USPAS Accelerator Physics 2019
(Virtually) Texas A&M University

Longitudinal Motion and Dispersion (continued)

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http://www.toddsatogata.net/2021-USPAS
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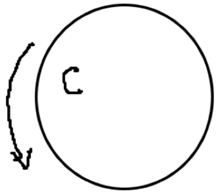
1/28/21 L5.5 Longitudinal Motion and Standard Map



HOW DO PARTICLES WITH $\delta \neq 0$ MOVE RELATIVE TO DESIGN (IN z)?

$\underline{\delta} \equiv \frac{dp}{p_0} = \frac{p-p_0}{p_0}$

DISTANCE TO TRAVEL

$$\Delta C_{path} = \delta \oint \frac{\eta(s)}{\rho(s)} ds = \delta C \left(\frac{1}{\gamma_T^2}\right)$$

$$\frac{1}{\gamma_T^2} \equiv \frac{1}{C} \oint \frac{\eta(s)}{\rho(s)} ds$$

SPEED

$$\Delta C_{SPEED} = C \frac{\Delta\beta}{\beta} = C \frac{\delta}{\gamma^2}$$

$\beta = v/c \leftarrow$ speed of light

$\gamma_T \equiv$ "gamma T", transition energy

$$\Rightarrow \Delta z = -\delta C \underbrace{\left(\frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}\right)}_{\eta_s \text{ "slip factor"}}$$

z-δ coupling M_{56}

$\eta_s > 0$ when $\gamma > \gamma_T$ "relativistic" $\leftarrow$ $v \approx c$ → careful!

< 0 when $\gamma < \gamma_T$ "nonrelativistic"

velocity dominates

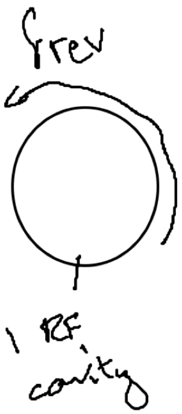
pathlength dominates

γ_T: "Transition energy"

$\gamma = \gamma_T$: Isochronous motion (like cyclotron)

② How does δ change? $(p, E, \ldots) \Rightarrow$ E fields, RF

$\underline{\text{RF cavities}}$: E electric field $\underline{\sin(\omega t)} \sim \sin\left(2\pi \frac{z}{\lambda_{RF}}\right)$



particle energy (total) $\rightarrow$ $E_{\underbrace{n+1}_{\text{turns}}} = E_{\underline{n}} + q V_{RF} \underline{\sin\left(2\pi \frac{z}{\lambda_{RF}}\right)}$

$$f_{RF} = \frac{c}{\lambda_{RF}} = h f_{rev}$$

$\uparrow$ h: $\underline{\text{HARMONIC NUMBER}}$

$\underline{\text{HW reminder}}$ $\underline{\frac{\Delta p}{p_0} = \frac{1}{\beta^2}\frac{\Delta E}{E}}$

$$\boxed{\delta_{n+1} = \delta_n + \frac{q V_{RF}}{\beta^2 E_0} \underline{\sin\left(\frac{2\pi z_n}{\lambda_{RF}}\right)}}$$

DIFFERENCE EQN FOR δ

$$\boxed{z_{n+1} = z_n - \eta_s \delta_n C}$$

DIFFERENCE EQN FOR z.

$\underline{\text{LINEARIZE}}$:

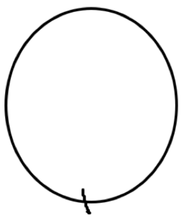
$$z_{n+1} = z_n - \eta_s \delta_n C$$

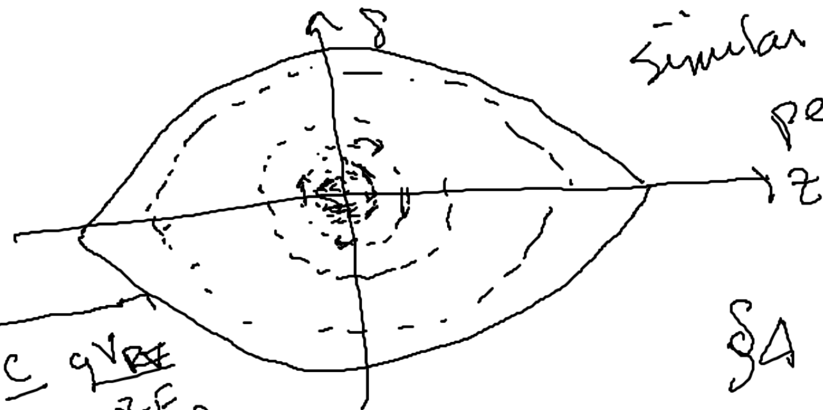
$$\delta_{n+1} = \delta_n + \frac{q V_{RF}}{\beta^2 E_0}\left(\frac{2\pi}{\lambda_{RF}}\right) \underline{\underline{z_n}}$$

$$\begin{pmatrix} z \\ \delta \end{pmatrix}_{n+1} = \begin{pmatrix} 1 & -\eta_s C \\ (\ ) & 1 \end{pmatrix} \begin{pmatrix} z \\ \delta \end{pmatrix}_n$$

③

$$\left.\begin{array}{l} z_{n+1} = a_z \sin(2\pi Q_s n) \\ \delta_{n+1} = a_\delta \cos(2\pi Q_s n) \end{array}\right\} \text{for small } z \text{ (linearized)}$$





similar to pendulum

$$Q_s = \sqrt{\frac{\eta_s}{2\pi}\,\frac{c}{\lambda_{RF}}\,\frac{qV_{RF}}{\beta^2 E_0}}$$

↑ synchrotron tune

§4.4 : standard map

HAMILTONIANS

GRAVITY PENDULUM