.

The second ptc tracking run is preceded by another line definition of a FODO cell with the same elements, but now with the periodic boundary conditions taken at the start of a dipole as defined in the line `FODO2`. Indeed, $(\beta(s), \alpha(s), \gamma(s))$ Courant-Snyder twiss parameters are different in this case, and the phase space ellipses for horizontal and vertical motion are tilted because $\alpha \neq 0$.

Run `madx` on this input file. It should produce several output files, including a file `PTC_TRACK.ps` that contains pictures of phase space ellipses produced by ptc tracking.

Questions

a) (2 points) What are $(\beta_x, \alpha_x, \beta_y, \alpha_y)$ at the edges of the two sequences `FODO` and `FODO2`? You can either read them from the `madx` output or you can just plot them using the `PLOT` command and read the values at $s = 0$.

b) (8 points) For the `FODO2` lattice with $\alpha_{x,y} \neq 0$, compare the phase space plots to Figure 5.1 of the text. (Be careful of the orientation of the (x, x') axes!) Find the J_x values of the innermost and outermost particles. Is that consistent with the outermost having an initial position that was 9 times as large as the innermost?